Energy Transmission by High Energy Electrons

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In considering the theory of energy transfer at close collisions between charged particles, one of which is of high energy, J. R. Oppenheimer has postulated the condition that in calculating the effect of the field of the high energy particle, one must omit from its appropriate Fourier analysis all of those harmonic terms whose frequency lies above a certain critical value characteristic of the charged particle acted upon. Some time ago, the writer proposed a modification of the fundamental wave-mechanical equation which was designed so as to lead statistically to the classical equation of motion of an electron including the so-called radiation reaction terms. It is now shown that the Oppenheimer condition is a direct analytical consequence of the modified wave-mechanical equation referred to. The significant feature of the matter is that the terms which in Oppenheimer's postulate are simply omitted are, on the writer's theory, rendered automatically impotent by having associated with them large denominators which kill their effects.

IN a recent publication, J. R. Oppenheimer,[1] in seeking harmonization with the experimental facts concerning the absorption of cosmic-ray energy has been led to make a certain postulate. This postulate is to the effect that in considering the action of a high energy charged particle upon an electron of an atomic system, we must analyze the field of the high energy particle into harmonic waves according to the Fourier principle and then discard those frequencies which lie above a certain value ν_m whose order of magnitude is given by

$$2\pi\nu_m = v/\rho = 3m_0c^2v/2e^2, \tag{1}$$

where ρ is the radius of the electron as given by classical dynamics, and is given in terms of the rest mass m_0 by the expression $\rho = 2e^2/3m_0c^2$. The velocity of the high energy particle is v, and is approximately equal to c, the velocity of light.

The purpose of the present paper is to show that the Oppenheimer postulate or its equivalent, is a direct consequence of the principles embodied in a generalization of wave mechanics which I published in 1933.[2]

The procedure adopted in my former publication is as follows:[3] According to the classical theory, with the neglect of so called radiation terms the acceleration $\ddot{s}$ of a particle of rest mass m_0 and charged e, is related to the electric field E the magnetic field H and the velocity of light c by the equation

$$m_0\ddot{s}/e = E + [\dot{s}H]/c. \tag{2}$$

It was recalled that, according to the theory of P. Ehrenfest and of E. H. Kennard, this equation could be represented wave mechanically in the sense that the centroid of the wave packet representing the moving charged particle moves according to (2), $\ddot{s}$ being of course the acceleration in question. Remembering that the complete classical equation of motion of an electron is of the form

$$m_0(\ddot{s} - a_1\dddot{s} + a_2\ddddot{s},\ \text{etc.})/e = E + [\dot{s}H]/c, \tag{3}$$

the attempt was made to modify the wave-mechanical equation for ψ so as to cause it to lead to an equation of this type by principles analagous to those which through Kennard's theorem caused the ordinary wave-mechanical equation to lead to (2).

Writing (3) in the form

$$m_0P\ddot{s}/e = E + [\dot{s}H]/c, \tag{4}$$

where P is the operator defined by

$$P = 1 - a_1D + a_2D^2,\ \text{etc.}, \tag{5}$$

D being written for d/dt, it was shown that the desired end could be secured by *replacing the scalar and vector potentials φ and $\bar{U}$ of the external field in the ordinary wave-mechanical equation by $P^{-1}\varphi$ and $P^{-1}\bar{U}$.*

[1] Oppenheimer, Phys. Rev. **47**, 44 (1935).
[2] W. F. G. Swann, Phys. Rev. **44**, 943 (1933): also W. F. G. Swann, J. Frank. Inst. **217**, 59 (1934).
[3] For the present purpose I omit certain refinements contained in the original papers, and having to do with the magnetic field and with relativistic invariance of the equations. These matters are of only secondary importance in the present discussion.

The particle whose equation of motion is under discussion is of course the electron whose ejection from the atom is being examined. Its velocity is small in contrast to that of the high energy electron responsible for the scalar and vector potentials φ, and $\bar{U}$. Thus, it is possible to replace d/dt by $\partial/\partial t$ in P; and, in what follows, we shall regard D as meaning $\partial/\partial t$.

