An Interpretation of Page's "New Relativity"

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Page's proposed extension of the theory of relativity is examined from the standpoint of the general kinematical theory developed by the present author, of which it is found to be a special case. The contention that this extension renders Einstein's theory untenable would seem to be without real foundation; for although such an attempt to treat of observers in motions other than those contemplated in the special theory of relativity must lead to a theory transcending this latter, Page's theory is shown to be readily understandable in terms of a simple, though somewhat artificial, space time of a type met in the general theory of relativity.

IN a recent issue of this *Review* L. Page[1] has developed a relativistic kinematics suitable for observers suffering constant relative accelerations, and has announced his intention of setting up a corresponding theory of the electromagnetic field. We here examine this program, in the light of previous related work, in order to determine whether or to what extent its results are incompatible with the existing relativistic theories. As a result of this examination we are led to the conclusion that Page's kinematics is included as a special case in the general kinematical theory formulated some months ago by the present author[2] for a different purpose, and that as such, although outside the frame of the special theory of relativity, it may readily be understood from the standpoint of the general theory. It is further pointed out that an electrodynamical theory, based on transformations of the kind employed by Page, has been developed by H. Bateman[3] and by E. Cunningham.[4] For the sake of conciseness and completeness the terminology of the theory of groups is at times invoked, but it is to be emphasized that all results are obtained by a straight-forward analysis, for an understanding of which a knowledge of the theory of groups is not required.

1. KINEMATICAL PRELIMINARIES

We begin, then, with a brief statement of the results of our kinematical analysis, referred to above,[2] which will be of service in the following. It is there assumed that, for the purposes in mind, the space-time region in question admits a three-parameter family of possible equivalent observers, in the sense that through each event there passes the world-line of one (and in general only one) such observer A, and that his view of the development of the whole, as obtained with the aid of a proper clock, light signals and a theodolite, is intrinsically indistinguishable from that of any other observer A′ in the family. It is then shown that under these conditions A can define operationally a set of coordinates $\eta^\mu \equiv (\tau\eta\theta\psi)$, and can introduce a Riemannian metric

$$\begin{aligned} ds^2 &\equiv d\tau^2 - \xi^2(\tau)du^2, \\ du^2 &\equiv d\eta^2 + \sigma^2(\eta)[d\theta^2 + \sin^2\theta d\psi^2], \end{aligned} \tag{1.1}$$

in which

$$\sigma(\eta) \equiv \sinh\eta, \quad \eta \quad \text{or} \quad \sin\eta \tag{1.2}$$

according as the Riemannian curvature k of the auxiliary three-space with metric du^2 is -1, 0 or $+1$, respectively; the function $\xi(\tau) \geq 0$ characterizes the type of relative motion between any two equivalent observers of the family, and is so far arbitrary. This metric ds^2 possesses the important properties:

(a) The world-line of each of the fundamental particle-observers A′ is a geodesic $\eta^\alpha = \text{const.}$ $(\alpha = 1, 2, 3)$ of ds^2, along which the interval s measures the clock time τ of A′.

(b) The world-lines of all light paths are the null-geodesics of the metric.

The coordinates η^μ of any event E in the region are determined operationally by A as follows. Let t_1 be the time at which A must send out a light signal in order that it pass through

[1] Page, Phys. Rev. **49**, 254 (1936).
[2] Robertson, Astrophys. J. **82**, 284 (1935).
[3] Bateman, Proc. Lond. Math. Soc. **7**, 70 (1908); **8**, 223 (1909).
[4] Cunningham, Proc. Lond. Math. Soc. **8**, 77 (1909). It is of interest to note that this paper bears the title "The Principle of Relativity in Electrodynamics and an Extension Thereof."

