just $\int|\varphi_2|^2 d\tau$ or unity. The integral in k of (16) is a summation of O_k^2 over some of these functions, and therefore equal to unity minus the summation over the rest of the functions, which are just the ones occupied by electrons in the atom, so that

$$\frac{\text{double ionization cross section}}{\text{single ionization cross section}} = 1-|\int\psi_2{}^*\varphi_2 d\tau|^2 - \sum_{n,\,l,\,m}|\int\psi_{nlm}{}^*\varphi_2 d\tau|^2 \quad (17)$$

in which the summation is to be extended over all the electrons of the atom except electrons numbers 1 and 2 themselves. This formula, with the summation missing, is the one which was used by Bloch in his "order of magnitude" calculations connected with the double-jump theory of satellites. The summation would probably be quite important in quantitative work.

The author wishes to express his gratitude to Professor Philip M. Morse of Massachusetts Institute of Technology for much helpful guidance and many valuable suggestions received from him during the course of this investigation.

Note added in proof: Dr. L. G. Parratt of Cornell University has kindly sent the writer some unpublished data of his on the relative intensities of the satellites of $K\alpha$. These were taken with a two-crystal spectrometer and an ionization chamber. They are more accurate than the data of Mrs. Pearsall, and cover a wider range of atomic numbers. When plotted on the graph of Fig. 3, the points so determined are somewhat closer to the theoretical curve, but still slightly above it. However, Parratt's experiments are so much better than my theory, that a more detailed comparison at this time is not worth while.

Since the manuscript was submitted for publication, the writer has had the privilege of discussing the subject on several occasions with Professor J. R. Oppenheimer. Professor Oppenheimer suggests that the interaction of the two *expelled* electrons may be of more importance than one might perhaps think at first. This interaction was of course entirely neglected in the theory, since Hartree wave functions were used for the doubly ionized state of the atom. This point cannot be quantitatively investigated until further calculations are made.

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An Apparent Failure of the Photon Theory of Scattering

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A test has been made of the photon theory of the scattering of high frequency radiation. The pairs of scattered photons and recoil electrons predicted by this theory have been looked for by means of specially designed Geiger-Müller counters. Coincident discharges in the electron and photon counters were recorded by means of a vacuum tube amplifying and adding circuit. The scatterers used were air, aluminum, beryllium, filter paper and paraffin. The radiation was the gamma-rays from radium C. Experiments were performed with the counters set at various angles, some where the photon theory predicts coincidences, and others where coincidences should not be expected. The experiments uniformly gave fewer coincidences in the correct positions than were expected, and those observed could in every case be accounted for as chance coincidences due to the finite resolving time of the apparatus. It has not been found possible to bring the results of these experiments into accord with the photon theory of scattering. The wave-mechanical theory of the scattering process has not yet been extended to include the gamma-ray region so that it is impossible to compare this theory with the present experiments. Unless it is shown that the two theories disagree in the gamma-ray region it does not seem possible to reconcile the present experiment with the Bothe-Geiger and Compton-Simon experiments.

Introduction

The discovery of the change in wave-length of x-rays when scattered by loosely bound electrons led A. H. Compton[1] to develop a photon theory based on the concept of light quanta to account for this phenomenon. This theory accounted for the interaction between radiation and matter by picturing the process as a mechanical collision between a light corpuscle and an electron which obeyed the laws of conservation of energy and momentum. At about the same time a virtual radiation theory was

[1] A. H. Compton, Phys. Rev. **21**, 483 (1923).

proposed by Bohr, Kramers and Slater.[2] This theory also accounted for the change in wavelength of the scattered radiation, and for the production of recoil electrons, which had been predicted by the photon theory and observed in the cloud expansion experiments of C. T. R. Wilson[3] and W. Bothe.[4]

It was evident, however, that the photon theory predicted four other phenomena, at that time unobserved, which the virtual radiation theory did not require. First, the recoil electron should appear at the same instant of time that the photon of radiation was scattered. Second, there should be a definite angular relationship between the direction in which the recoil electron is ejected and the direction in which the scattered photon travels away. Third, the momentum vectors of the incident photon, the scattered photon, and the recoil electron should be coplanar. Fourth, as a corollary of the second and third points, the momenta of the recoil electrons should be statistically distributed in magnitude and direction according to the angular distribution of the scattered photons.

