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Selection Rules for the β-Disintegration

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§1. The selection rules for β-transformations are stated on the basis of the neutrino theory outlined by Fermi. If it is assumed that the spins of the heavy particles have a direct effect on the disintegration these rules are modified. §2. It is shown that whereas the original selection rules of Fermi lead to difficulties if one tries to assign spins to the members of the thorium family the modified selection rules are in agreement with the available experimental evidence.

§1.

ACCORDING to the theory of β-disintegration given by Fermi[1] no change of the total nuclear spin should occur in the most probable transformations, i.e., in transformations located on the first Sargent curve.[2] The transformations corresponding to the second Sargent curve approximately 100 times less probable should correspond to changes ± 1 or 0 of the angular momentum of the nucleus. One may expect the existence of still lower curves for higher changes in the nuclear spin. This selection principle is based on the assumption that the spin of the heavy particles does not enter in the part of the Hamiltonian which is responsible for the β-disintegration. The same assumption was made in the modified theory of Konopinski and Uhlenbeck[3] who introduced the derivative of the neutrino wave function in the Hamiltonian in order to get a better fit with the experimental curves of the energy distribution in β-spectra. We should like to note here that this selection rule will be changed if the spins of the heavy particles are introduced into the Hamiltonian, a possibility proposed in many discussions about this subject.

We shall first give the derivation of Fermi's selection rule in a somewhat generalized form. The probability of β-disintegration is proportional to the square of the matrix element.

$$M_1=\sum_l \int (\Omega_l^{N,\,P}\psi_i)\psi_f^*\delta q_l\{0(\psi_\nu^*\psi_\epsilon^*)\}. \quad (1)$$

Here ψ_i and ψ_f are the proper functions of the heavy particles, protons and neutrons, for the initial and final state, respectively. These functions depend on the positions of the heavy particles, on their spins, and on a third variable[4] which corresponds to the charge of the heavy particles and which is capable of two values, in a manner similar to the spin variable, the value 1 corresponding to a proton and the value 0 to a neutron. The operator $\Omega_l^{N,\,P}$ acts on this last variable converting the lth particle in ψ_i into a proton if it was a neutron and giving $\Omega_l^{N,\,P}\psi_i=0$ if the lth particles is already a proton. The integration in (1) includes summation over the spin and charge coordinates of the heavy par

[1] Fermi, Zeits. f. Physik **88**, 161 (1934).
[2] Sargent, Proc. Roy. Soc. **A139**, 659 (1933).
[3] Konopinski and Uhlenbeck, Phys. Rev. **48**, 7 (1935).

[4] Introduced by Heisenberg, Zeits. f. Physik **77**, 1 (1932).

ticles. ψ_ν and ψ_ϵ are the proper functions of the neutrino and the electron. O is an operator acting on these functions but not involving the heavy particles and the delta function δq_l substitutes the position coordinate q_l of the lth heavy particle for the coordinates of the electron and neutrino.

In Fermi's paper the operator O was simply a summation over certain products of the four Dirac components of the electron wave function and components of the neutrino wave function. The Konopinski and Uhlenbeck operator involved in addition the first derivative of the neutrino wave function. In both cases, however $\delta q_l\{O(\psi_\nu{}^*\psi_\epsilon{}^*)\}$ is a scalar function of q_l. This is necessary since in $(\Omega_l{}^{N,\,P}\psi_i)\psi_f{}^*$ the summation over the spins of the heavy particles gives also a scalar and the integral in (1) must be a scalar.

Supposing at first that ψ_ν and ψ_ϵ are plane waves, the same will be true for $\delta q_l\{O(\psi_\nu{}^*\psi_\epsilon{}^*)\}$. If we expand this wave in spherical harmonics, and suppose that the nuclear radius r_0 is small compared to the wave-length λ, then the amplitudes of the zeroth, first, second . . . spherical harmonics within the nucleus will have the ratio $1 : (r_0/\lambda) : (r_0/\lambda)^2 \cdots$. Neglecting all but the zero-order spherical harmonic M_1 will be different from zero only if the angular momentum i of the nucleus does not change during the β-transformation and if the nuclear proper function is even with regard to reflection on the mass center before and after the disintegration or if it is odd before and after. These transitions will correspond to the first Sargent curve.

