The Photoelectric Effect of the Deuteron

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(Received April 18, 1936)

Theoretical cross sections for the dissociation of the deuteron by absorption of γ-rays, the Chadwick-Goldhaber effect, have been calculated, by using a square well law of potential, both of the ordinary type and of the Majorana type. The curves of cross section as a function of energy for various assumed widths are given. For widths less than 2×10^{-13} cm they are quite similar in shape for either type of interaction. For greater widths the ordinary potential shows a fairly sharp peak at 4.7 MEV (for a width of 4×10^{-13} cm) whereas the Majorana shows for the same width a much flatter maximum at 6.2 MEV. It is pointed out that suitable measurements of relative cross sections would give a means of telling which type of interaction is obeyed and give an approximate figure for the range of interaction.

THE problem of the nature of the interaction between proton-neutron and proton-proton is basic to all theoretical work in nuclear physics and therefore is deserving of detailed consideration from every point of view. There seem to be at present three possible ways of getting interaction laws: (1) Study of angular distribution and magnitude of scattering, such as the scattering of protons and neutrons in hydrogen, (2) interpretation of mass defects of atoms, and (3) the study of the Chadwick-Goldhaber effect[1] or photodissociation of nuclei. Already there is quite an extensive literature devoted to the problems presented by the first two methods. In this paper we examine the situation with regard to the information about proton-neutron interaction that is obtainable from studies of the cross section for the photodissociation of deuterons by γ-rays of more than 2.2 MEV energy.

The deuteron is the simplest of nuclei heavier than H^1 and occupies among them a position analogous to that of the hydrogen atom in the theory of atomic spectra. One may expect that a study of its properties will be essential for the development of views about the structure of nuclei and that the relative simplicity of the mathematical concepts involved in its treatment will make the information derivable from its study more definite than that obtainable from studies of heavier nuclei. We have thought it of interest, therefore, to calculate the theoretical cross section for photodissociation in some detail so as to make it possible to plan experiments which can throw light on the type of interaction involved as well as on the spatial extension of the forces between neutrons and protons.

Calculations of the cross section of the deuteron for this process have already been made by Bethe and Peierls[2] for interaction forces of extremely short range. Independently, Massey and Mohr[3] have made some calculations for finite ranges of interaction, using both the square potential well and a law of force of exponential type. They found the cross section to be substantially the same for the two types of force law and they also found that an increase in the range of force brings about an increase in the expected cross section. Their calculations are the most complete of those that have been published so far but they are not complete enough inasmuch as only the Wigner types of forces are used and the variation of the cross section with radius is not discussed in much detail. Later, Mamasachlisof[4] published an extension to the Bethe-Peierls calculations which takes into account the nuclear radius to the first order but unfortunately the first term of his formula is too large by a factor 2 and the second by a factor 4. The first-order correction is correctly given by Hall[5] whose results are in agreement with those of Massey and Mohr as well as those presented in this paper.

This first-order correction is entirely due to the effect of the range of interaction on the shape of the S wave function of the bound state. The discussions of this state by the Wigner and Majorana interactions are identical and it is thus impossible

[1] Chadwick and Goldhaber, Nature **134**, 237 (1934); Proc. Roy. Soc. **A151**, 479 (1935).

[2] Bethe and Peierls, Proc. Roy Soc. **A148**, 146 (1935).

[3] Massey and Mohr, Proc. Roy. Soc. **A148**, 206 (1935); Nature **133**, 211 (1934).

[4] Mamasachlisof, Physik. Zeits. Sowjetunion **8**, 206 (1935).

[5] Hall, Phys. Rev. **49**, 401 (1936).

to distinguish between these types of interaction by considerations of the first-order effect. Higher order effects involve also changes in the p state with range and for this state the use of Majorana's operator brings about effectively a repulsion as contrasted with the attraction which would be used in Wigner's treatment. It will be seen as the result of the calculations given below that the higher order effects are large for sufficient energies of the γ-rays and that it should be possible to make use of them in distinguishing between the two types of interaction.

Interpretation of observed cross sections in terms of the force-law is complicated by the fact that dissociative transitions caused by magnetic dipole interaction of the γ-rays with the deuteron are of importance comparable with the more familiar electric dipole interaction. The importance of the magnetic dipole contribution for the inverse effect—capture of neutrons by protons with emission of γ-rays—has already been emphasized by Fermi[6] who also gives a formula for the magnetic dipole cross section of the photodissociation of deuterons by γ-rays. The magnetic dipole effect has also been investigated by Bethe, Peierls, Teller and Wigner in a paper which, unfortunately, is not being published. The magnitude of the magnetic dipole cross section may be one-half or one-fourth the electric dipole effect, according to whether the ^1S level of the deuteron (which interpretation of scattering cross sections places within 50 kv of the dissociation limit) is unstable or stable.