E, and let t_2 be the time at which a signal sent out from E reaches the world-line of A. Then the coordinates τ, η assigned the event E by A are obtained from his clock readings t_1, t_2 by means of the equations

$$2F(\tau)=F(t_2)+F(t_1), \qquad 2\eta=F(t_2)-F(t_1), \quad (1.3)$$

where
$$F(\tau)\equiv\int d\tau/\xi(\tau) \quad (1.4)$$

is any primitive of $1/\xi(\tau)$. The angles θ and ψ are those made by the direction of either light signal with the zenith and meridian, respectively, of A's theodolite. The transformation $\eta^\mu\rightarrow\eta'^\mu$ from the coordinates employed by A to those employed by any other fundamental observer A′ is given by $\tau'=\tau$ and a transformation $\eta^\alpha\rightarrow\eta'^\alpha$ of the spatial coordinates which leaves du^2, and therefore ds^2, invariant in form as well as in fact; that this latter transformation is a member of a *six*-parameter group G_6 of motions is accounted for by the fact that each of the ∞^3 particle-observers may orient his theodolite in any one of ∞^3 ways. For the case $k=0$, of greatest interest for our present purposes, this group G_6 is the group of Euclidean motions, consisting of the three-parameter group T_3 of Euclidean translations and the group R_3 of rotations; we may then for most purposes ignore the latter, and confine ourselves to the group T_3 of translations between observers with similarly oriented theodolites.

2. Constancy of Light Velocity

The kinematical theory given in résumé above is clearly the general solution of the equivalence problem stated by Page in Part 1 of his paper. However he, following E. A. Milne, would have the theory expressed in terms of a time t and a distance r assigned to the event E *as in the special theory of relativity*, i.e. he would replace our τ, η or t_1, t_2 by their expressions in terms of the variables

$$t\equiv\tfrac{1}{2}(t_1+t_2), \qquad r\equiv\tfrac{1}{2}c(t_2-t_1). \quad (2.1)$$

This in itself is of course no restriction on the generality of the solution, although it leads to the more cumbersome expression

$$ds^2=[\xi^2(\tau)/\xi(t_1)\xi(t_2)]\{dt^2 -[dr^2+R^2(d\theta^2+\sin^2\theta d\psi^2)]/c^2\} \quad (2.2)$$

for the line element (1.1), where

$$R^2(t,r)\equiv c^2\xi(t_1)\xi(t_2)\sigma^2(\eta) \quad (2.3)$$

and τ, η, t_1, t_2 are to be expressed in terms of t, r with the aid of (1.3), (2.1).

Now the path of any light signal which intersects the world-line of A is given by

$$r=\pm c(t-t_0), \qquad \theta=\text{const.}, \qquad \psi=\text{const.},$$

and may therefore be interpreted as a radial straight in the Euclidean (xyz)-map constructed from the Cartesian coordinates (xyz) obtained from the polar coordinates $(r\theta\psi)$ in the usual way; hence any light signal sent by A or received directly by A is propagated rectilinearly in the (xyz)-map with the constant *coordinate* velocity $dr/dt=\pm c$. However, it is readily seen from (2.2) that other light paths do not in general possess this property, and that to follow Milne and Page in requiring that they do, places a severe restriction on the function $\xi(\tau)$ characterizing the relative motions of the fundamental particle-observers. We remark in passing that this result in no way contradicts that obtained in section 3 of the paper referred to above,[2] where it is shown that, at least in the open models $k=-1, 0$, coordinates $(TXYZ)$ may always be introduced in which light is propagated rectilinearly with constant coordinate velocity c; these latter are defined in quite another way than the $(txyz)$ here employed, and are such that the transformations from one particle-observer to another constitute a six-parameter subgroup of the general Lorentz group.

In order to interpret Page's work in terms of the kinematics outlined above, we must first investigate the restriction imposed by his requirement that *all* light signals, and not only those which A is in position to observe directly, are propagated rectilinearly with constant coordinate velocity c in the Euclidean map constructed from the coordinates (xyz). It is seen immediately from (2.2) that, in order for all null elements $ds=0$ to be null elements in the Minkowskian $(txyz)$-map, the function $R(t,r)$ defined by (2.3) must be identically equal to r; that this condition is also sufficient follows from the fact that the null geodesics of a conformally-flat manifold are at the same time null geodesics of the associated flat. On replacing $\xi(\tau)$ by its value $1/F'(\tau)$ in terms of the derivative F' of F, the condition $R\equiv r$ yields the differential equations

$$F'(a)F'(\tau)(\tau-a)^2=\{2\sigma[(F(\tau)-F(a))/2]\}^2 \quad (2.4)$$

for $F(\tau)$, where we have expressed all variables in terms of t_1, t_2 and have then set $t_1=a$, $t=\tau$. The solution $F(\tau)$ is given, to within an additive constant, by a quadrature, and the corresponding forms of $\xi(\tau)$ by differentiation. The results of this computation may be expressed in terms of the general linear fractional function