Taking into account the first-order spherical harmonic in the development of $\delta q_l\{O(\psi_\nu{}^*\psi_\epsilon{}^*)\}$ further transitions become possible. The selection rules for these additional transitions are those valid for a polar vector: The change in angular momentum Δi is ± 1 or 0 (but not $i=0\rightarrow i=0$) and one of the two combining states is even, the other odd. For all these cases, however, the matrix element M_1 will be smaller by r_0/λ than for the zero-order spherical harmonic and consequently the transitions will be less probable by $(r_0/\lambda)^2$. Now for most β-disintegrations r_0/λ is about 10^{-2} and the transformations arising from the first-order harmonic are ten thousand times less probable than those arising from the zero-order harmonic.

Actually the proper function of the electron is not a plane wave because of the Coulomb interaction between the nucleus and the electron. Fermi has shown that for the heavy elements, where this interaction is the greatest, the result will be to increase the probability of emitting an electron with unit angular momentum, this event being only about 100 times less probable than the emission of the light particles with zero angular momentum, thus giving the second Sargent curve. The situation will be similar if we accept the Hamilton term introduced by Konopinski and Uhlenbeck or any other expression of the type given in the matrix element M_1.

We have therefore from a generalized treatment of Fermi's theory the following selection rules,

First Sargent curve: (1) $\Delta i=0$; (2) proper functions, even-even, or odd-odd.
Second Sargent curve: (1) $\Delta i=0$ or ± 1; (2) proper functions, even-odd.

If we now assume that the spin of the proton and neutron enters into the Hamilton term which is responsible for the transformation we may substitute M_1 by the more complicated expression

$$M_2=\sum_\xi\sum_l\int(\Omega_l{}^{N,\,P}\alpha_l{}^\xi\psi_i)\psi_f{}^*\delta q_l\{O^\xi(\psi_\nu{}^*\psi_\epsilon{}^*)\}. \quad (2)$$

Hereby $\alpha_l{}^\xi$ operates on the spin of the lth heavy particle and signifies the three Pauli matrices:[5]

$$\alpha^x=\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix};\quad \alpha^y=\begin{vmatrix}0 & i\\ -i & 0\end{vmatrix};\quad \alpha^z=\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}. \quad (3)$$

The summation over ξ is meant to include the three values x, y, z. The three operators O^ξ are the three components of a vector. This means that by a coordinate transformation the three operators are transformed in the same way as the x, y and z components of a vector. It is then seen that the matrix element M_2 will again be scalar quantity.

[5] A similar expression was introduced by Fermi (in order to insure relativistic invariance) as an additional term. In his expression, however, the α's stood for the Dirac matrices which give only a small contribution as long as the velocity of the heavy particles are small compared to c. It should also be noted that Dirac's α's are the components of a polar vector whereas Pauli's α's form an axial vector.

Expanding the expressions $\delta q_l\{O^\xi(\psi_\nu{}^*\psi_\epsilon{}^*)\}$ into spherical harmonics and retaining only the zero-order functions integration over the coordinates of the heavy particles shows that transitions are possible if (1) $\Delta i=0$ or ± 1 (but not $i=0\rightarrow i=0$), and if (2) The transition is of an odd-odd or even-even type. These are the same selection rules as those valid for an axial vector. The corresponding transitions would be located on the first Sargent curve.

From the first-order spherical harmonics we obtain transitions with (1) $\Delta i=0$; ± 1 or ± 2, (2) the transitions are of the odd-even type.

Either the matrix element M_1 or the matrix element M_2 or finally a linear combination of M_1 and M_2 will have to be used to calculate the probabilities of the β-disintegrations. If the third possibility is the correct one, and the two coefficients in the linear combination have the same order of magnitude, then all transitions which would lie on the first Sargent curve according to any one of the two sets of selection rules mentioned above would now lie on the first curve. This would mean that the selection rules are the same as for an axial vector with the addition that also the $i=0\rightarrow i=0$ even $\rightarrow$even or odd$\rightarrow$odd transitions are allowed.