Now the experimental value for the cross section obtained by Chadwick and Goldhaber is actually smaller than that given by theory for the electric dipole effect alone, using extremely short range neutron-proton forces. So inclusion of the magnetic dipole effect only makes matters worse. The situation is not that of a definite contradiction between theory and experiment since the experimental measure of the proton energy is apparently uncertain by 80 kv in 240 kv. If the proton energy is really only 160 kv instead of 240 kv, this would correspond to a reduction by a factor $\frac{2}{3}$ of the corresponding theoretical electric dipole cross section.

When sources of much harder γ-rays become available, however, these difficulties become less important. The magnetic dipole effect drops off rapidly with increasing energy so that for 6-MEV γ-rays it contributes only about 3 percent of the cross section. For such γ-rays the energy of the protons would be about 2 MEV and so could be accurately determined by measuring their range if the γ-ray energy were not already known from other work. In the hope of stimulating the interest of experimentalists in this problem we have therefore worked out the theoretical cross sections in their dependence on energy and the assumed force-law in some detail.

The calculation is arranged below so as to have explicit formulas for the most general type of interaction laws. Closed expressions for contributions to the matrix elements due to regions outside the interaction region will be given. These are often the main parts of the matrix elements. The formulas will then be specialized for the case of a "square well." The notation used is as follows.

σ = collision cross section for the incident photon;
M = mass of proton;
E = sum of kinetic energies of proton and neutron after dissociation;
ϵ = absolute value of binding energy of the deuteron,
V = potential energy;
D = constant depth of potential hole if it is "square";
a = nuclear radius = distance between proton neutron beyond which their mutual potential energy vanishes;
$h\nu$ = energy of photon;
v = relative velocity of proton and neutron after dissociation;
$\gamma = h\nu/\epsilon$; $\alpha = (M\epsilon)^{\frac{1}{2}}/\hbar$; $\beta_2 = (ME)^{\frac{1}{2}}/\hbar$.

For "square" hole:
$\beta = M^{\frac{1}{2}}(D-\epsilon)^{\frac{1}{2}}/\hbar$; $\beta_1 = M^{\frac{1}{2}}(D+E)^{\frac{1}{2}}/\hbar$;
r = distance between proton and neutron; $z = \beta r$, $z_1 = \beta_1 r$; $z_2 = \beta_2 r$;
u = regular solution of radial wave equation for s terms:

$$\frac{d^2u}{dr^2}+\frac{2}{r}\frac{du}{dr}+(M/\hbar^2)(E-V)u=0;$$

N = normalization factor for s terms defined by

$$4\pi N^2\int_0^\infty r^2u^2dr=1;$$

$F = \sin z_2/z_2 - \cos z_2$; $G = \cos z_2/z_2 + \sin z_2$;
$\bar{F}/r$ = regular solution of radial wave equation for p terms normalized so as to make $\bar{F}$ asymptotic to a sine wave of unit amplitude at infinity;
F_i/r = any (not normalized) regular solution of the radial equation for p terms in $0<r<a$;
$F' = dF/dz_2$, $F_i' = dF_i/dz_2$, etc.; unless otherwise specified the $'$ stands always for d/dz_2;
h = Planck's constant; $\hbar$ = Dirac's constant.

[6] Fermi, Phys. Rev. 48, 570 (1935).

For any shape of "hole" one finds by standard methods making use of Einstein's absorption probability expressed in terms of matrix elements:

$$\sigma=(16\pi^3/3)(e^2/\hbar c)(\nu/v)N^2\left|\int_0^\infty r^2 u\bar{F}dr\right|^2. \quad (1)$$