$$f(\tau)\equiv(A\tau+B)/(C\tau+D),\quad \Delta\equiv AD-BC>0, \quad (2.5)$$

in the various cases k as follows:

$$F=f,\qquad \xi=(C\tau+D)^2/\Delta,\qquad (k=0); \quad (2.6)$$

$$F=\log f,\qquad \xi=(A\tau+B)(C\tau+D)/\Delta,\qquad (k=-1); \quad (2.6')$$

$$F=2\tan^{-1}f,$$

$$\xi=[(A\tau+B)^2+(C\tau+D)^2]/2\Delta,\quad (k=+1). \quad (2.6'')$$

Now in a certain sense

$$\lambda\equiv c\xi(\tau)\sigma(u) \quad (2.7)$$

may be considered as a significant measure of the "distance" at "time" τ between two particle-observers with the fixed u-interval u, and we define

$$v\equiv d\lambda/d\tau,\qquad \alpha\equiv d^2\lambda/d\tau^2, \quad (2.8)$$

as the corresponding measures of the relative "velocity" and "acceleration," respectively. It now follows from the results of the preceding paragraph that the requirement of constant coordinate-velocity of light in the (xyz)-map implies that *the relative motion of any two fundamental particle-observers must be such that their relative α-acceleration is constant.*

An examination of the three sets of equations (2.6) shows that the only case in which the relative velocity v can be zero for all τ is that in which $k=0$, $C=0$; we then have

$$\xi=D/A;\qquad t=\tau,\qquad r=Dc\eta/A, \quad (2.9)$$

and the original line element (1.1) is itself Minkowskian on change of spatial scale. The only other case in which the acceleration α vanishes throughout the motion is that in which $k=-1$, $C=0$, whence

$$\xi=\tau;\qquad t=\tau\cosh\eta,\qquad r=c\tau\sinh\eta, \quad (2.10)$$

and the new line element (2.2) is the Minkowskian form on which the special theory of relativity is based; the original metric (1.1) is in this case the "polar" form implied in the work of Milne.

Finally, to come to the case adopted by Page, we note that he requires (pp. 260, 263) that the relative motion of two particle-observers be such that at some instant they coincide spatially and are then momentarily at relative rest. The first of these conditions allows us to reject the case (2.6″), for in it the distance function $\xi(\tau)$ has only complex roots; the second condition then throws out the hyperbolic case (2.6′), in which the two roots of $\xi(\tau)$ are real but distinct. Thus we find, as *the only case in which two particle-observers are at some time momentarily in coincidence and at relative rest*, the parabolic case (2.6), in which $\xi(\tau)$ has a double root and the spaces $\tau=$const. are Euclidean. The remainder of our investigation will be devoted to a discussion of this case.

3. Transformation Theory

On taking as the origin of τ that event at which all ∞^3 particle-observers momentarily coincide, the invariant metric (1.1) defined by (2.6) may be written

$$ds^2=d\tau^2-(\varphi\tau^2/2c)^2[d\eta_1{}^2+d\eta_2{}^2+d\eta_3{}^2], \quad (3.1)$$

where the η_a $(a=1,\ 2,\ 3)$ are the Cartesian coordinates associated with the polar coordinates $(\eta\theta\psi)$ in the usual way, and φ is the (constant) acceleration between two particle-observers whose (constant) u-distance is unity. Our first task is that of discovering the transformation $(\tau\eta_a)\rightarrow(tx_a)$ which throws (3.1) into the conformally-Minkowskian form (2.2), where the x_a are the Cartesian coordinates associated with $(r\theta\psi)$. We may now take

$$F(\tau)=-2c/\varphi\tau, \quad (3.2)$$

and the Eqs. (1.3), (2.1) yield as the required transformation

$$\tau=(t^2-r^2/c^2)/t,\qquad \eta_a=2x_a/(t^2-r^2/c^2)\varphi; \quad (3.3)$$

the form (2.2) of the metric is then found to be

$$ds^2=(\tau/t)^2[dt^2-(dx_1{}^2+dx_2{}^2+dx_3{}^2)/c^2]. \quad (3.4)$$

The transformation (3.3) has merely the effect of allowing the given observer A at $\eta=0$ to introduce a new coordinate mesh, of the kind demanded by Page, in which A is at the origin $r=0$ of the new spatial coordinates x_a, and in which the new temporal variable t is also a direct measure of proper-time along his world-line. In order to find the relations between the coordinates (tx_a) employed by A and the coordinates $(t'x_a')$ similarly introduced by another particle-observer A′ at $\eta_a=\beta_a$, we must consider the group of motions admitted by the metric (3.1); as indicated above, we may here confine ourselves to the subgroup T_3 of translations between observers with similarly oriented theodolites. Hence the transformation A→A′ is given by