We shall show now that if exchange forces of the Majorana type[6] are acting between protons and neutrons and if these forces have to be explained by a β-disintegration of the neutron and a following capture of the electron and neutrino by the proton then the actual matrix element to be used is the sum of the matrix elements M_1 and M_2. Indeed if we should have only M_1 then the charges would be exchanged with the spins remaining unaffected, i.e., we should obtain Heisenberg forces. If on the other hand M_2 were the correct expression then considering a system of one neutron and one proton represented by $\psi_N(q_1\uparrow)\psi_P(q_2\uparrow)$[7] and applying first to $\psi_N(q_1\uparrow)$ the operator corresponding to M_2 and then the inverse operator to $\psi_P(q_2\uparrow)$, the expression $\psi_P(q_1\uparrow)\psi_N(q_2\uparrow)$ would be obtained. By a similar procedure $\psi_N(q_1\uparrow)\psi_P(q_2\downarrow)$ is transformed into $2\psi_P(q_1\downarrow)\psi_N(q_2\uparrow)-\psi_P(q_1\uparrow)\psi_N(q_2\downarrow)$. Now $\psi_N(q_1\uparrow)\psi_P(q_2\uparrow)$ is according to both Majorana and Heisenberg in exchange interaction with $\psi_P(q_1\uparrow)\psi_N(q_2\uparrow)$ whereas $\psi_N(q_1\uparrow)\psi_P(q_2\downarrow)$ exchanges with $\psi_P(q_1\uparrow)\psi_N(q_2\downarrow)$ according to Heisenberg and with $\psi_P(q_1\downarrow)\psi_N(q_2\uparrow)$ according to Majorana. The matrix element M_2 will correspond therefore to a superposition of the Majorana and the Heisenberg forces in the ratio 2 to -1. If we want to obtain pure Majorana forces we must add M_1 and M_2 with equal coefficients.

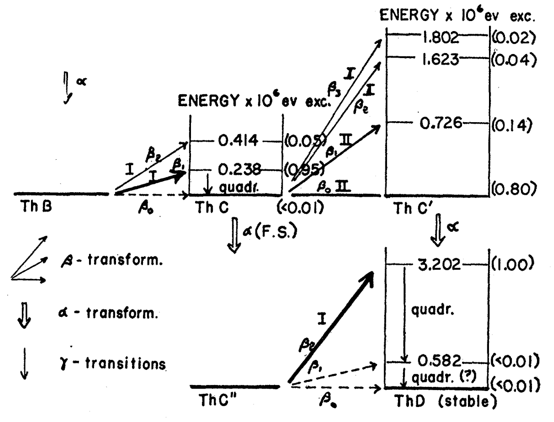

FIG. 1. Schematic representation of the radioactive α- and β-disintegrations from Th B to Th D, indicating various β-transformations leading to excited states of product-nuclei.

§2.

We can now show that the new selection rules help us to remove the difficulties which appeared in the discussion of nuclear spins of radioactive elements[8] by using the original selection rule of Fermi.

We shall discuss the sequence of transformations in the thorium family leading from Th B to Th D (thorium lead) which is represented schematically in Fig. 1. First of all we can conclude with a rather high degree of certainty that the normal states of Th B, Th C′ and Th D nuclei, possessing *even* atomic numbers and *even* mass numbers, have the spin $i=0$.[9] The transformation Th B$\rightarrow$Th C gives rise to a continuous β-spectrum with the observed upper limit

[6] Majorana, Zeits. f. Physik **82**, 137 (1933).

[7] The arrows in $\psi_N(q_1\uparrow)$ and $\psi_P(q_2\uparrow)$ represent the spins of the neutron and proton.

[8] Gamow, Proc. Roy. Soc. **A146**, 217 (1934); Physik. Zeits. **35**, 533 (1934).

[9] For four elements of this type (${}_2He^4$; ${}_6C^{12}$; ${}_8O^{16}$; ${}_{16}S^{32}$) the absence of spin is directly shown by the band spectra; other 13 investigated elements of this type (${}_{48}Cd^{110}$; ${}_{48}Cd^{112}$; ${}_{48}Cd^{114}$; ${}_{56}Ba^{136}$; ${}_{56}Ba^{138}$; ${}_{80}Hg^{200}$; ${}_{80}Hg^{202}$; ${}_{80}Hg^{204}$; ${}_{82}Pb^{204}$; ${}_{82}Pb^{206}$; ${}_{82}Pb^{208}=$Th D) do not show any hyperfine structure which makes it very probable that their spin is also zero.

$E_\beta = 0.362$ MV and is accompanied by a very strong γ-line $h\nu = 0.238$ MV along with several much weaker lines. The number of secondary electrons (due to internal conversion of the γ-line 0.238 MV in the K level) is according to Ellis and Mott[10] about $N_\beta = 0.25$ per disintegration from which these authors conclude that it must correspond to a quadripole radiation. In fact the internal conversion coefficients for this frequency are, according to calculations of Mott and Taylor,[11] $\alpha_d = 0.026$ and $\alpha_q = 0.205$ for dipole and quadripole radiation, respectively. Thus for the total number of γ-quanta radiated by nuclei $N_\gamma = N_\beta/\alpha$ we should have according to these two possibilities 9.6 or 1.2. Since this number should not be larger than unity we must exclude the possibility of dipole radiation and consider the γ-line in question as due to quadripole transition with the intensity almost one quantum per disintegration. The fact that the observed value is 20 percent larger than unity must be due to errors in the measurements of N_β or the calculation of α. Accordingly we admit with Ellis and Mott that in this case we have 100 percent excitation of the quantum level 0.238 MV of the Th C nucleus. The total energy of the transformation is $0.362 + 0.238 = 0.600$ MV and the observed upper limit of the β-energies corresponds to the transformation Th $\mathrm{B}_{\mathrm{norm}} \rightarrow$ Th $\mathrm{C}_{\mathrm{exc}}$.

