

(3) The moment of the double layer at the surface is relatively small, in agreement with the conclusions reached in reference I. This, of course, means that the ordinary electrostatic potential in the interior is small.

(4) Approximate agreement is obtained between the work function of the free-electron model and the experimental values of the work function of sodium.

The writer wishes to express his gratitude to Professor E. Wigner, under whose direction this work was carried out, and to Professor J. H. Van Vleck, Professor J. C. Slater, and Dr. F. Seitz for interesting discussions.

APPENDIX

Evaluation of the integral:

$$I = \int \int \frac{e^{i[(k_2-l_2)(y-y_1)+(k_3-l_3)(z-z_1)]}}{((\xi-\xi_1)^2+(y-y_1)^2+(z-z_1)^2)^{\frac{3}{2}}} dy_1 dz_1.$$

This integral is not absolutely convergent, and in evaluating it one must be careful to introduce polar coordinates,  $\theta$ ,  $\rho$  in place of  $y_1$  and  $z_1$  and then integrate first over  $\theta$ . If one chooses the origin of  $\theta$  properly, the integral takes the form:

$$I = \int_0^\infty \rho d\rho \int_{-\pi}^\pi \frac{e^{i\lambda\rho \cos \theta}}{((\xi-\xi_1)^2+\rho^2)^{\frac{3}{2}}} d\theta$$

with  $\lambda^2 = (k_2-l_2)^2 + (k_3-l_3)^2$ . The integration over  $\theta$  gives Bessel's function  $J_0(\lambda\rho)$ . We thus have:

$$I = 2\pi \int_0^\infty \frac{\rho J_0(\lambda\rho) d\rho}{((\xi-\xi_1)^2+\rho^2)^{\frac{3}{2}}} = (2\pi/\lambda) e^{-\lambda|\xi-\xi_1|}.$$

Eq. (22) follows immediately.

The Variation of Young's Modulus with Magnetization and Temperature in Nickel

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(Received March 3, 1936)

The relation between Young's modulus, magnetization, and temperature in annealed polycrystalline nickel, 99.7 percent pure, is determined for values of  $J$  between zero and saturation, and for values of  $T$  between 23.5°C and 400°C. The percent increase in Young's modulus is proportional to  $J^2$  between zero and about 0.4 saturation, at all temperatures below 311°C. The total increase in the modulus from the demagnetized to the saturated state is 6.7 percent at 23°, reaches a maximum of 18.7 percent at 185°, and decreases to zero at the Curie point. The results indicate that the theory of this phenomenon offered by

Akulov is probably essentially correct but requires modification. The method employed for the measurement of Young's modulus yields at the same time a measure of the coefficient of internal friction of the material. The internal friction decreases with increasing magnetization at all temperatures below the Curie point. In the demagnetized material it reaches a maximum value at the same temperature as does the change in elastic modulus. Its value at magnetic saturation is the same order of magnitude as that of a nonferromagnetic substance.

THE domain theory of ferromagnetism, first suggested by Webster in connection with magnetostriction and since developed by Heisenberg, Akulov and others, is noteworthy for its success in correlating different ferromagnetic phenomena in both single and polycrystalline materials. For example, Akulov<sup>1</sup> has shown that the change of Young's modulus with magnetization in a polycrystalline substance can be described solely in terms of the initial susceptibility and one magnetostrictive constant of a single crystal. The present paper is a report of an experimental investigation of the relation between Young's modulus and magnetization in annealed polycrystalline nickel at various tem-

peratures below the Curie point. The results are compared with the formulae of Akulov's theory.

EXPERIMENTAL METHOD

The research consists in ascertaining the value of the Young's modulus,  $E$ , corresponding to an intensity of magnetization,  $J$ , and a temperature,  $T$ , for values of  $J$  lying between zero and technical saturation, and values of  $T$  ranging from 20°C to 400°C. Accordingly the experimental method must permit the simultaneous measurement of  $E$ ,  $J$ , and  $T$ . To this end a preliminary determination is made of the relation between  $J$ ,  $T$  and the normal magnetic induction,  $B$ , so that in the subsequent procedure a knowledge of  $B$  and  $T$  suffices to fix the value of  $J$ .

<sup>1</sup> Akulov, *Zeits. f. Physik* **85**, 661 (1933).