$$\tau'=\tau, \qquad \eta_a'=\eta_a-\beta_a, \tag{3.5}$$

and the transformation which it induces on the corresponding Minkowskian coordinates is defined implicitly by

$$\begin{aligned}(t'^2-r'^2/c^2)/t'&=(t^2-r^2/c^2)/t\ (=\tau)\\ x_a'/t'&=x_a/t-\varphi\tau\beta_a/2.\end{aligned} \tag{3.6}$$

Now this transformation must leave (3.4) invariant in form as well as in fact, and it must therefore be a member of the general conformal group in four dimensions; hence it must be compounded of at most dilatations, translations, rotations (including reflections), and inversions. The explicit form, as found directly from (3.6), is

$$\begin{aligned}t'&=b^2t/R^2,\\ x_a'-b_a&=b^2[x_a-(1+2\Sigma b_ex_e/b^2)b_a]/R^2,\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}&R^2(tx_a;b_a)\equiv\Sigma(x_a+b_a)^2-c^2t^2,\\ &b_a\equiv2c^2\beta_a/\beta^2\varphi, \qquad \beta^2\equiv\Sigma\beta_a^2, \qquad b^2\equiv\Sigma b_a^2;\end{aligned} \tag{3.8}$$

this transformation may in fact be broken down into the reflection

$$\bar{x}_a=x_a-2(1+\Sigma b_ex_e/b^2)b_a \tag{3.9}$$

in the plane through $x_a=-b_a$ orthogonal to the vector b_a, followed by the space-time inversion

$$t'=b^2t/R^2, \qquad x_a'-b_a=b^2(\bar{x}_a+b_a)/R^2 \tag{3.10}$$

in the pseudo-sphere

$$R^2(t\bar{x}_a;b_a)\equiv R^2(tx_a;b_a)=b^2 \tag{3.11}$$

of radius b and center at the event $(0,-b_a)$. Thus the group I_3 induced by T_3 is that subgroup of the general conformal group which consists of the ∞^3 transformations $\mathrm{t}(b_a)$ of the form (3.7), and which possesses the differential invariant (3.4); the law of composition of this (Abelian) group I_3 is given by

$$\mathrm{t}(b_a)\mathrm{t}(c_a)=\mathrm{t}(b_a+c_a). \tag{3.12}$$

That (3.7) is indeed the explicit form of the transformation derived by Page can be seen by comparing it with his Eqs. (46), (47), (68), on going over to the variables there employed. The particle-observer A′ at $x_a'=0$ moves radially away from A in the direction of the vector b_a with a coordinate-velocity v which satisfies the equation

$$v(1-v^2/c^2)^{-\frac{1}{2}}=\beta\varphi t, \tag{3.13}$$

as can be found directly from (3.7); hence the t rate of change of the left-hand side of (3.13), which Page calls the "relativity acceleration" of A′ relative to A, is the constant $\beta\varphi$—and is the same as their relative α-acceleration, as defined by (2.8). The geometrical theorems derived by Page in Part 4 of his paper are consequences of general theorems on inversions, as applied to the spatial inversion defined by (3.10) at time $t=t'=0$.

We also see, in retrospect, that the transformation $(\tau\eta_a)\rightarrow(tx_a)$, defined by (3.3) for a given observer A, is itself interpretable as an inversion of the variables

$$b_0^2/c^2\tau, \qquad b_0\eta_a \qquad (b_0\equiv2c^2/\varphi)$$

in the pseudo-sphere of imaginary "radius" ib_0 about the origin.

