TEMPERATURE CHANGES ACCOMPANYING THE ADIABATIC COMPRESSION OF STEEL.

By K. T. Compton and D. B. Webster.

Introduction.

THE formulæ of Clapeyron, which apply to reversible transformations of a body whose state is a function of two independent variables, lead to the conclusion that all substances which expand when heated will experience a rise in temperature when mechanically compressed, and vice versa. It was first shown by Lord Kelvin[1] that, as a result of the second law of thermodynamics, the rise in temperature $\Delta\theta$ produced in a rod or wire by an increase ΔF in the stretching force should be expressed by the equation

$$\Delta\theta = -\frac{a\theta}{\pi r^2 \rho s J}\Delta F,$$

where r and ρ are the radius and density, respectively, of the wire, a is its thermal coefficient of expansion, s is its specific heat, θ is the absolute temperature and J is the mechanical equivalent of heat.

Joule[2] first attempted a verification of this equation by measuring the temperature changes produced by suddenly stretching or compressing various liquids, metals, wood, rubber, etc. Although in a general way Joule's observations agree with the theory, there is an average discrepancy between theory and experiment amounting to about 15 per cent.

Later Edlund,[3] experimenting with metal wires, showed that the *relative* temperature changes in different metals may be accurately predicted by Thomson's formula, but failed to prove the *absolute* accuracy of the formula. In the case of steel, for instance, the apparent temperature increase was only 63 per cent. of that predicted by the formula.

By using a greatly improved method of measuring small temperature changes in wires and by taking into account the effect of possible variations in the thermal coefficient of expansion of the wire at different tensions,[4] Haga[5] succeeded in verifying Thomson's formula within 2.54

[1] Edinb. Trans., 20, p. 283, 1883; Winkelmann, Handbuch der Physik, 2, Vol. 3, p. 637.
[2] Proc. Roy. Soc., 8, p. 353, 1857; Phil. Trans., 149, p. 91, 1859.
[3] Pogg. Ann., 126, p. 539, 1865.
[4] Dahlander, Pogg. Ann., 145, p. 147, 1872; Winkelmann, Hand. d. Phys., 2, Vol. 3, p. 60.
[5] Ann. d. Phys. u. Chem., 15, p. 1, 1882; Winkelmann, Hand. d. Phys., 2, Vol. 3, p. 637.

per cent. in the case of steel and within 0.25 per cent. in the case of german silver for changes of 21.715 and 17.134 kg. respectively in the stretching force.

Each of these investigators measured the change in the temperature of the wire by means of a thermocouple of which one junction was soldered to the stretched part of the wire and the other to an unstretched portion of the same wire. The disagreement in the results of investigations of this phenomenon are largely due to the difficulties involved in accurately measuring the small temperature changes. These changes are small, amounting to about 0.5° C. in the case of steel suddenly stretched to its elastic limit. But the quantity of heat liberated or absorbed is very small, owing to the small heat capacity of the metal wire, and this heat is so rapidly lost by surface conduction and radiation from the wire that the galvanometer, with its period of swing of several seconds, is unable to register the total initial change in temperature.

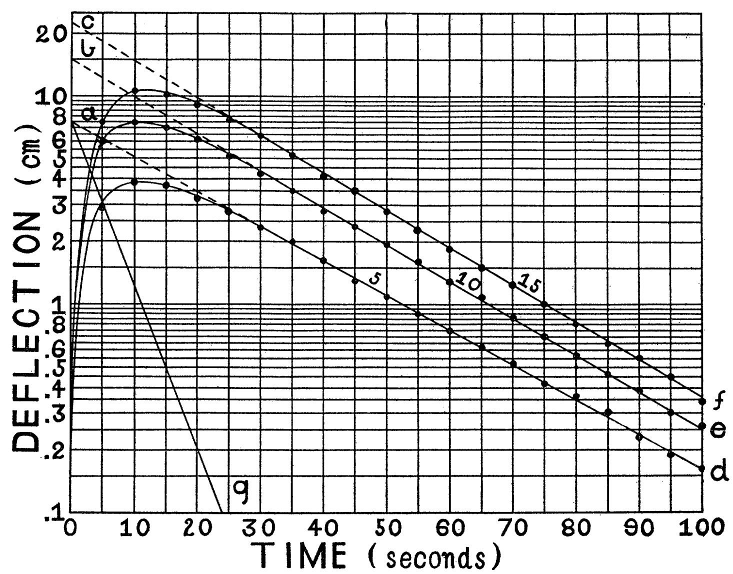

Fig. 1.