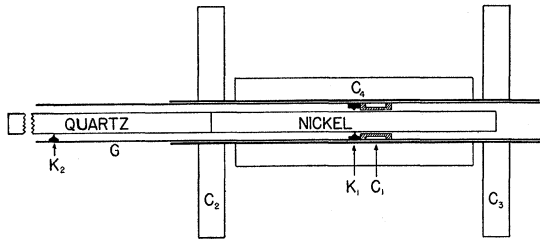


FIG. 1. Cross section of the assembly.

A specimen is prepared in the form of a right circular cylinder 0.464 cm in diameter and 7.2 cm long terminated by the end sections of a coaxial ellipsoid of revolution. The form approximates that of an ellipsoid of revolution whose major axis is 17.8 cm and minor axis is 0.464 cm. The specimen is mounted axially in a tubular electric furnace surrounded by a solenoid, and the variation of the normal magnetic induction with temperature and applied magnetizing field,  $H_0$ , is measured with a fluxmeter galvanometer. Corresponding values of  $J$  are then calculated with the formula  $J = (B - H_0)/(4\pi - N)$ , where  $N$  is the demagnetization factor of the ellipsoid, assumed the same as that of the specimen. The latter assumption is justified by the observed uniformity of  $B$  over the length of the cylinder, and by the fact that, since  $N = 0.028$ , an error in the first significant figure would alter the calculated value of  $J$  by less than 0.1 percent. After these measurements are completed the cylindrical part of the specimen is sawed out and its ends ground flat.

The method employed for the measurement of Young's modulus is a modification of that previously described in this journal by Zacharias.<sup>2</sup> The specimen cylinder of nickel forms one part of a composite piezoelectric oscillator constructed by cementing to one end a cylinder of crystalline quartz of square cross section equal in area to that of the specimen. The quartz cylinder is so cut from the crystal that the optic axis lies perpendicular to one pair of opposite faces and an electric axis to the other, and the latter pair is coated with gold leaf. An harmonically varying potential difference of con-

<sup>2</sup> J. Zacharias, Phys. Rev. **44**, 116 (1933). Other papers on the use of composite piezoelectric oscillators for the measurement of elastic constants are: L. Balamuth, Phys. Rev. **45**, 715 (1934); and F. C. Rose, Phys. Rev. **49**, 50 (1936).

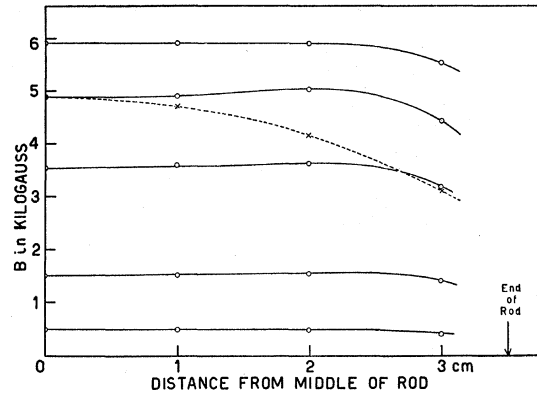


Fig. 2. Distribution of magnetic flux in the specimen. The dotted curve shows the distribution when the specimen is placed in a long solenoid.

stant amplitude is established between the gold leaf electrodes, and means are provided for measuring the amplitude of the resulting current. In consequence of the harmonically varying piezoelectric stress in the quartz, produced by the electric field between the electrodes, the entire oscillator assumes a state of forced longitudinal vibration, the frequency of which is the same as that of the applied potential difference.

The amplitude of the current which flows to the quartz varies critically with frequency in the neighborhood of certain "resonance frequencies" at which the amplitude of the vibration passes through a maximum. If  $f_0$  denote one of these frequencies, then at a frequency  $f'$  slightly less than  $f_0$  the current passes through a maximum, and at a frequency  $f''$  slightly greater than  $f_0$  it passes through a minimum. It is shown in Zacharias's paper that if  $I'$  and  $I''$  denote, respectively, the maximum and minimum values of the current amplitude, then  $f_0$  is given by the formula<sup>3</sup>

$$f_0 = f' + I''(f'' - f') / (I' + I''). \quad (1)$$

Furthermore, the fundamental frequency,  $f_1$ , of free longitudinal vibration of the nickel cylinder alone is related to  $f_0$  by the formula

$$m_1 f_1 \tan(\pi f_0 / f_1) + m_2 f_2 \tan(\pi f_0 / f_2) = 0, \quad (2)$$

where  $m_1$  and  $m_2$  are the masses, respectively, of the nickel and quartz cylinders, and  $f_2$  is the fundamental frequency of free longitudinal vibration of the quartz cylinder alone. The value of  $f_2$