More recently Wentzel[5] has discussed the problem by the methods of wave mechanics and obtained results for the ordinary x-ray region that agree in general with the older photon theory, but with a less definite physical picture of the scattering process. According to this theory there is a high probability that the scattered photon and the recoil electron will travel in the directions required by the photon theory so that both energy and momentum will be conserved. For an initially free electron this probability approaches unity in agreement with the older theory. The wave-mechanical theory does not require exact time coincidence between the appearance of the scattered photon and recoil electron since the motion of these corpuscles is determined by probability laws. However, the probable value of the time difference would be so small that it could hardly be experimentally detected.

The diverse predictions of these theories can only be cleared up by an appeal to experiment.

[2] N. Bohr, H. A. Kramers and J. C. Slater, Phil. Mag. **47**, 785 (1924).
[3] C. T. R. Wilson, Proc. Roy. Soc. **A104**, 1 (1923).
[4] W. Bothe, Zeits. f. Physik **16**, 319 (1923).
[5] G. Wentzel, Zeits. f. Physik **43**, 1 (1927).

Previous Experiments

Experiments designed to test the predictions of the theories were soon reported. The angular distribution of the recoil electrons was definitely established as being in statistical agreement with the photon theory by the cloud expansion experiments of Compton and Simon.[6] Further experiments by these investigators were carried out to determine whether the predictions of this theory were realized in the individual scattering events as well.[7] Within the rather large experimental uncertainties inherent in this type of experiment there was apparently an exact individual relationship between the angles as required by the photon theory.

At about the same time Bothe and Geiger[8] performed an experiment which was designed to determine if the recoil electron and scattered photon appear simultaneously. The primary radiation was supplied by an x-ray tube operated at 70 kv. Hydrogen was used to scatter the radiation, and point counters to record the coincidences. Under normal operating conditions with this apparatus the recoil electron counter discharged at the rate of about 400 times per minute, while the photon counter discharged about twice per minute. About one recoil electron counter discharge in 2000 was coincident with a photon counter discharge to an accuracy of about 1/1000 sec. These coincidences were not exact, for the discharge in one counter always seemed to lag behind that in the other to a measurable degree; but Bothe and Geiger concluded that this lag was not due to inductive effects between the counters, and calculated by probability theory that there was only one chance in 400,000 that the number of coincidences observed could be due to chance.

While this experiment was in progress, a similar experiment was undertaken at the University of Chicago by Bennett.[9] His apparatus consisted of a vacuum drum in which two Geiger point counters were mounted on spectrometer

[6] A. H. Compton and A. W. Simon, Phys. Rev. **25**, 309 (1925); also F. Kirchner, Ann. d. Physik **83**, 969 (1927).
[7] A. H. Compton and A. W. Simon, Phys. Rev. **26**, 289 (1925).
[8] W. Bothe and H. Geiger, Zeits. f. Physik **32**, 639 (1925).
[9] R. D. Bennett, Ph.D. Dissertation, University of Chicago (1925).

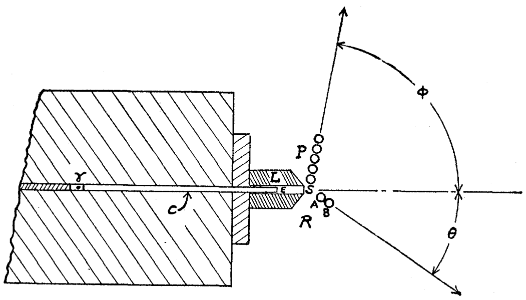

FIG. 1. Schematic arrangement of apparatus.

arms so that they could be set at the angles to the incident beam required by the photon theory. One counter was designed to record recoil electrons and the other to detect scattered photons. The primary radiation was supplied by an x-ray tube operated at 180 kv and was filtered through 7 mm of brass. In some of Bennett's experiments the scatterer was a thin piece of oiled paper or mica; in others it was air at atmospheric pressure. Because, as in the Bothe-Geiger experiment, there appeared to be a time lag in the action of the point counters so that the "coincidences" observed were not exact, the results of these experiments did not give a conclusive answer to the question.