Haga employed a Thomson galvanometer whose period of swing was 6 seconds and whose damping factor was less than the critical value, so that it possessed an oscillatory swing. From the equation of motion of the galvanometer and observations of the first three swings following a change in the tension of the wire, he was able to calculate the deflection which would have been observed instantaneously after changing the tension if the inertia of the moving system of the galvanometer had been zero. To this was added an approximate correction for the deflection produced during the time (2 seconds) required to make the change in the tension. From the deflection thus calculated, the sensitiveness of the galvanometer, the resistance of the circuit and the constants of the thermocouple the initial change in the temperature of the wire could be calculated.

In the present investigation we discarded the thermocouple method of measuring the temperature change and employed a resistance method. The stretched wire formed one arm of a Wheatstone's bridge and the temperature change was calculated from the change in the resistance of the wire accompanying the removal of the stretching force. There are several features of this method which commend it as preferable to the thermocouple method. In the first place it is far more sensitive. A calculation of the current through the galvanometer when the bridge is thrown out of adjustment by a slight change in the temperature of the wire compared with the current which would be produced through the galvanometer by the same temperature change in an iron-platinum thermocouple shows that the resistance method may be made several hundred times more sensitive than the thermocouple method. In our apparatus the sensitiveness was about thirty-five times that of Haga's apparatus. In the second place the resistance method is free from possible errors due to the Peltier effect, which is a disturbing factor in all thermocouple work except that in which a balanced or potentiometer method can be employed. The Peltier effect would be of especial significance in this work, since the supply of heat near the junction is so minute. Such an error would tend to decrease the apparent temperature change, and is therefore in a direction to account for the fact that practically all the temperature changes observed by investigators have been smaller than those predicted by the theory. Finally, by using a strongly damped galvanometer, the computations were greatly simplified and instead of rather complicated calculations the results may be obtained graphically, as is shown later in the paper. The resistance method, however, can be employed only in the case of metals and then only for stretching forces so far within the elastic limit that there is no elastic lag. For this reason we chose steel piano wire as the material for investigation, and carefully tested it with a micrometer microscope to prove the absence of appreciable elastic lag within the region of the stretching forces which we employed.

Method.—One end of a steel piano wire was soldered into a heavy lug which was held rigidly in a clamp, while the other end was passed over a pulley and was attached to a constant weight of 2.5 kg. which maintained a taut horizontal length of 160 cm. of wire between the lug and the pulley. About 10 cm. from the pulley a short piece of No. 10 copper wire was soldered to the steel wire and dipped into a mercury cup. The lug and the mercury cup were the terminals of the 150 cm. length of wire which formed one arm of a Wheatstone's bridge. The balancing resistance was a calibrated standard 1.0050 ohm. The resistance of the bridge wire was 0.01032 ohm per cm., and this wire was extended by resistances of 40 and 41 ohms respectively at its ends. At room temperature, 24° C., the bridge balanced with the sliding contact at 68.0 cm., whence

$$R = 1.005 \frac{41 + 68 \times 0.01032}{40 + 32 \times 0.01032} = 1.03918 \text{ ohms}$$

is the resistance of the steel wire. The battery terminals joined the two large to the two small resistances since this arrangement gave the largest deflections for a given current through the steel wire.

The galvanometer was calibrated with respect to temperature changes of the steel wire by the following simple method. The bridge was accurately balanced with the galvanometer at zero. A standard of 0.001 ohm resistance was then introduced in series with the steel wire. This caused the galvanometer to deflect 18.72 cm. Since the current through the galvanometer is proportional to the change in the resistance of one arm, if the change is small, we may consider each centimeter deflection as indicating a change $\Delta R = 0.00005345$ ohm in the resistance of the steel wire. Thus if a small change in the resistance of the wire is produced by a temperature change $\Delta\theta$, $\Delta R = R\alpha\Delta\theta$. We carefully determined the temperature coefficient of resistance α for the wire and found $\alpha = 0.002820$ ohm per ohm per degree at 24° C. Thus

$$1 \text{ cm. deflection} \Leftrightarrow \frac{0.00005345}{1.03918 \times 0.002820} = 0.01825^\circ \text{ C.}$$

change in the temperature of the wire. There remains the problem of determining the deflection of the galvanometer, following a given change in the tension of the wire, which would occur if the galvanometer could respond *instantly* to temperature changes.