The β-transformation from Th C to Th C′ corresponds to the upper limit of the β-spectrum $E_\beta = 2.25$ MV and is accompanied with only very weak γ-radiation. Thus we conclude that in this case the main transformation, 80 percent, takes place between normal states Th $\mathrm{C}_{\mathrm{norm}} \rightarrow$ Th $\mathrm{C}'_{\mathrm{norm}}$.

Finally in the β-transformation between Th C″ and Th D the level 3.202 MV of Th D nucleus is (according to Ellis and Mott) excited to almost 100 percent, the transition to the normal state occurring by emission of two γ-lines 0.582 MV and 2.620 MV both with the absolute intensities of the order unity. Thus the observed upper limit of the β-spectrum $E_\beta = 1.79$ MV corresponds to the transformation Th $\mathrm{C}''_{\mathrm{norm}} \rightarrow$ Th $\mathrm{D}_{\mathrm{exc}}$. The total energy of transformation being $1.79 + 3.202 = 4.99$ MV.

In Fig. 2 the logarithms of the partial decay constants of different subgroups of β-spectra are plotted against the logarithms of the corresponding upper energy limits.[12]

The curves I and II correspond to Sargent's permitted and nonpermitted transformations as estimated from different members of three radioactive families. We see that the main transformation Th $\mathrm{B}_{\mathrm{norm}} \rightarrow$ Th $\mathrm{C}_{\mathrm{exc}}$ corresponds to the curve I whereas the main transformation Th $\mathrm{C}_{\mathrm{norm}} \rightarrow$ Th $\mathrm{C}'_{\mathrm{norm}}$ corresponds to the curve II. From the original Fermi selection rule: Curve I: $\Delta i = 0$; Curve II: $\Delta i = 0$ or ± 1. We conclude that $i(\text{Th C}_{\mathrm{exc}}) = i(\text{Th B}_{\mathrm{norm}}) = 0$. Since the γ-line represents a quadripole transition we have further $i(\text{Th C}_{\mathrm{norm}}) = 2$. In this case the transformation Th $\mathrm{C}_{\mathrm{norm}} \rightarrow$ Th $\mathrm{C}'_{\mathrm{norm}}$ should correspond to $\Delta i = 2$, i.e., must belong to the third Sargent curve, which is in contradiction with experimental evidence, this transformation belonging to the curve II.

The difficulty will be still not removed if we take the possibility into account that γ-line 0.238 mv corresponds to a magnetic dipole radiation. For this case the coefficients for internal conversion have been calculated by Fisk and Taylor[13] and are considerably larger than the corresponding coefficients for electric radiation. Accepting this possibility we obtain for the number of γ-quanta $h\nu = 0.238$ mv a value small compared with unity (weak excitation) and

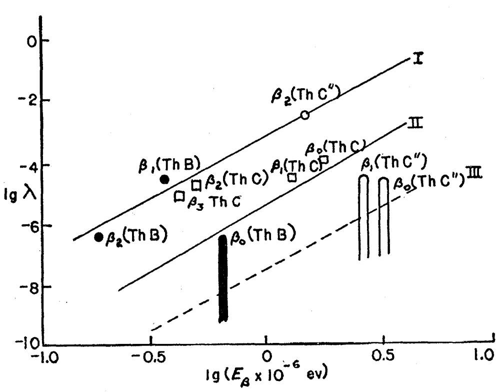

FIG. 2. Logarithmic plot of the relation between partial decay constants and corresponding upper energy limits for various components of complex β-ray spectra.

[10] Ellis and Mott, Proc. Roy. Soc. **A139**, 369 (1933).

[11] Mott and Taylor, Proc. Roy. Soc. **A138**, 665 (1932).

[12] Upper energy limits of different β-subgroups are obtained by subtracting the excitation-energies from the total energy of transformation; partial decay constants are estimated from the total decay constant and relative excitation of different nuclear levels.