<sup>3</sup> Misprints which occur in Eqs. (11) and (12) of Zacharias' paper are here corrected.

is obtained by observing the current which flows to the quartz without the specimen attached. Lastly, the quantity  $f_1$  is related to the Young's modulus of the material by the formula

$$E = 4l^2 f_1^2 \rho, \quad (3)$$

where  $l$  is the length of the specimen cylinder and  $\rho$  is its density.

This method for measuring Young's modulus yields at the same time the value of the logarithmic decrement,  $\delta$ , of free longitudinal vibration of the specimen cylinder. Thus

$$\delta = 2\pi(1 + m_2/m_1)\{(f'' - f_0)(f_0 - f')\}^{1/2}/f_0. \quad (4)$$

The coefficient of internal friction of the material is equal to  $\delta/E$ .

Errors introduced by the adhesive and by inequality of areas at the interface are minimized by adjusting the lengths of the cylinders so that  $f_1 = f_2$  within 10 percent. These frequencies lie in the neighborhood of 36 kilocycles.

Fig. 1 is a cross-sectional diagram of the assembly upon which the measurements are made. The composite oscillator is mounted in a thin glass tube,  $G$ , upon an annular duralumin knife-edge,  $K_1$ , and a straight knife-edge,  $K_2$ , located at displacement nodes of vibration. The temperature of the specimen is measured with a Chromel-P *vs.* Alumel thermocouple in contact with the knife-edge  $K_1$ . The magnetic induction is measured with a fluxmeter galvanometer connected to the coil  $C_1$ , constructed by winding 90 turns of No. 40 chromoxide covered copper wire on a thin cylinder of baked lavite. The magnetizing coils  $C_2$ ,  $C_3$ , and  $C_4$  are designed to secure as nearly as possible a uniform magnetization over the length of the specimen cylinder.  $C_2$  and  $C_3$  are each 92 turns of copper ribbon 0.023 cm thick and 0.635 cm wide, insulated with thin strips of mica.  $C_4$  is 340 turns of No. 20 chromoxide covered copper wire wound in 6 layers 5 cm long on a Pyrex tube 1 cm in diameter. The layers of this coil are also separated by mica sheet. The magnitudes and directions of the end and middle coil currents are so adjusted that  $B$  is uniform to 5 percent over eight-tenths the length of the specimen, at all temperatures and for all values of  $B$ . Typical distribution curves for  $B$  are shown in Fig. 2.

The assembly shown in Fig. 1 is placed in a rectangular duralumin oven, the interior dimensions of which are 7 in.  $\times$  7 in.  $\times$  12 in. The oven is heated electrically over its entire outer surface with grids formed by laying No. 22 constantan wire along parallel grooves cut in sheet transite. The outer thermal lagging is corrugated asbestos paper built up to a thickness of 1.75 in.

The voltage applied to the quartz crystal is supplied by a vacuum tube oscillator whose frequency can be varied in the neighborhood of  $f_0$  by means of a variable condenser. As noted above, the determination of  $E$  requires the measurement of the maximum and minimum piezoelectric oscillator current amplitudes, and the frequencies at which these values occur. In previous applications of the method, the recording device for measuring the currents has been a d'Arsonval galvanometer, used in connection with a vacuum thermocouple or other rectifier. In the present instance, however, the heat produced by the current in the magnetizing coils changes the temperature of the specimen too rapidly to permit the use of this instrument. The entire set of observations must be made within six seconds after the magnetizing field is applied. This is accomplished in the following manner:

The difference of potential across a non-reactive resistance in series with the crystal is amplified and rectified with a vacuum tube rectifier. The rectified voltage is, in turn, ampli-