This formula should hold for γ-rays having a wave-length large in comparison with the dimensions of the deuteron provided it is correct to represent the effect of the γ-ray by a term $-e\mathbf{E}\cdot\mathbf{r}$ in the Hamiltonian. Here $\mathbf{E}$ is the electric vector of the γ-ray and $\mathbf{r}$ is the displacement of the proton. In fact such a term gives the above formula in Schrödinger's treatment of absorption. Its use is correct and logical if the interaction between neutrons and protons arises from an ordinary potential. For Majorana forces one may formally also use such a term. The complete consequences of doing so do not appear to have been investigated from a theoretical point of view. One may give arguments in favor of using such a procedure. Thus it gives correct results for cases in which the binding between the proton and neutron is weak and the use of the same form of the interaction energy for cases of finite coupling between protons and neutrons appears to be the simplest mathematical procedure. For finite binding the effect of a constant field is very reasonably represented by the same interaction term. It should be noted, however, that the use of this or any other type of interaction with radiation is speculative and that the usual connection of quantum theory with the classical is established with more difficulty for forces of the Majorana type than usually. Thus a wave packet formed out of Majorana wave functions will describe a condition in which the proton and the neutron parts of the wave packet change places at a rate determined by the binding between the proton and neutron. Under such conditions the classical analogy breaks down and one does not have a real justification for the use of standard interaction energies. According to an observation of Feenberg's kindly communicated by him to us the f sum rule of Thomas and Kuhn does not hold for systems with Majorana's forces because the classical relations $\dot{x}=p_x/m$ are violated in such systems. This fact throws additional doubt on the use of $-e\mathbf{E}\cdot\mathbf{r}$ because the interaction of photons having energies high in comparison with the binding is as a consequence definitely nonclassical. There is thus no point at which a close connection with classical theory can be established in the same sense as for forces of the ordinary potential type.

It has been suggested by Massey and Mohr following a related discussion of Taylor and Mott that dipole radiation should disappear altogether for Majorana systems. This is a too stringent point of view since for loosely bound systems one may discuss conditions satisfactorily by usual means.

The use of an interaction energy $-e\mathbf{E}\cdot\mathbf{r}$ is equivalent for long wave-lengths to using $-(e/c)\mathbf{A}\cdot\dot{\mathbf{r}}$ where $\mathbf{A}$ is the vector potential. The latter form of the interaction energy is correct whenever $e\dot{\mathbf{r}}$ can be identified with the electric current. For systems obeying ordinary interaction laws such an identification is very reasonable. For systems governed by exchange forces it is not so immediate because the operators $e\dot{\mathbf{r}}$ represent only the part of the current due to the motion of the separate particles. It is conceivable that the exchange of particles also contributes to the electric current in a manner not directly describable by following the motion of the charged particle. A complete understanding of this question presumably requires a better insight into the nature of heavy particles and their interactions than that which we have at present. Being unable to see the situation more fully we use Eq. (1) even though one cannot be absolutely sure of its validity. For a "hole" of any shape and for either the Wigner or Majorana type of interaction

$$\bar{F}=N_pF_i(r)/F_i(a) \qquad (r<a), \quad (2a)$$

$$\bar{F}=N_p[F_a'G-G_a'F+(F_i'/F_i)_a(FG_a-F_aG)] \qquad (r>a), \quad (2b)$$

where N_p for p states is given by

$$N_p=[1-z_2^{-2}+z_2^{-4}+2z_2^{-3}(F_i'/F_i)+(1+z_2^{-2})(F_i'/F_i)^2]_a^{-\frac{1}{2}}. \quad (2c)$$

The correctness of (2a), (2b) to within the common factor N_p is verified most easily using the relation $F'G-FG'=1$. The form of the factor N_p has been specialized to p states and to a force free condition in $r>a$. In the above formulas the suffix a indicates that the quantity is evaluated at $r=a$. As a rule the most important part of the integral in (1) is from a to ∞. A general form of this integral is obtained with (2b) and is given by

$$\int_a^\infty r^2u\bar{F}dr=N_p\beta_2^{-2}(1+\alpha^2\beta_2^{-2})^{-2}u(a)\{2z_2^{-2}+\alpha^2\beta_2^{-2}-1+2\alpha\beta_2^{-1}z_2^{-1}+\alpha\beta_2^{-1}(1+\alpha^2\beta_2^{-2})z_2 +(F_i'/F_i)[z_2+2z_2^{-1}+\alpha^2\beta_2^{-2}z_2+2\alpha\beta_2^{-1}]\}_a. \quad (3)$$

To this one must add the integral from 0 to a which depends on the shape and size of the hole and which can usually be estimated for low energies and radii with sufficient accuracy without precise calculation.