A similar experiment was performed at the University of Chicago by J. A. Bearden. In this experiment the recoil electron counter was first placed in a position R_a where coincidences should have been observed according to the photon theory, and then in a second position R_b where coincidences should not have been observed. The position of the photon counter P was unchanged. The results were: "In position R_a there were 71 counts in P and 31,000 in R with only 8 coincidences. In position R_b there were 112 counts in P and 42,400 in R with 9 coincidences. Thus there were just as many coincidences in the wrong position according to the theory as in the correct position."[10]

The chief difficulties in these experiments are the time lag in the action of point counters and the fact that the recoil electrons produced by x-rays do not have as much energy as is desirable. The present experiment is an attempt to give a more definite answer to the question of the mechanism of scattering. To obtain recoil electrons of greater energy than in the experiments previously described, gamma-rays from radium C were used as the primary radiation. As an additional improvement specially designed Geiger-Müller counters were used to record the recoil electrons and scattered photons.

Description of the Experiment

The general arrangement of the apparatus used in this experiment is shown in Fig. 1.

The source of gamma-rays (radon tube) was placed at γ and the beam of radiation which fell upon the scatterer at S was collimated by the system of lead shields shown. The Geiger-Müller counters at P and R were further protected from direct and scattered radiation by additional lead shields and lead shot which, for simplicity, have been omitted from the figure. To prevent the rays scattered from the end of the collimating tube at E from passing through the counters at P and R the lead cylinder L was used in the position shown. The arrangement was such that all parts of L were in the geometrical shadow of the bundle of rays collimated by the cylindrical hole C, thus to a large degree eliminating unwanted scattering.

The fraction F of the gamma-rays emitted by the radon tube which will strike the scatterer at S is determined by the solid angle subtended at γ by the scatterer. In the apparatus shown in Fig. 1 the distance γE is 28 cm and the diameter of the hole C is 0.80 cm. Therefore, $F=5.13 \times 10^{-5}$. The fraction of the incident beam of gamma-rays scattered to the photon counters at P will be determined by several factors: the effective volume of the scatterer, the scattering coefficient for the radiation used, the distribution in angle of the scattered radiation, and the solid angle subtended by the counters. This fraction has been calculated for each of the experiments performed and used in computing the expected number of coincidences given in Tables I and II below. A photon scattered into the counter at R will record itself only when it ejects a secondary electron into the electric field of the counter. Since the absorption by the gas in the counter will be negligible, these secondary electrons must come from the walls of the counter cylinder. The

[10] Letter to A. H. Compton, July, 1935. In this letter Bearden reports also a repetition of these experiments with findings which confirm the earlier negative results and also those found in the present experiment.

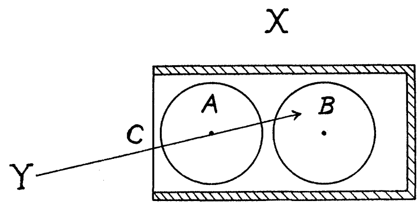

FIG. 2. β-ray counter.

cylinders used for the photon counters were gold tubes 3 cm long, 1 cm inside diameter, and of wall thickness $\frac{1}{2}$ mm. The reason for selecting gold as the material for constructing these counter cylinders was the following: The gamma-ray may be absorbed by the scattering process with the production of a recoil electron, or a photoelectron may be ejected when the whole quantum is absorbed. The number of recoil electrons that escape from the walls into the interior of the counter will increase only slowly with atomic number, because the increased scattering by elements of higher atomic number will be nearly canceled by their increased stopping power for beta-particles. On the other hand the absorption by the photoelectric process increases very markedly with atomic number. For elements of atomic number as high as that of lead or gold the photoelectric absorption for gamma-rays will be very important. Except for lead, which is slightly radioactive, the most convenient metal of high atomic number to use is gold. To further increase the chance of recording the photons, five of these gold counters were placed in a line and electrically connected so that if the photon were absorbed in any one of the five the discharge would be effective in contributing to a possible coincidence with a discharge in the recoil electron counter. The efficiency E of the photon counter can be determined by experiment by observing the discharge rate of the counter when a known number of gamma-ray quanta are passing through it in unit time. It was found that the chance of a photon's being absorbed was about one in 125, giving a value of $E=8\times10^{-3}$. The photon counters used in these experiments were filled with dry air at a pressure of 10.5 cm of mercury and were operated at about 1400 volts. Various experimenters have shown that the entire inside diameter of the counter is effective but that an end correction must be applied to allow for the decreased intensity of the electric field at the ends of the cylinder. Experiment proved that the effective volume of the cylinder was 0.92 of the geometrical volume.