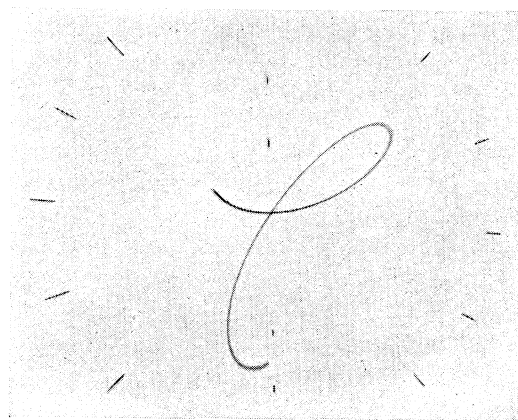


FIG. 3. Polar oscillogram of the piezoelectric oscillator current amplitude.

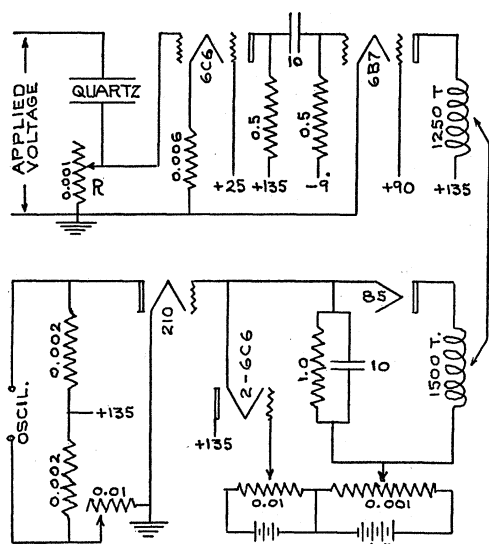


FIG. 4. The amplifying circuit. Resistances are in megohms and capacities in millimicrofarads.

fied and energizes a General Electric Company Type PM-11 supersensitive oscillograph element. The light beam from the oscillograph passes through a cylindrical lens and falls upon a sheet of bromide paper attached to a vertical circular disk. This disk is mounted centrally on one end of the horizontal shaft of the variable condenser which controls the exciting frequency. An electric motor rotates the condenser at a rate which carries the frequency through  $f'$ ,  $f_0$ , and  $f''$  in 4 seconds. During this time the light beam traces on the sensitized paper a polar diagram of the piezoelectric current amplitude, in which the angular coordinate is proportional to frequency and the radial coordinate to current. The condenser is frequency calibrated by comparison with the output of a piezoelectric clock,<sup>4</sup> and frequency calibration marks are placed on the record by illuminating it locally for an instant through short diametrically opposed radial slits.<sup>5</sup> A typical oscillogram is reproduced in Fig. 3.

Fig. 4 is a diagram of the amplifying circuits employed. The rectified voltage corresponding to an oscillator current amplitude between  $I'$  and  $I''$

<sup>4</sup> Quimby, Phys. Rev. 39, 345 (1932).

<sup>5</sup> It should, perhaps, be noted that this polar oscillograph is made fully automatic in operation by employing the various features of an RCA model 86 remote control radio unit. The different station selector contacts serve to start and stop the motor, flash the light source, open the source slit, etc.

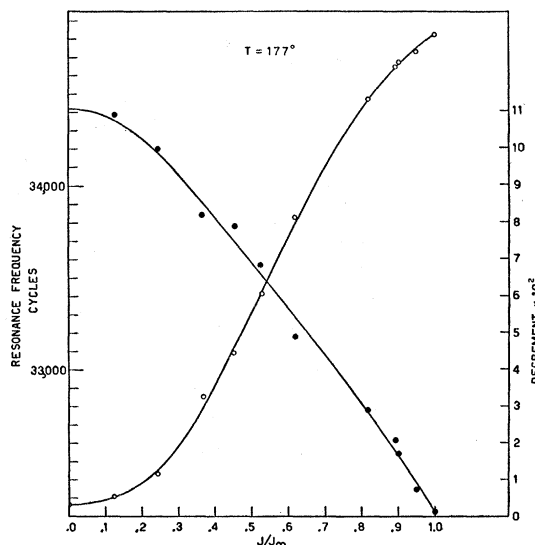


FIG. 5. Curves showing the resonance frequency (circles) and decrement (dots) of the piezoelectric oscillator as functions of magnetization at 177°C.