For a square hole the s state is described by

$$ru=\sin\beta r \quad (r<a); \qquad ru=\sin\beta a\cdot e^{-\alpha(r-a)} \quad (r>a). \quad (4)$$

The boundary conditions at $r=a$ give

$$z\cot z=-\alpha a. \quad (5)$$

The normalizing factor N on using (4) and (5) can be put into the form

$$2\pi(1+a\alpha)N^2=\alpha. \quad (6)$$

For ordinary interactions one has as a result of calculation making use of Eq. (5):

$$\int_0^a r^2\bar{u}\bar{F}dr=N_p\sin z\left\{\frac{2\beta_1^2}{(\beta_1^2-\beta^2)^2}-\frac{\alpha a}{\beta_1^2-\beta^2}-\frac{z_1\sin z_1}{F_i}\cdot\frac{2(1+\alpha a)+a^2(\beta_1^2-\beta^2)}{a^2(\beta_1^2-\beta^2)^2}\right\} \quad (7)$$

all quantities on the right side being taken for $r=a$. Adding this to the integral given by (2) the expressions simplify on using

$$\beta_1^2-\beta^2=\alpha^2+\beta_2^2. \quad (8)$$

By substituting the value of N_p and by using in it the expressions for F_i appropriate to the square hole it is found that

$$\sigma=\sigma_{BP}\frac{[(\alpha a)^2+(\beta a)^2](\beta a)^2}{(1+\alpha a)[((z_1\sin z_1)/F_i-z_2^2)^2+z_2^2((z_1\sin z_1)/F_i)^2]_a}, \quad (9a)$$

where

$$\sigma_{BP}=(8\pi/3)(e^2/\hbar c)\alpha^{-2}\gamma^{-3}(\gamma-1)^{\frac{3}{2}} \quad (9b)$$

is the value of σ for $a=0$ obtained by Bethe and Peierls. In applying (9a) it is convenient to use

$$z_1^2=(\beta a)^2+\gamma(\alpha a)^2; \quad z_2^2=(\alpha a)^2(\gamma-1); \quad F_i=\sin z_1/z_1-\cos z_1. \quad (9c)$$

For $a=0$ both βa and z_1 approach $\pi/2$ and σ approaches σ_{BP} as it should. Eq. (5) determines $z=\beta a$ for a given αa most easily graphically by plotting $z\cot z$ or else by an expansion.[7] Eqs. (9c) give z_1, z_2, and F_i.[8] Numerical calculations can be checked by using the sum rule

$$\int_0^\infty \sigma(\nu)d(h\nu)=\pi e^2h/2Mc. \quad (10)$$

For the Majorana potential there is effectively repulsion in the p state which tends to push the p wave function out of the region $r<a$. The kinetic energy E may be either smaller or greater than the potential energy in this region. Only the first case is considered here in detail because it has the greater practical interest. For this case the quantity

[7] Wigner, Zeits. f. Physik **83**, 253 (1933).

[8] The function F_i is tabulated in Yost, Wheeler and Breit, J. Terr. Magn. and Atmos. Elec. **40**, 443 (1935). See Table I_1 and Table I_0 for $z\cot z$.

$$\alpha_1 = M^{\frac{1}{2}}(D-E)^{\frac{1}{2}}/\hbar \tag{11}$$

is real. It is convenient to use the abbreviation:

$$Q = \alpha_1 a \sinh \alpha_1 a[(\sinh \alpha_1 a)/\alpha_1 a - \cosh \alpha_1 a]^{-1}. \tag{12}$$

It is then found that:

$$\sigma/\sigma_{BP} = \tfrac{1}{4}\gamma^4(\gamma-1)^{-2}\alpha^4(1+\alpha a)^{-1}\left|\int_0^\infty r^2 u\bar{F}dr\right|^2, \tag{13}$$

$$\alpha^2\int_a^\infty r^2u\bar{F}dr = N_p \sin \beta a\cdot\gamma^{-2}\{-2(\gamma-1)+\alpha a\gamma - Q[\gamma+2(1+\alpha a)(\alpha a)^{-2}]\}, \tag{13'}$$

$$\alpha^2\int_0^a r^2u\bar{F}dr = N_p \sin \beta a(2D/\epsilon-\gamma)^{-2}\{(2D/\epsilon)(\alpha a-1)+2(\gamma-1)-\gamma\alpha a+Q[2(1+\alpha a)(\alpha a)^{-2} +\gamma-2D/\epsilon]\}, \tag{13''}$$

$$N_p^2 = z_2^4[(Q+z_2^2)^2+z_2^2Q^2]^{-1},$$

which give on substitution into (1)

$$\sigma/\sigma_{BP} = \tfrac{1}{4}\sin^2 \beta a(1+\alpha a)^{-1}(1-\gamma\epsilon/2D)^{-4}[(Q+z_2^2)^2+z_2^2Q^2]^{-1}\cdot\{(\gamma\epsilon/2D)[(\alpha a)^2(3\gamma-4-\gamma\alpha a) +Q(\gamma\alpha^2a^2+4(1+\alpha a))]+(\alpha a)^2[-2(\gamma-1)+\alpha a\gamma]-Q[\gamma(\alpha a)^2+2(1+\alpha a)]\}^2. \tag{14}$$