Now following the foregoing principles and taking, for simplicity, but not of necessity, as illustration, the case where we confine our attention to a classical Eq. (3) which has no more than two terms on the left-hand side, our corresponding operator P is $(1-a_1D)$. The wave-mechanical principle applied according to the Kennard method to produce (2) must now be modified by replacing φ and $\bar{U}$ by φ_1 and $\bar{U}$, where

$$\varphi_1=(1-a_1D)^{-1}\varphi, \quad \bar{U}_1=(1-a_1D)^{-1}.\bar{U} \qquad (6)$$

Following Oppenheimer, we expand the field, or the field potentials, in the form of a Fourier integral.[4] Thus, for example,

$$\varphi=\int A_\nu e^{2\pi i\nu t}d\nu,$$

where it is to be understood that the real parts are to be taken. While (6) with a corresponding expression for $\bar{U}$ gives the expansion of the potentials, the effective potentials required by our theory are φ_1 and $\bar{U}_1$.

Thus,
$$\varphi_1=\int \frac{A_\nu e^{2\pi i\nu t}}{1-a_1D}d\nu.$$

Now any function of D operating on $e^{2\pi i\nu t}$ may be replaced by the same function of $2\pi i\nu$ operating on $e^{2\pi i\nu t}$. Hence,

$$\varphi_1=\text{Real part of} \int \frac{A_\nu e^{2\pi i\nu t}}{1-2\pi a_1\nu i}d\nu,$$

i.e.,
$$\varphi_1=\int \frac{A_\nu \cos(2\pi\nu t+\theta_\nu)d\nu}{(1+4\pi^2a_1{}^2\nu^2)^{\frac{1}{2}}}, \qquad (7)$$

where $\tan\theta_\nu=2\pi a_1\nu$.

When $4\pi^2a_1{}^2\nu^2$ becomes appreciably larger than unity the value given by (7) for the contribution of the corresponding frequency to φ_1 becomes small compared with the contribution of that frequency to φ. In other words, the frequencies which in Oppenheimer's theory are discarded, are here rendered *impotent by the natural consequences of the theory*. The theory thus provides a basis for the Oppenheimer conditions. Without such a basis the most that can be said concerning frequencies greater than the critical value is that the contributions which the ordinary theory would cause them to make to the story are probably wrong. This, alone, does not provide for the actual contributions being negligible, however.

On the classical theory, the quantity a_1 occurring in (3) and in subsequent equations would be equal to ρ/c, where ρ is the classical radius of the electron. Thus, the limiting frequency ν_m is given by

$$2\pi\nu_m=c/\rho=3m_0c^3/2e^2. \qquad (8)$$

Thus, since $v=c$ for high energy particles, we see by comparison with (1) that (7) not only reproduces the qualitative significance of the Oppenheimer conditions but provides for them quantitatively. It must be admitted, however, that it would be straining matters too much to hold to the magnitude of a_1 predicted by the classical theory, particularly as there is no direct experimental verification of the magnitude of a_1 as there is for the mass of the electron. The significant thing is that there is *some* value which is appropriate for this coefficient.

In conclusion, I wish to thank Dr. A. Bramley for calling the Oppenheimer conditions to my attention.

[4] We only need expression of the integral in its dependence on the time; for, as the high energy particle passes by the electron to be ejected, the latter experiences, in view of the small velocity to which it is limited, variations in φ which are concerned only with the time variations of that function. The complete expression of the field involves a coefficient A_ν' of the form given by Oppenheimer,

$$A_\nu'=(2\pi^2ie\nu/2\epsilon v^2)H_1{}^{(1)}(2\pi\nu pi/\epsilon v)e^{-2\pi i\nu z/v}$$

where v is the velocity of the high energy particle, $\epsilon=(1-v^2/c^2)^{-1/2}$, where the high energy particle moves parallel to the axis of z, and the origin is at the electron to be ejected. p is the perpendicular distance from this electron to the path of the high energy particle, and $H_1{}^{(1)}$ is the Hankel function of the first kind.