is balanced out with the 10,000-ohm potentiometer, so that only the fluctuation associated with  $I'$  and  $I''$  is recorded by the oscillograph. The deflection of the oscillograph mirror varies linearly with the amplitude of the potential difference across the resistance,  $R$ , in series with the oscillator, and so with that of the current. Furthermore, the current is independent of the magnitude of  $R$ , since the latter is small in comparison with the impedance of the oscillator. Accordingly each oscillogram is current calibrated by noting the values of  $R$  required to duplicate the deflections corresponding, respectively, to  $I'$  and  $I''$ , the current being held constant at the intermediate value. The actual working formula for the calculation of  $f_0$  is thus, from Eq. (1),

$$f_0 = f' + R''(f'' - f') / (R' + R''), \quad (5)$$

where  $R'$  and  $R''$  correspond to the deflections associated, respectively, with  $I'$  and  $I''$ .

The experimental procedure is as follows: The temperature of the oven is stabilized, the specimen is placed in the desired state of magnetization on the normal induction curve by repeated reversals of the magnetizing current, and the resonance current record is taken immediately. The flux density,  $B$ , is then measured, the oven is allowed to resume its former equilibrium

temperature, and the process is repeated for various values of  $J$ . The oscillograph records are reduced with the aid of Eqs. (5) and (4), and curves are plotted, of which those shown in Fig. 5 are typical, showing  $f_0$  and  $\delta$  as functions of  $J/J_\infty$ , where  $J_\infty$  denotes the saturation magnetization at the temperature of observation. Lastly, isothermals of this sort are obtained at various temperatures.

RESULTS

The specimen material upon which the observations here reported were made is Mond nickel containing 0.295 percent impurity distributed as follows:<sup>6</sup> Co, 0.00; Si, 0.01; Cu, 0.02; Fe, 0.11; Mn, 0.00; C, 0.049; Mg, 0.11; S, 0.005. The specimen was annealed at 1100°C for 4 hours in hydrogen at atmospheric pressure and allowed to cool to room temperature in 18 hours. Microscopic examination reveals it to be an array of crystals whose average linear dimensions are 0.4 mm. The coercive force is 0.3 oersted, and the remanent intensity of magnetization is in the neighborhood of 9 gauss. The Curie point is 353°C, as determined to within one degree by the magnetic measurements and by the change of Young's modulus with magnetization and temperature. The variation of  $J_\infty$  with temperature is given by the first two columns of Table I.

The variation of Young's modulus with magnetization and temperature is shown by the curves of Fig. 6. The points on these curves were calculated with Eqs. (2) and (3), and values of  $f_0$  secured from smooth curves of the sort shown in Fig. 5.

Table II gives the variation with magnetization and temperature of the quantity  $\Delta E/E$  defined by the formula,  $\Delta E/E = (E_J - E_0)/E_0$ .

TABLE I. Variation of  $J_\infty$  with temperature.

$T$	$J_\infty$	$E_0 \times 10^{-11}$	$T$	$J_\infty$	$E_0 \times 10^{-11}$
23.5	485	20.82	270	332	18.54
70.0	472	20.09	311	260	19.10
106	459	19.26	340	162	19.63
147	440	18.37	353	39	19.79
177	422	17.93	365	2	19.70
197	408	17.81	369	1	19.68
200	405	17.78	385	—	19.57
217	371	17.85	397	—	19.49

<sup>6</sup> This analysis, together with the specimen material itself, was furnished by the International Nickel Company.

$E_J$  and  $E_0$  denote the values of Young's modulus when the value of the magnetization is, respectively,  $J$  and zero.  $E_0$  is given as a function of the temperature in the third column of Table I.

The variation of the logarithmic decrement with magnetization and temperature is shown by the curves of Fig. 7.

ACCURACY OF THE MEASUREMENTS

Temperature

The Chromel-P vs. Alumel thermocouple was calibrated before and after the measurements against the melting points of ice, Sn, Pb and Zn. The two calibrations agreed to 0.25°C. The temperature variation *in the oven* over the region occupied by the specimen is less than 0.3°C. That along the specimen is, of course, considerably less. The temperature of the thermocouple in contact with the knife-edge (Fig. 1) agrees with that of the specimen to better than 0.1°C. Accordingly the error in the temperature measurement is not greater than 0.5°C.