Another form is

$$\sigma/\sigma_{BP} = \tfrac{1}{4}(\alpha a)^4 \sin^2 \beta a(1+\alpha a)^{-1}(A+B)^2/C, \tag{15}$$

where

$$A = \gamma^2(2D/\epsilon-\gamma)^{-1}\{\alpha a - 2\alpha_1^2(\beta^2+\alpha_1^2)^{-1} - Q[1-2(1+\alpha a)(\alpha a)^{-2}(2D/\epsilon-\gamma)^{-1}]\};$$

$$B = 2(1+\alpha a)(\alpha a)^{-2}+2-\gamma+\gamma\alpha a+P\beta_2 a[2(1+\alpha a)(\alpha a)^{-2}+\gamma];$$

$$C = 1-z_2^2+z_2^4+2Pz_2+(1+z_2^2)P^2z_2^2;$$

$$P = F_i'/F_i.$$

Here A is proportional to the contribution to the integral from 0 to a, while B is proportional to the contribution from a to ∞. Numerical calculations can be made either by means of (13) combined with (13′) and (13″) or else by (14) or by (15). The forms (13) and (15) keep track of the contributions to the integral due to $r<a$ and due to $r>a$.

As the formulas are too complicated for one to be able to recognize their properties by inspection we have calculated the cross section for the range of values likely to be of interest experimentally, namely, for γ-ray energies from 2.2 MEV to 11 MEV and for the widths $a=1$, 2, 3 and 4×10^{-13} cm. The results are shown in Figs. 1 to 5. All calculations for these graphs were made with the same value of α (0.231×10^{13}) and

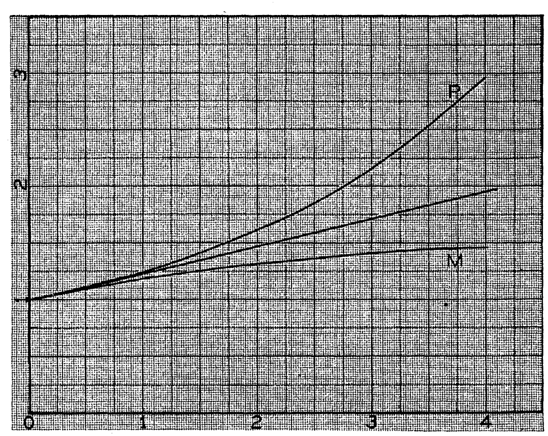

FIG. 1. Dependence of cross section on range and type of interaction law at the photoelectric threshold, $E=0$. Ratio σ/σ_{BP} is plotted against a in 10^{-13} cm. P refers to ordinary, M to Majorana, interaction.

