Effect of Electron Pressure on Plasma Electron Oscillations

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A general equation for electron motion in a plasma is developed which includes a term arising from electron gas pressure. The resulting expression is

$$\partial^2\xi/\partial t^2+(4\pi ne^2/m)\xi=(kT/m)\nabla^2\xi,$$

where ξ is electron displacement, n electron density, and T electron gas temperature. From this it is found that the possible frequencies of free vibration form a series given by $f_i=(kT/\lambda_i^2m+ne^2/\pi m)^{\frac{1}{2}}$. The lower limit corresponds to the Tonks-Langmuir value $(ne^2/\pi m)^{\frac{1}{2}}$, while the other frequencies depend upon the possible standing waves which may exist. The theory explains the observed variation of frequency with electron gas temperature.

IN their investigation of electron oscillations in ionized gases, Tonks and Langmuir[1] did not take into consideration forces arising from electron gas pressure. The writer has found that if this force is included the previously not understood variation of resonance frequency with electron temperature may be explained.

In order to determine the effect of electron pressure, consider the equation of motion,

$$m\ddot{\xi}=-eE+\partial p/n\partial x, \tag{1}$$

where the last term represents the force arising from pressure. The electron displacement is denoted by ξ. The electric field E arises from electron displacements. In an undisturbed plasma the electron and positive ion densities are uniform and equal, and the net charge in any element of volume is zero, hence the plasma is field free. However, if electron displacements occur, the net charge in a volume element is no longer zero, in general, and electric forces are set up. The expression for this force, is[1]

$$E=4\pi ne\xi. \tag{2}$$

Departures from a uniform distribution will give rise also to pressure gradients. It will be shown that the forces thus arising may be of the same order of magnitude as the force E, and that their inclusion in the theory gives a variation of frequency with electron gas temperature.

If the undisturbed electron density is taken to be n_0, the change due to any displacement $\xi(x)$ is

$$\delta n=n_0\partial\xi/\partial x, \tag{3}$$

so that

$$n=n_0(1+\partial\xi/\partial x). \tag{4}$$

[1] L. Tonks and I. Langmuir, Phys. Rev. **33**, 195 (1929).

Now

$$p=nkT,$$

hence

$$\frac{\partial p}{\partial x}=kT\frac{\partial n}{\partial x}=n_0kT\frac{\partial^2\xi}{\partial x^2}. \tag{5}$$

Substituting (5) and (2) in (1) yields

$$\frac{\partial^2\xi}{\partial t^2}+\frac{4\pi ne^2}{m}\xi=\frac{kT}{m}\frac{\partial^2\xi}{\partial x^2}. \tag{6}$$

A more general equation can be derived from Maxwell's equations, and a force equation similar to (1), wherein $\partial p/\partial x$ is replaced by grad p. The procedure is the same as that given by Tonks and Langmuir,[1] except that the term involving pressure is carried through. The resulting equation is the same as (6) if $\partial^2\xi/\partial x^2$ be replaced by $\nabla^2\xi$.

For a plane wave the possible frequencies are given by,

$$f_i=(kT/\lambda_i^2m+ne^2/\pi m)^{\frac{1}{2}}, \tag{7}$$

where the λ_i's are the possible wave-lengths determined by the standing wave systems which may exist. For a given n and T, there is therefore a series of possible frequencies, the lower limit of which is the Tonks-Langmuir frequency $(ne^2/\pi m)^{\frac{1}{2}}$. This series, however, appears experimentally as a single broad band due to the breadth of the individual resonances and the closeness of spacing.

It is assumed that the possible range of wave-lengths lies between a length comparable to the size of the containing vessel, and a length comparable to the Debye distance for the plasma. It will be shown that the experimental data can be fitted by choosing a wave-length within this region.

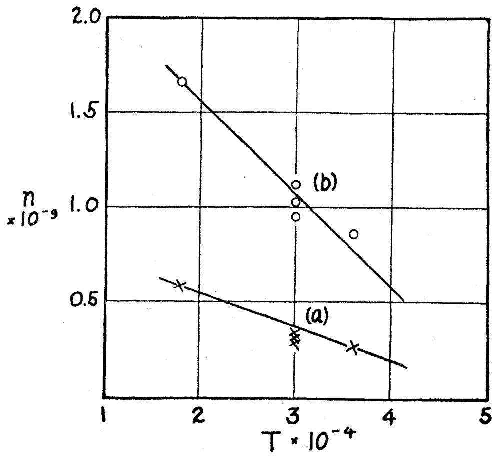

FIG. 1. Showing combinations of n, the number of electrons per cc, and T, the electron gas temperature, which give resonance under applied frequency of 1.6×10^8.

In Fig. 1 are plotted data, some of which were taken from Tonks' paper,[2] and some of which, previously unpublished, were kindly supplied to the writer by him. These represent combinations of n and T, which gave resonance when the plasma was placed between condenser plates to which a fixed frequency (1.6×10^8) was applied. The linear relation is represented by (7) if both sides are squared, and f is regarded as a constant. The two lines, (a) and (b), correspond to two resonances which were observed, but for which there is no explanation. They apparently represent two modes of oscillation, and may be related to plasma shape.[3] It will be noted that there is a very definite effect due to electron temperature, the values of n required for resonance varying by as much as 2 : 1, over the temperature range investigated.

The slopes of these lines should be, according to (7),

$$dn/dT = -\pi k/e^2\lambda^2 \qquad (8)$$

To fit the data it is found that the effective wave-lengths must be $\lambda_a=0.36$ cm, and $\lambda_b=0.22$ cm. The plasma diameters in the two cases were about 1.9 cm and 2.5 cm, respectively. The discharge did not fill the entire cross section of the tube, which was 3 cm in diameter. The Debye distances, $\lambda_D=(kT/4\pi ne^2)^{\frac{1}{2}}$, are 0.064 and 0.038 cm, respectively. It is evident that the necessary λ's lie within the required range in both cases.

The width of the resonance band may be estimated as follows: Assume the small wave-length limit to be given by $\alpha\lambda_D$, where α is a small numerical constant, found by Langmuir[4] to be 3.31. Since $\lambda_D=(kT/4\pi ne^2)^{\frac{1}{2}}$ we may make use of (7) and write the limiting frequency as

$$f_m=\left(\frac{4\pi^2}{\alpha^2}+1\right)^{\frac{1}{2}}\left(\frac{ne^2}{\pi m}\right)^{\frac{1}{2}}=2.14\left(\frac{ne^2}{\pi m}\right)^{\frac{1}{2}}.$$

Hence the maximum frequency is 2.14 times the lowest frequency, giving a band width of about one octave, which is in agreement with experiment.

The contribution of the pressure term to the frequency may be computed by using the above-determined values of λ. It is found that the frequencies predicted for the (a) and (b) resonances are 50 percent and 45 percent greater, respectively, for a given electron density, than when pressure is not considered. The available data do not permit a decisive check of this because of the unexplained fact that several resonances are observed. The agreement is better with the (a) type but worse with the (b).

It should be noted that Eq. (8) for the slope is independent of any constant factor multiplying f in (7), for example, to correct for plasma configuration as was done by Tonks.[2] The expression for slope would be unchanged by any such factor.

[2] L. Tonks, Phys. Rev. **37**, 1458 (1931).
[3] L. Tonks, Phys. Rev. **38**, 1219 (1931).
[4] I. Langmuir, Proc. Nat. Acad. Sci. **14**, 633 (1928).