Magnetic

The measurement (on the approximate ellipsoid) of the saturation magnetization is accurate to 1 percent. Values of  $J$  between zero and saturation are reliable to 2 percent, and errors arising from this uncertainty are reduced by the smoothing process represented by the curves of Fig. 5.

Elastic

The relation (Eq. (3)) between  $E$  and  $f_1$  involves the length and density of the specimen cylinder at the temperature of observation. These are evaluated with accuracies of 0.02 and 0.05 percent, respectively, from measurements

TABLE II.  $(E_J - E_0)/E_0$ .

$T$	$J/J_\infty$	0.2	0.4	0.6	0.8	1.0
23.5		0.0040	0.015	0.035	0.055	0.067
70.0		.0065	.025	.052	.077	.096
106		.0088	.033	.071	.109	.131
147		.0109	.048	.094	.136	.170
177		.0084	.046	.106	.156	.186
197		.013	.053	.117	.163	.186
200		.013	.055	.116	.161	.188
217		.013	.053	.111	.152	.176
270		.0092	.039	.073	.098	.109
311		.0063	.023	.046	.055	.058
340		.0025	.0061	.011	.013	.016

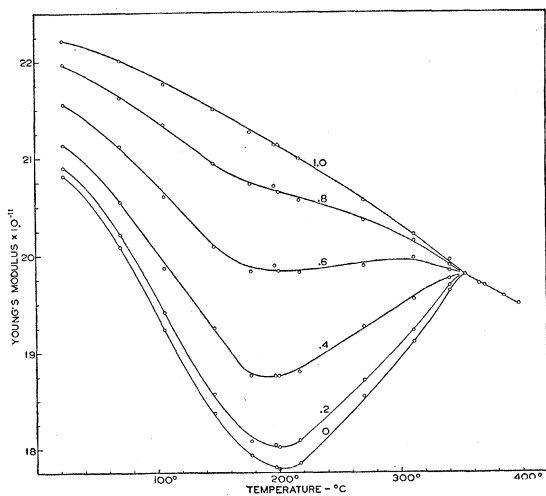


FIG. 6. Curves showing the variation of Young's modulus with magnetization and temperature in nickel. The numbers on the curves give the magnetization in fractions of the saturation value.

made at 30°C, together with values of the coefficient of thermal expansion of nickel reported by Williams.<sup>7</sup>

A possible source of error in the measurement of Young's modulus lies in the fact that about ten percent of the length of the specimen cylinder at each end is in a lower state of magnetization than that observed at its center. The maximum effect of the consequent elastic inhomogeneity on  $f_1$  is readily obtained by an analysis similar to that which leads to Eq. (2). It follows that if the value of  $E$  over the end sections is assumed to be 0.9 the value over the central eight-tenths of the cylinder, then the resulting error in the calculated value of  $E$  is only 0.25 percent. This is merely a consequence of the fact that the fundamental frequency of free longitudinal vibration of a rod is principally determined by the *inertia* of its terminal parts and the *elasticity* of its central part. If the above-mentioned inhomogeneity occurred at the center of the cylinder the resulting error in  $E$  would be 4.2 percent.

The major part of the uncertainty in  $E$  arises from the flatness of the maxima and minima shown on the oscillograms when the decrement,  $\delta$ , is large. The error thus introduced into the determination of  $f'$  and  $f''$  can affect the calculated value of  $E$  by as much as 0.2 percent.

<sup>7</sup> Williams, Phys. Rev. **46**, 1011 (1934).

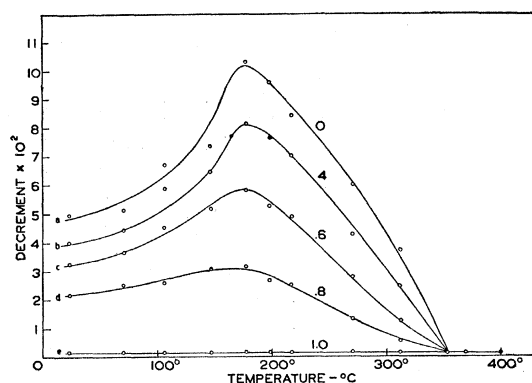


FIG. 7. Curves showing the variation of logarithmic decrement with magnetization and temperature in nickel. The numbers on the curves give the magnetization in fractions of the saturation value.