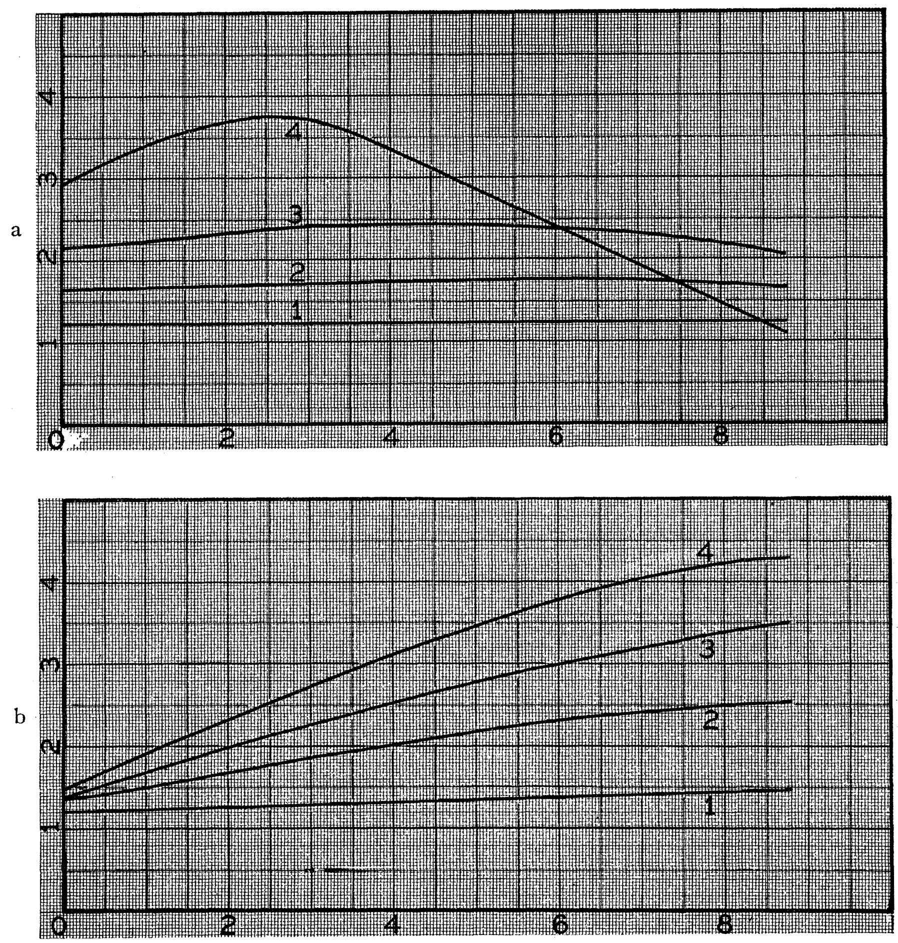

FIG. 2. Dependence of cross section on range and type of interaction law for energies up to $E=8.8$ MEV. Ratio σ/σ_{BP} is plotted against E in MEV. Curves are labeled by value of a in 10^{-13} cm. Fig. 2a is for ordinary, 2b for Majorana, interaction.

of ϵ (2.2 MEV). It is easy to change these graphs to other values of ϵ or universal constants by remembering that σ/σ_{BP} is a function only of γ and αa since αa determines βa and D/ϵ. Thus if ϵ is changed to ϵ' each graph gives the right value of σ/σ_{BP} for a radius $a'=a(\epsilon/\epsilon')^{\frac{1}{2}}$ and the γ-ray energy for each point is $h\nu'=(\epsilon'/\epsilon)h\nu$.

Fig. 1 shows the effect of the range of the interaction force on the cross section in the limit of energies near the "photoelectric threshold," $E=0$. The curves are the ratio σ/σ_{BP} plotted against a, at $E=0$, the one marked P being for ordinary potential, that marked M for the Majorana law. Both are tangent to the line, $1+\alpha a$, at small values of αa. This straight line corresponds to the values given by Hall.

Figs. 2a and 2b are graphs of σ/σ_{BP} as a function of E in MEV for various values of a, Fig. 2a referring to ordinary and Fig. 2b to Majorana potential. These bring out clearly that the two kinds of interaction behave quite differently if the range of interaction exceeds 2×10^{-13} cm but for narrow ranges of interaction the results are pretty much the same.

Figs. 3a and 3b are graphs of the electric dipole cross section, σ, as a function of E in MEV for various values of a, Fig. 3a referring to ordinary and Fig. 3b to Majorana interaction. For wide ranges of interaction the curves are quite different for the two forms.

Perhaps the experimental possibilities are best brought out by considering what information