[13] Fisk and Taylor, Proc. Roy. Soc. **A146**, 178 (1934).

should be forced to accept that the observed upper limit of β-spectrum corresponds to transformation Th $B_{norm} \to$ Th C_{norm}. This will lead again to contradiction with Fermi's original selection rule first because the transformation Th $B_{norm} \to$ Th C_{norm} and Th $C_{norm} \to$ Th C'_{norm} corresponding to the same spin-change (because $i(\text{Th } B_{norm}) = i(\text{Th } C'_{norm}) = 0$) would belong to different Sargent curves and secondly because in this case both transformations Th $B_{norm} \to$ Th C_{norm} and Th $B_{norm} \to$ Th C_{exc} being of the first Sargent's class would lead to the conclusion $i(\text{Th } C_{norm}) = i(\text{Th } C_{exc}) = i(\text{Th } B_{norm}) = 0$ which would exclude the possibility of any γ-transition.

Applying our modified selection rule, curve I $\Delta i = 0$ or ± 1 curve II $\Delta i = 0$, ± 1 or ± 2 we have the following possibilities

$$\begin{array}{r|c|c|c|c|} \text{nucleus:} & \text{Th } B_{norm} & \text{Th } C_{exc} & \text{Th } C_{norm} & \text{Th } C'_{norm} \\ \text{spin:} & 0 & 0 \text{ or } \pm 1 & 0, \pm 1 \text{ or } \pm 2 & 0 \end{array}. \quad (4)$$

The possibility $i(\text{Th } C_{norm}) = 0$ or ± 1 must, however, be excluded as in this case the transformation Th $B_{norm} \to$ Th C_{norm} would correspond to the curve I and consequently, because of larger energy, be more probable than Th $B_{norm} \to$ Th C_{exc}. There remains only the possibility $i(\text{Th } C_{norm}) = 2$ which is in good agreement with the quadripole-character of the γ-transition. The transformation Th $B_{norm} \to$ Th C_{norm} corresponding to $\Delta i = 2$ cannot belong now to the first Sargent curve, i.e., it must be at least 100 times weaker than the main transition, as can be seen from Fig. 2. This accounts for the fact that the corresponding "long range" component of the continuous β-spectra of Th B has never been observed. *Thus we see that the new selection principle removes the difficulty originated in the case of the older rule.*

It must be pointed out, however, that according to the above considerations it is not possible to assign even proper functions to all nuclei with even atomic number and even mass number. Because if the proper function of Th B_{norm} is even, the same is true for Th C_{exc} (since the transition Th $B_{norm} \to$ Th C_{exc} lies on the first curve) and also for Th C_{norm} (since Th $C_{exc} \to$ Th C_{norm} is a quadripole transition). But Th $C_{norm} \to$ Th C'_{norm} lies on the second curve and therefore the proper function of Th C'_{norm} is odd. This is unsatisfactory since it would be nice to substitute the rule that nuclei with even atomic number and even mass number have $i = 0$ by the rule that the proper functions of these nuclei remain unchanged during *any* symmetry operation.

Turning our attention to the β-transformation leading from Th C″ to Th D we see that the main transformation Th $C''_{norm} \to$ Th D_{exc} corresponds to the first Sargent curve from which we conclude that $i(\text{Th } C''_{norm}) - i(\text{Th } D_{exc}) = 0$ or ± 1. It can also be seen from Fig. 2 that the transformation Th $C''_{norm} \to$ Th D_{norm} belongs at least to third, or still higher order, curve which excludes the possibilities of $i(\text{Th } C''_{norm})$ being 0, or ± 1. The excited level 3.202 mv of Th D nucleus is connected with the normal level by two γ-transitions, 0.582 mv and 2.620 mv, from which the second is surely quadripole. This indicates that its spin will not be larger than 4, because by each γ-transition $\Delta i \leqslant 2$. This gives for $i(\text{Th } C''_{norm})$ the upper limit ≤ 5. Thus for the spin of normal state of Th C″ nucleus we have the choice between 2, 3, 4 and 5; it seems however, to be necessary to accept the largest possible value $i(\text{Th } C''_{norm}) = 5$ in order to have a sufficiently large spin difference between Th $C_{norm}(i = 2)$ and Th C''_{norm} to explain the presence of strong fine structure of α-rays in the Th C $\to$ Th C″ transformation.[14]

[14] Gamow and Rosenblum, Comptes rendus **197**, 1620 (1933).