According to the photon theory, with each photon scattered in the angular range $\phi\pm\Delta\phi$, a recoil electron should be ejected with the initial direction in the range $\theta\pm\Delta\theta$. The recoil electron counter was set at the mean value of θ in this range. In estimating the number of associated recoil electrons which will enter the recoil electron counter it is necessary to take account of the fact that the primary beam of gamma-rays is inhomogeneous and that the recoil electrons will be scattered by the matter between their point of origin and the recoil electron counter. These effects were taken into consideration in the computations which give the expected number of coincidences recorded in the tables below.

The problem of designing a counter to record the recoil electrons is very different from that discussed above for the photon counters. Here it is necessary to make the walls, etc., so thin that the beta-particles will be able to enter the counter easily. The final type of counter developed for this work is illustrated by Fig. 2.

The cylinders A and B are made of thin Al foil ($t=0.0007$ cm) supported upon a skeleton brass tube. The beta-rays entered the counter through a thin cellophane window C ($t=0.0017$ cm). The effective cross section of this counter was 3.2×0.9 sq. cm. This counter was used in two ways. First, counter A alone was used in such a way that coincidences were recorded when a recoil electron discharged counter A and at the same instant a scattered photon discharged any one of the gold counters P. This arrangement gave the data recorded below under the heading of Double Coincidence Experiments. In the second method, coincidences were recorded when a recoil electron passed through both A and B, discharging both, and at the same time a photon discharged one of the counters at P. The data from this type of experiment are listed below as Triple Coincidence Experiments. The principal advantage of the latter arrangement over the former is that it records a much smaller number of chance counts. These chance coincidences are due to the finite resolving time τ of the adding circuit of the vacuum tube amplifier. In addition to recoil electrons entering the counters A and B, there will come from the scatterer gamma-radiation which will eject secondary electrons from the walls of the counter. These discharges will not contribute to true coincidences but will be effective in producing a certain number of false or chance coincidences. For the double and triple arrangements the number of these chance coincidences will be respectively:

$$D = 2\tau N_A N_P, \quad (1)$$

$$T = 2\tau(N_{AB}N_P + N_{AP}N_B + N_{BP}N_A) + 3\tau^2 N_A N_B N_P. \quad (2)$$

Here N_A, N_B and N_P are the individual counting rates in counters A, B and P, respectively. N_{AB} is the number of true coincidences between counters A and B etc.; while τ is the resolving time, which was determined for the circuit used, to be $\tau = 4.4 \times 10^{-6}$ min. The chance coincidences listed in column four of the tables were computed by these formulae.

Test experiments were performed to show that this double counter functioned properly by comparing the number of coincident discharges in A and B with the single counting rates of A or B. When this was done with a source of beta-rays at Y, the double discharges were found to be a large fraction of the single rates—showing that the beta-particles were actually passing through both counters. However, when a source of gamma-rays was placed at X, the coincidences were much fewer than the single discharge rates in either counter. The actual number of coincidences was found to agree with the prediction of Eq. (1) as would be anticipated because in this case only chance coincidences should be possible.

Tests were made to show that the counters and circuits as described were functioning properly and would record coincidences of the kind sought for in this experiment. These tests were performed by arranging the counters A, B and P in a vertical plane so that they could be used to record the passage of cosmic-ray particles. It was found that the number of triple coincidences observed agreed well with the number to be expected from the experiments of Hsiung[11] and others, showing that the counters and circuit were functioning properly. Another experiment similar to that described by Street and Woodward[12] was performed to determine the efficiency of the counters and circuits in recording coincidences when the single counting rates were approximately the same as in the main experiment. This test experiment showed the efficiency of both the photon counters and the electron counters for fast ionizing particles was about 85 percent and that the circuit was working properly to record coincidences.