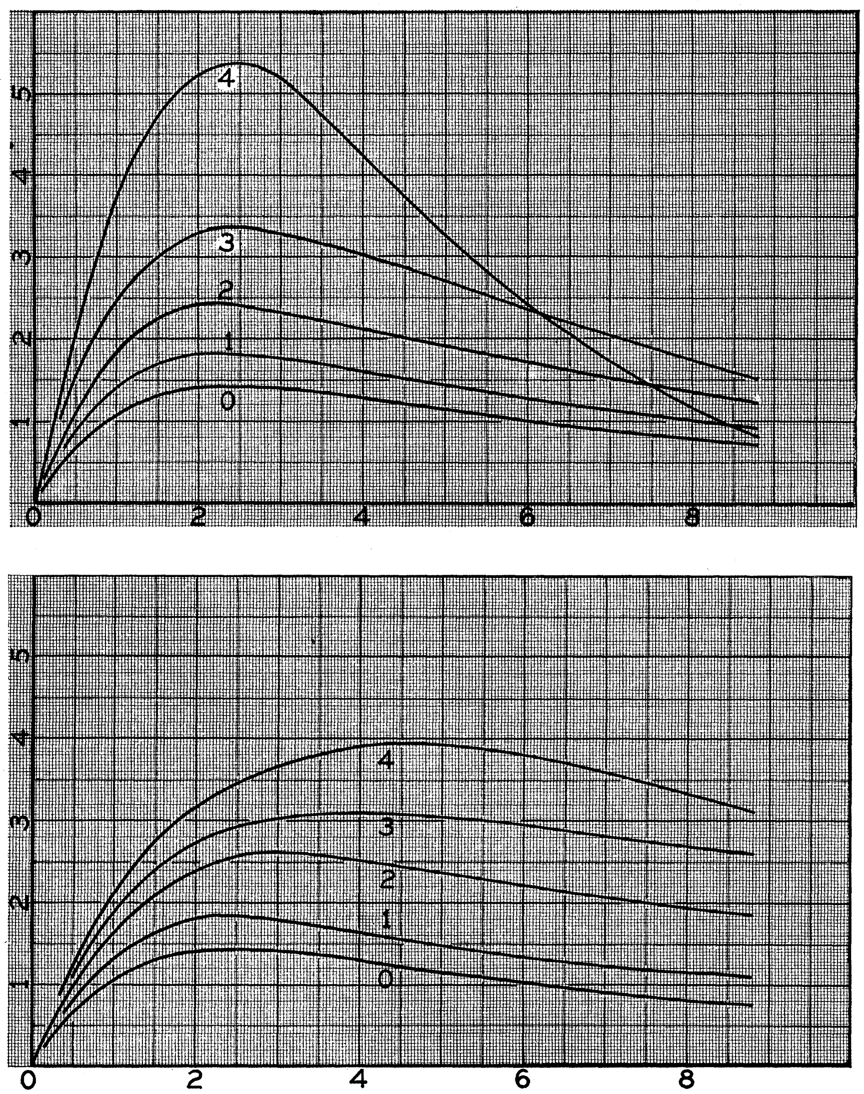

FIG. 3. Dependence of cross section on range and type of interaction for energies up to $E=8.8$ MEV. Absolute electric dipole cross section is plotted with 10^{-27} cm^2 as unit, against E in MEV. Curves are labeled by value of a in 10^{-13} cm. Fig. 3a is for ordinary, 3b for Majorana, interaction.

could be obtained by a measurement of the relative cross section for γ-rays of quite different amounts. In Fig. 4 we have plotted the ratio[9] of the cross section for 8.8-MEV γ-rays to that for 4.4-MEV γ-rays, against the range of interaction, a. Clearly if the experimental value of the ratio came out definitely greater than 0.65 one could conclude that the interaction is of Majorana type and could get an estimate of a. If the experimental value were close to 0.65 the conclusion would not be so definite but again if it were definitely lower one could conclude in favor of the ordinary potential and know that the range was between 3 and 4×10^{-13} cm.

Finally, Fig. 5 shows the ratio of the cross section for magnetic dipole to that for electric dipole in the limit $a=0$ calculated from a formula

[9] Of course similar results will be obtained for any two γ-ray energies of this general order; these values were picked simply for convenience, not because of any special properties.

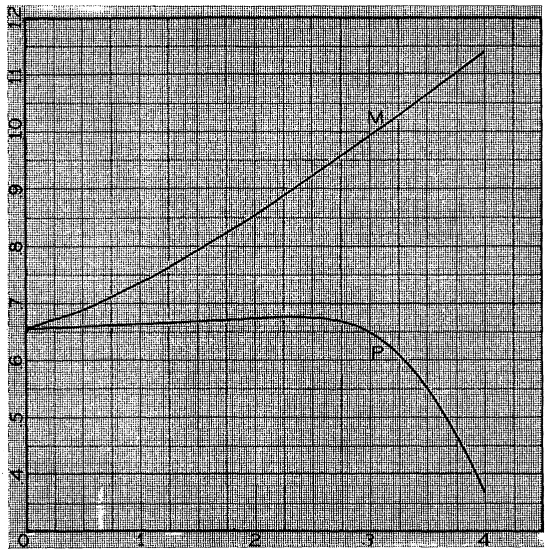

FIG. 4. Showing dependence on range and type of interaction of ratio of cross section at $h\nu = 8.8$ MEV to that at $h\nu = 4.4$ MEV. Ratio is plotted against a in 10^{-13} cm. Curve P is for ordinary, M for Majorana, interaction.