Finally, the reproducibility of the results is indicated by the observations at 197° and 200° as plotted in Fig. 6. The second set were taken a month later than the first. As a further test, the oscillator was demounted, disassembled, reassembled, and remounted, and a new set of data was taken with the specimen demagnetized and magnetized to saturation. The lowest and highest curves of Fig. 6 were reproduced with maximum accidental errors in  $E$  of 0.3 percent and 0.16 percent, respectively.

The values of  $\delta$  obtained by the present method are reliable to 10 percent.

#### THEORETICAL DISCUSSION

The domain theory of ferromagnetism is based on the assumption that a ferromagnetic crystal is composed of a large number of regions, each of which is spontaneously magnetized to an intensity which is a function of the temperature alone. When the holomagnetization is zero these micromagnetizations are aligned in equal numbers along the various directions of "easy magnetization" of the crystal. The effect of a magnetizing field is, in the first instance, simply to alter this uniform distribution, so that a preponderant number of micromagnetizations lie in that direction of easy magnetization which most nearly coincides with the field. When this process is complete a further increase in the field strength produces a rotation of the micromagnetization of each domain into the direction of the field.

Each domain is deformed in a manner which

depends upon the direction of its magnetization. Consequently any alteration in the directional distribution of the micromagnetizations throughout the crystal is accompanied by a deformation of the crystal as a whole. When the alteration is produced, as above, by a magnetizing field alone, the resultant deformation constitutes the ordinary phenomenon of magnetostriction.

Now it is well known that the holomagnetization of a ferromagnetic body in a given magnetizing field is, in general, altered by the application of a stress, and furthermore, that *the magnitude of this alteration depends upon the magnetization*, being nil when the substance is magnetized to saturation. In terms of the domain theory this means that a given applied stress alters the distribution of the micromagnetizations, and hence the linear dimensions of the body, by an amount which depends upon the holomagnetization. This is the qualitative explanation of the phenomenon dealt with in this paper.

The problem is attacked quantitatively by Akulov,<sup>1</sup> who employs the analysis invented by Heisenberg<sup>8</sup> to resolve the simpler problem of single crystal magnetostriction. The procedure is simply to calculate the most probable distribution of the micromagnetizations for a given applied field and stress, subject to the conditions that the total number of domains and the quantity  $U$  are constant, where  $U$  is defined by the formula

$$U = -(H_1J_1 + H_2J_2 + H_3J_3) - F\lambda.$$

In this expression  $H_i$  and  $J_i$  are the components of the field and magnetization respectively,  $F$  is the applied force per unit area, and  $\lambda$  is that part of the change in length per unit length parallel to  $F$  which depends upon the distribution of micromagnetizations. The calculation yields the distribution in terms of  $H$ ,  $J$ ,  $F$ , and the magnetostrictive coefficient of a domain for a direction of easy magnetization.  $\lambda$  is evaluated in terms of the same quantities by summation over the array of domains, and from this the variation of the elastic modulus with magnetization. The behavior of a polycrystalline material is obtained by averaging over all directions in the crystal.

The calculation cannot be completed without

<sup>8</sup> Heisenberg, *Zeits. f. Physik* **69**, 287 (1931).

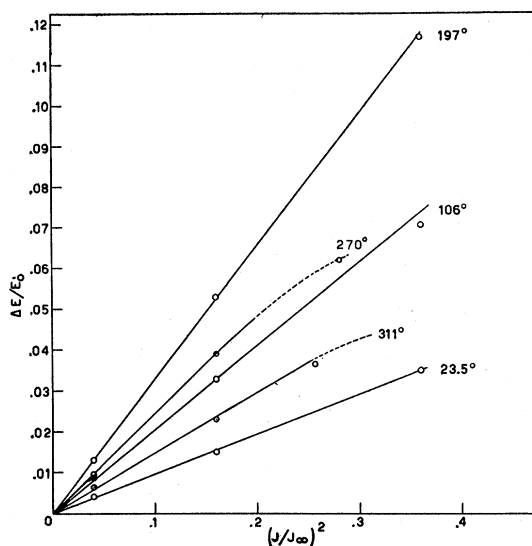


Fig. 8.  $\Delta E/E_0$  is proportional to the square of the magnetization for values of  $J$  up to 0.4 saturation.