Our approach has been based on the existence of an invariant interval ds^2 which satisfies the conditions (a), (b) stated in section 1 above, and which therefore offers an invariantive description of the only elements so far involved in the theory—namely, the equivalent particle-observers, their proper clocks and theodolites, and all light signals. In the light of this, and considering the rather arbitrary definition of t and r, it is difficult to see why Page's "physical interval" $c^2dt^2-dx^2$ should be expected to appear as an invariant. It is to be noted, in this connection, that the *content* of Page's Eq. (32) is equivalent to the fact that our ds^2 is invariant in the two-dimensional case $\theta=$const., $\psi=$const.; his expressions (31), (32) are not, however, invariants of the transformation $(tr)\rightarrow(t'r')$ in the technical sense, for they depend on the parameter φ of the transformation. On the other hand, the differential expression (69) which he writes down for the full four-dimensional case is not at all equivalent to our differential invariant (3.4); even if it be modified by the same treatment as that accorded (32) in bringing it into invariant form, it could at best be expected to be invariant under a one-parameter subgroup of I_3, and not under the full group.

4. Associated Particle-Observers

It remains to clarify the role of the ∞^3 possible particle-observers which Page associates with each given particle-observer A of the type dealt with above. These auxiliary observers have as their world-lines the curves $x_a=\text{const.}$; in order to avoid confusion with the primary particle-observer A we shall refer to the former as test-particles. Computation shows that the world-lines $x_a=\text{const.}$ are not geodesics of the metric ds^2, unless $x_a=0$—hence, in Page's terminology, the reference systems employed by two observers A and A′, while *equivalent*, are not *homogeneous*. Now we have examined,[5] in connection with the general kinematical theory described in section 1 above, the possible modes of motion of such a six-parameter family of test particles in a space time possessing the uniformity properties here implied, and have found that they must satisfy differential equations of the form

$$\frac{d^2\eta^\mu}{ds^2}+\left\{\begin{matrix}\mu\\ \nu\sigma\end{matrix}\right\}\frac{d\eta^\nu}{ds}\frac{d\eta^\sigma}{ds}=\Gamma\left(\tau,\frac{d\tau}{ds}\right)\left[\delta_0{}^\mu-\frac{d\tau}{ds}\frac{d\eta^\mu}{ds}\right], \tag{4.1}$$

where $\eta^0\equiv\tau$, the Christoffel symbols are computed from the coefficients $g_{\mu\nu}$ of (1.1), and Γ is some function of at most τ and $d\tau/ds$. On evaluating these expressions for the metric (3.1) and taking η^α as defined by (3.3), in which the x_a are constants, these equations of motion reduce to

$$dt/ds=t/\tau,\qquad (\Gamma+1/\tau)x_a d\tau/ds=0; \tag{4.2}$$

hence the ∞^3 world-lines $x_a=\text{const.}$ associated with *each* of the particle-observers A are solutions of (4.1) provided we set

$$\Gamma(\tau,d\tau/ds)\equiv-1/\tau. \tag{4.3}$$

These results allow us to conclude, in accordance with the general theory developed in section 5 of the paper cited above,[5] that the ∞^6 solutions of (4.1) fall into classes characterized by a parameter $d\geq 0$. The class $d=0$ consists of the ∞^3 world-lines of the fundamental particle-observers, the class $d=\infty$ of the ∞^5 light paths in space time, and each class $d(\neq 0,\infty)$ of the ∞^5 test particles which are at the fixed x distance d from the particle-observer with which each of them is associated. Further, the motion of each member of such a class d is such that its λ-distance, as defined by (2.7), from the observer A with which it is associated, approaches asymptotically the value d.

Finally, these considerations unearth a surprising formal connection between Page's work and the transformation theory of the de Sitter universe in the general theory of relativity. To exhibit this connection, we first note that in any case in which Γ is a function of τ alone, the equations (4.1) may be interpreted as the equations of the ∞^6 geodesics of the metric

$$dS^2\equiv e^{2\int\Gamma d\tau}ds^2 \tag{4.4}$$

conformal to the original metric (1.1). For the case (3.4) of interest here, this new metric may, on subjecting t to the transformation

$$t\equiv(b_0/c)e^{cz/b_0},\qquad (b_0\equiv 2c^2/\varphi) \tag{4.5}$$

be taken in the form

$$dS^2=dz^2-e^{-2cz/b_0}(dx_1{}^2+dx_2{}^2+dx_3{}^2). \tag{4.6}$$

This invariant metric satisfies the conditions (a) and (b) of section 1 above, except that it is no longer a direct measure of clock time along the geodesic paths of the fundamental particle-observers. It is, on the other hand, *the stationary form of a contracting de Sitter universe* of constant Riemannian curvature $-c^2/b_0{}^2$; the relation between a particle-observer A and an associated test particle $x_a=\text{const.}$ in the space time (3.4) of interest here is, to within the time transformation (4.5) above, formally the same as that between two associated free observers in the de Sitter universe (4.6), and the relation between

[5] Robertson, Astrophys. J. **83**, 187 (1936); in particular, sections 4 and 5.