In order to reduce the rate of loss of heat from the wire it was passed axially through a polished tin tube whose ends were loosely plugged with cotton and which was covered with a thick layer of cotton to protect it from the effect of slight temperature variations in the room. This reduced the rate of loss of heat to less than half the rate when freely exposed.

After stretching the wire several times to avoid the anomalous effects of the first stretch the experiment was conducted as follows. The bridge was carefully balanced while the wire was kept taut by the constant tension of 2.5 kg. Then the galvanometer key was opened and the stretching weight added to the end of the wire. After standing several minutes the stretching weight was suddenly released by a snap and simultaneously the galvanometer circuit was closed and the deflections at five-second intervals were recorded. After the initial outward swing the galvanometer moved slowly back to zero as the wire cooled to room temperature. These readings were repeated five times each for changes of 5, 10 and 15 kg. respectively in the stretching force. The table of observations with five kg. shows the consistency of the results, especially

Time Sec.	Deflection (Cm.).						Time Sec.	Deflection (Cm.).					
	1	2	3	4	5	Ave.		1	2	3	4	5	Ave.
0	0.00	0.00	0.00	0.00	0.00	0.00	65	0.60	0.60	0.65	0.65	0.60	0.62
5	3.10	2.00	3.20	3.40	2.90	2.92	70	0.50	0.50	0.55	0.55	0.50	0.52
10	3.95	3.65	4.00	4.00	3.70	3.86	75	0.40	0.42	0.45	0.45	0.40	0.43
15	3.80	3.60	3.80	3.80	3.50	3.70	80	0.35	0.37	0.40	0.40	0.30	0.36
20	3.30	3.30	3.30	3.30	3.05	3.25	85	0.30	0.30	0.30	0.35	0.27	0.30
25	2.80	2.80	2.85	2.80	2.70	2.79	90	0.23	0.24	0.23	0.28	0.20	0.23
30	2.40	2.40	2.40	2.40	2.20	2.36	95	0.20	0.20	0.18	0.22	0.17	0.19
35	2.00	2.00	2.00	2.00	1.90	1.98	100	0.15	0.18	0.15	0.20	0.15	0.16
40	1.70	1.60	1.65	1.60	1.50	1.61	110	0.14	0.10	0.10	0.15	0.10	0.13
45	1.30	1.30	1.30	1.35	1.20	1.29	120	0.10	0.08	0.10	0.10	0.05	0.09
50	1.10	1.10	1.10	1.10	1.00	1.08	130	0.05	0.00	0.05	0.05	0.00	0.03
55	0.90	0.90	0.90	0.90	0.85	0.89	140	0.00	0.00	0.00	0.05	0.00	0.01
60	0.75	0.75	0.75	0.80	0.70	0.75	150	0.00	0.00	0.00	0.00	0.00	0.00

after about ten seconds, when any effect of not pressing the galvanometer key exactly with the release of the weight becomes negligible. The logarithms of these averages, and also those for 10 and 15 kg., are plotted with the time in the accompanying figure.

The first deflections depend upon the moment of inertia and damping factor of the galvanometer as well as upon the rise in temperature of the wire. But after about thirty seconds the deflection at any instant accurately records the temperature of the wire at that instant, as is shown by the following analysis. Thus the curves after thirty seconds represent the cooling curves of the wire following these three changes in tension, and the points a, b, c at which these curves, extended, intersect the axis $t = 0$ give the deflections which would be recorded if the galvanometer could reach a steady deflection before the wire loses part of the heat developed.

The validity of this statement is proved by the solution of the equation of motion of the galvanometer,

$$I\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Mx = ifM,$$

where x is the deflection, I is the moment of inertia and f the sensitiveness of the galvanometer, D is the moment of damping at unit rate of deflection, M is the moment of restoring force due to the suspension and i is the current through the galvanometer. The current at any instant t is given in terms of the initial rise in temperature θ_0, the resistance R of the wire and its temperature coefficient of resistance α and a constant k depending on the other resistances and the electromotive force in the bridge by

$$i = kR\alpha\theta_0 e^{-\frac{\sigma}{ms}t},$$

where σ is the thermal coefficient of surface conductivity of the wire and ms is its heat capacity per unit length. Putting