Experimental Results

The results of the present series of experiments are given in Tables I and II. The first column of these tables lists the scatterer and the second column the strength in millicuries of the radon tube used as a source of gamma-rays. In the third column are recorded the azimuths of the recoil electron counter measured from the direction of the incident beam of radiation. The

TABLE I. *Triple coincidence experiments.*

Scatterer	Source (mc)	θ	Chance Triples (hr.$^{-1}$)	Observed Triples (hr.$^{-1}$)	Expected Triples (hr.$^{-1}$)
Air	135	35°	0.4	0.0±0.5	13
Al t 0.0015 cm	140	35°	1.7	2.4±0.5	23
Al t 0.004	124	35°	4.9	4.7±1.5	48
Paraffin t 0.05	138	35°	9	14 ±3	69
Paraffin t 0.05	129	j35°	9	12 ±4	9
Be t 0.02	133	35°	8	9.5±2	48
Be t 0.02	131	j35°	8	9.5±2	8

TABLE II. *Double coincidence experiments.*

Scatterer	Source (mc)	θ	Chance Doubles (min.$^{-1}$)	Observed Doubles (min.$^{-1}$)	Expected Doubles (min.$^{-1}$)
Air	105	35°	0.67	0.92±0.07	0.89
Air	102	j35°	0.67	0.87±0.08	0.67
Filter paper t 0.015 cm	195	25°	2.5	1.9 ±0.18	5.7
"	191	−25°	2.5	2.2 ±0.28	2.5
Paraffin t 0.05	97	35°	1.5	1.8 ±0.09	2.8
"	94	j35°	1.5	1.8 ±0.17	1.5
"	95	35°	4.5	5.6 ±0.15	9.9
"	93	j35°	4.5	6.0 ±0.8	4.5
"	81	Hor	0.94	0.85±0.11	2.4

[11] D. S. Hsiung, Phys. Rev. **46**, 653 (1934).

[12] J. C. Street and R. H. Woodward, Phys. Rev. **46**, 1029 (1934).

angles given without a symbol before them indicate that the counter was set at the angle required by the photon theory. A negative sign before the value of θ means that the counter was set on the same side of the incident beam as the photon counter; in which position, of course, no coincidences should be expected. When the symbol j is before the value of the angle, it signifies that the recoil electron counter was rotated about the primary beam as an axis into a position 90° from the plane containing the incident beam and the centers of the photon counters. The photon theory predicts no coincidences in this position because momentum could not be conserved. Column four gives the chance coincidences calculated from Eqs. (1) and (2) and the individual discharge rates of the several counters. Column five gives the observed coincidence rates together with the probable error. For each experiment a calculation was made of the number of coincidences to be expected from the photon theory, with the strength of the source, counter efficiency and geometrical arrangement known. This number added to the chance rate is given in the last column of each table and should be the number of coincidences per unit time which should be expected if the photon theory is correct. The experiment summarized in the last row of Table II was performed with all counters in the horizontal plane. This was to minimize the effect of the inhomogeneity of the incident rays by permitting a wide range of values of both ϕ and θ to be effective.

An examination of the tables will reveal the following facts: The number of coincidences observed are always fewer than predicted by the photon theory, and in fact agree rather well in each case with the number of chance coincidences to be expected. Further, when the recoil electron counter is set in the position $-\theta$ or $j\theta$ the number of coincidences observed is as great as in the correct position θ. This strengthens the view that all the observed coincidences are due only to chance and that the predictions of the photon theory are not verified by this experiment. Thus the present series of experiments, in common with the experiments of Bennett and of Bearden discussed above, yields results contrary to those obtained by Bothe and Geiger and by Compton and Simon. It seems difficult to understand how the present experiment could have failed to show the coincidences if they were real; but all the experiments performed by the writer have uniformly yielded a negative result.

The predictions of the photon theory should apparently apply to the scattering of gamma-rays studied in the present experiments. However, the wave-mechanical theory as developed by Wentzel contains approximations which limit its strict validity to the ordinary x-ray region, for which $h\nu \ll mc^2$. The further development of this theory to include gamma-ray scattering may give a less definite angular relationship than the photon theory predicts. Coincidences would be much more difficult to observe if the angular relationship is not valid because of the much smaller chance of capturing the scattered quantum. There is thus a possibility that the results of the present experiment will not be inconsistent with those of Bothe and Geiger and of Compton and Simon. In any event, the photon theory in its present form does not agree with the experiments reported here.

This experiment was suggested to the writer by Professor Arthur H. Compton and its completion has been possible because of his generosity and stimulating advice. It is also a pleasure to acknowledge the helpful criticism and discussion given by Professor Ralph D. Bennett of Massachusetts Institute of Technology; Professor J. A. Bearden of Johns Hopkins University; Professor W. Bothe of the Kaiser Wilhelm-Institut, Heidelberg; and Professor H. Geiger of the University of Tübingen.