[6] Cf. the discussion of the de Sitter universe by the present author in Phil. Mag. **5**, 835 (1928). Eq. (11) of this discussion are in fact equivalent to those defining Page's transformation (68) between equivalent particle-observers.

two equivalent observers A, A′ here is formally the same as that between two free observers in general relative motion in the de Sitter universe.[6]

5. Summary and Conclusion

This examination of Page's work, from the standpoint of our general kinematics, has shown that it deals with one of the three possible cases in which there exists a set of ∞^3 equivalent particle-observers with respect to whom light is propagated rectilinearly with a constant coordinate velocity c in a coordinate system (tx_a) defined as in the special theory of relativity. In each of these cases there exists a significant Riemannian metric, of the type employed in the cosmological applications of the general theory of relativity, which is invariant under the group G_6 of motions which describe the equivalence; it is apparent *a priori* that this group is a six-parameter subgroup of the general conformally-Minkowskian group of transformations on the variables (tx_a), and that it is the direct product of the group R_3 of Euclidean rotations on the spatial variables x_a and a three-parameter group I_3 which has as its minimal invariant varieties the spaces $\tau=$const. We have in particular examined this group I_3 in the case adopted by Page, and have shown how it may be generated from translations, reflections and simple inversions.

The three cases thus found are characterized by the fact that in them the fundamental particle-observers undergo a constant relative α-acceleration, which in Page's case agrees with the t rate of change of the "relativistic" velocity defined by the left-hand side of (3.13); we must regard this agreement as a happy coincidence, however, for it is difficult to see what significance is to be attributed to Page's procedure outside the special relativistic theory of electrodynamics, where it leads to the usual classical expression for the ponderomotive force (and not to the relativistic four-vector by which it should even there be replaced). It would seem that, insofar as the present line of attack is concerned, this leaves little justification for Page's "hope of finding a rational detailed description of atomic structure" by an investigation of "equivalent reference systems having other types of motion [than with constant acceleration], particularly relative rotation"; indeed, it has long since been pointed out by E. Cunningham[7] that "no such relation [as that defined by the conformal group] can express anything corresponding to a rotational motion of the space-frame of reference." It would further seem, particularly in view of the severe restrictions which their introduction implies, that the quasi-Minkowskian coordinates $(txyz)$ are of no very great intrinsic significance, although possibly of convenience for some purposes. But, above all, it is to be remembered that we are here on the *kinematical* level of description, and that the question of whether or not two observers are *dynamically* equivalent can be decided only in terms of the more complete physical theory of which the kinematics is but a noncommittal tool; thus, while we are willing to entertain the view that a general kinematical theory such as that outlined in section 1 above may play a useful role in comological speculations, we should not be disposed to attempt to reconcile it with the obvious dynamical inequivalence of observers in general relative motions in terrestial or atomic problems. Otherwise stated, either these more general types do represent possible motions of dynamically equivalent observers, in which case we should expect to have recourse to methods and concepts of the type met in the general theory of relativity, or their dynamical inequivalence should be expected to set serious limitations to the usefulness of the kinematics.

Insofar as electrodynamics is concerned, we confine ourselves to the remark that the problem of transforming the electromagnetic field under the general conformal group has been given a detailed solution by Bateman and by Cunningham in the papers cited above[3, 4]; we do not here go into the question of whether or to what extent their theory is equivalent to the phenomenological theory of electrodynamics based on the line element (3.4) as in the general theory of relativity.

Finally, I should like to thank Princeton University for granting me sabbatical leave during the present academic year, and to express my appreciation to the California Institute of Technology for the facilities for work which it has placed at my disposal during this time.

[7] Cunningham, *The Principle of Relativity* (Cambridge, 1914), p. 89. Cf. also p. 79 of reference 4 above.