$$\frac{D}{I} = A, \qquad \frac{\sigma}{ms} = c,$$

$$\frac{M}{I} = B, \qquad \frac{2}{A - \sqrt{A^2 - 4B}} = T_1,$$

$$\frac{fMkR\alpha}{I} = C, \qquad \frac{2}{A + \sqrt{A^2 - 4B}} = T_2,$$

the equation reduces to

$$\frac{d^2x}{dt^2} + A\frac{dx}{dt} + Bx = C\theta_0 e^{-ct}, \tag{1}$$

of which the solution is

$$x = \frac{C}{C^2 - 2AC + 4B}\theta_0 e^{-ct} + c'e^{-\frac{t}{T_1}} + c''e^{-\frac{t}{T_2}},$$

where c' and c'' are the constants of integration.

The time constants T_1 and T_2 may be determined from the case where the right member of equation (1) is zero. To do this a portion of the bridge wire was short-circuited so that the galvanometer maintained a steady deflection at 7.5 cm. The short-circuiting key was suddenly removed and the readings noted at short intervals as the galvanometer returned to zero. The results are shown in the figure by the curve *ag* whose equation is

$$x = c_1 e^{-\frac{t}{T_1}} + c_2 e^{-\frac{t}{T_2}}.$$

T_1 is evidently too small to be detected (showing that the damping moment is very large compared with the restoring moment due to the suspension). It is certainly less than 0.5 second so that this component of the curve may be neglected. T_2 is the time taken for x to fall to $\frac{7.5}{e}$, and is about 5.5 seconds. It is seen from the curve that the effect of this term is certainly negligible after 30 seconds.

Beyond thirty seconds, therefore, the deflection is given by

$$x = \frac{C}{C^2 - 2AC + 4B}\theta_0 e^{-ct} = x_0 e^{-ct}.$$

Thus the curves beyond 30 seconds represent the true cooling curves and the points a, b, c, at which these straight lines intersect the axis $t = 0$, are the true values of the initial deflections which would have been observed if the galvanometer had responded instantly to the initial currents.

The most probable straight line through the points beyond 30 seconds was determined in each of the three cases by the method of least squares and the probable error calculated. The three initial deflections a, b and c were thus found to be 7.5 ± 0.037 cm., 15.0 ± 0.047 cm. and 22.5 ± 0.056 cm. respectively. When these deflections are multiplied by the calibration constant 0.01825 we find that temperature changes of 0.1369° C., 0.2737° C. and 0.4106° C. were produced in the wire by changes of 5, 10 and 15 kg. respectively in the stretching force.

Comparison of Experiment with Theory.—In order to take into account possible variations in the thermal coefficient of linear expansion of the wire due to tension, we measured the coefficient of expansion between temperatures of 13.6° C. and 32.2° C. under tensions of 4.5, 7.0 and 9.5 kg., which were the average tensions in the three cases. The wire was passed axially through a cylindrical water jacket and the expansion was measured by micrometer microscopes. These results, together with the other constants of the wire are given in the following table:

Radius of wire	$r = 0.0310$ cm.
Density of wire	$\rho = 7.930$
Coefficient of linear expansion (4.5 kg.)	$a = 0.00001109$
(7.0 kg.)	$a = 0.00001111$
(9.5 kg.)	$a = 0.00001115$
Specific heat of wire	$s = 0.1178$
Room temperature	$\theta = 297.0°$ K.
Mechanical equivalent of heat	$J = 4.185\ (10)^7$
Acceleration of gravity	$g = 980.6$

The values of $\Delta\theta$ calculated from Thomson's formula by substitution of these quantities are given in the following table and compared with the experimental results. The theory is verified much more closely and consistently than heretofore. If the mean of the experimental

F(kg.).	$\Delta\theta$ °C. (Calculated).	$\Delta\theta$ °C. (Observed).	Discrepancy, Per Cent.	Probable Error, Per Cent.
− 5.0	0.1366	0.1369	−0.22	±0.50
−10.0	0.2738	0.2737	+0.04	±0.32
−15.0	0.4122	0.4106	+0.39	±0.25
			Ave. = 0.07	

results is used to calculate the mechanical equivalent of heat we find $J = 4.188\ (10)^7$ ergs per calorie. The best results for steel previously obtained (by Haga) lead to the value $J = 4.290\ (10)^7$ ergs per calorie, in which the error is about thirty-five times that in the present work.

PHYSICAL LABORATORY,
REED COLLEGE.