given in the unpublished paper of Bethe, Peierls, Teller and Wigner. The formula is

$$\sigma_m/\sigma_{BP} = \frac{(g_p - g_n)^2}{4Mc^2/\epsilon}\gamma^2(\gamma-1)^{-1}\frac{[1\pm(\epsilon'/\epsilon)^{\frac{1}{2}}]^2}{\epsilon'/\epsilon+\gamma-1},$$

where ϵ' is the magnitude of the energy of the deuteron in its lowest ^1S level. The positive sign of $(\epsilon'/\epsilon)^{\frac{1}{2}}$ is to be used for the case in which the ^1S level is unstable, the negative sign in the case of stability. Fig. 5 is constructed for the case $\epsilon' = 46$ kev and $g_p - g_n = 5$. The positive and negative signs of the square root correspond respectively to curves marked $+\epsilon'$ and $-\epsilon'$. The relative importance of the magnetic effect diminishes rapidly with γ so there is a considerable advantage in working with 4.4-MEV γ-rays instead of the 2.6 MEV γ-rays. For 10 MEV the γ-ray wave-length is 120×10^{-13} cm, so even a nuclear size of 4×10^{-13} cm, a probable upper limit, is only 1/30 of the γ-ray wave-length and corrections for retardation are probably not

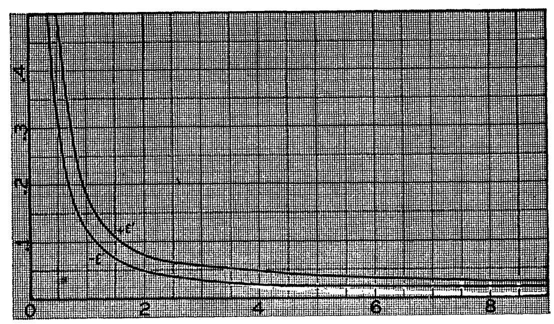

FIG. 5. Ratio of magnetic dipole cross section to electric dipole cross section at $a=0$, plotted against E in MEV. Curve $+\epsilon'$ refers to unstable, $-\epsilon'$ to stable, level for ^1S level of deuteron at 46 kev from dissociation limit.

important here.[10] Potentials with long ranges can be made to influence p states even for low energies. By means of them it would be possible to decrease the theoretically expected cross sections.

We are indebted to Mr. F. L. Yost and Mr. L. Eisenbud for checking some of the arithmetical calculations.

[10] In discussing retardation effects for such problems it is convenient to use a somewhat more general method than is customary for atomic spectra. The interaction energy between light and matter is proportional to $\mathbf{A}\cdot\dot{\mathbf{r}}$. The vector potential $\mathbf{A}$ contains in it the factor e^{ikz} where it has been supposed that the light wave propagates in the z direction. This factor can be expanded

$$e^{ikz} = \sum_0^\infty i^n(2n+1)P_n(\cos\theta)(\pi/2kr)^{\frac{1}{2}}J_{n+\frac{1}{2}}(kr),$$

where the P_n are Legendre functions of order n and the $J_{n+\frac{1}{2}}$ are Bessel functions of order $n+\frac{1}{2}$. Each term in the sum when multiplied by $\dot{x}$ or $\dot{y}$ gives rise to a linear combination of two spherical harmonics of order $n\pm1$. The first term of the sum gives rise only to a spherical harmonic of order 1. It can cause only transitions between states obeying the selection rule $\Delta L = \pm1$ in accordance with the triangle rule for Gaunt's integrals of products of three spherical harmonics. The values of the matrix elements are modified by the presence of $J_{\frac{1}{2}}(kr)$ in the integrand of the matrix elements rather than the first term of its power series expansion. The ratio of the second term to the first is $-k^2r^2/6 = (2\pi^2/3)(r^2/\lambda^2)$, where λ is the wave-length of the γ-ray. For $r/\lambda = 1/30$ this quantity is $\sim -1/150$ and the correction for retardation to the dipole effect due to this cause is still of little importance. The only other term in the sum for e^{ikz} which need be considered for an $s-p$ transition is that corresponding to $n=2$ because $n\pm1$ cannot be equal to 1 for any other term. The ratio of this term to the main one is found to be, on performing the angular integrations, $k^2r^2/15$ to the first order of k^2r^2 and it may also be neglected.