approximation. Akulov obtains two formulae, one of which is valid when the applied field is small, and the other when the field is sufficiently large that the effect of stress on the distribution is nil. Thus, for a polycrystalline specimen of nickel in which the distribution of microcrystalline axes is random,

$$\frac{E_J - E_0}{E_0} = \frac{81}{35} \frac{\chi_0}{J_\infty^2} \lambda_{111}^2 (J/J_\infty)^2, \quad (6)$$

provided  $J/J_\infty$  is small, while if  $J = J_\infty$ ,

$$\frac{E_\infty - E_0}{E_0} = \frac{9\chi_0\lambda_{111}^2 E_0 / 15J_\infty^2}{1 - 9\chi_0\lambda_{111}^2 E_0 / 15J_\infty^2}, \quad (7)$$

where  $E_\infty$  denotes the value of Young's modulus when the material is magnetically saturated. In arriving at these expressions  $H$  has been expressed in terms of  $J$  by the formula,  $J = \chi_0 H$ . Hence  $\chi_0$  is the initial magnetic susceptibility of a single crystal of nickel.  $\lambda_{111}$  is the saturation magnetostrictive coefficient for the direction of easy magnetization in the crystal. The measurements of Masiyama<sup>9</sup> give  $\lambda_{111} = 27 \times 10^{-6}$  and those of Becker and Kersten<sup>10</sup> give  $\chi_0 = 20$ , both at room temperature. These formulae and data will now be compared with the results of the present research.

<sup>9</sup> Masiyama, *Tohoku Imp. Univ. Sci. Rep.* **17**, 945 (1928).

<sup>10</sup> Becker and Kersten, *Zeits. f. Physik* **64**, 660 (1930).

The straight lines shown in Fig. 8 indicate that, in accordance with Eq. (6),  $(E_J - E_0)/E_0$  varies linearly with  $(J/J_\infty)^2$  up to temperatures at least as high as 311°C, and for values of  $J/J_\infty$  as large as 0.4.

The calculated value of the coefficient of  $(J/J_\infty)^2$  in Eq. (6) is 0.30. The observed value at 23.5°C is 0.10. The value of  $(E_\infty - E_0)/E_0$  calculated from Eq. (7) is 0.085. The observed value at the same temperature is 0.067. An approximate test of the correctness of Eqs. (6) and (7), independent of the values of  $\chi_0$ ,  $\lambda_{111}$  and  $E$  can be obtained by noting that the value of the numerator on the right-hand side of Eq. (7) is only 0.078. Hence the ratio of this quantity to the coefficient of  $(J/J_\infty)^2$  in Eq. (6) should equal 35/135, or 0.26, with an accuracy of 8.5 percent whatever the value of  $\chi_0$ ,  $\lambda_{111}$ , and  $E$ . The observed ratio is 0.67.

These discrepancies cannot be ascribed to a nonrandom distribution of microcrystals in the specimen. The possible values of the ratio noted in the preceding paragraph range from 0.20 for a (111) direction in a single crystal to 0.33 for a (110) direction. Nor can they be ascribed to the neglected effect of temperature, for it is observed that the ratio decreases with *increasing* temperature from the value 0.67 at 23.5°C to the value 0.39 at 311°C.

von Auwers<sup>11</sup> has investigated the  $\Delta E$  effect in nickel at room temperature by a method different from that here employed. He obtains the values 0.30 and 0.43 for the ratio, as observed on specimens characterized, respectively, as "soft" and "very soft."

In conclusion, therefore, it is evident that, while the domain theory is probably essentially correct, the greatly simplified calculation of Akulov requires emendation of such nature as to alter at least the numerical coefficients in Eqs. (6) and (7).

The interpretation of the decrement measurements awaits the development of a theory of internal friction in ferromagnetic substances. It may be noted that the observed decrement of the magnetically saturated nickel (curve *e* of Fig. 7) is the same order of magnitude as that of non-ferromagnetic materials.

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<sup>11</sup> von Auwers, *Ann. d. Physik* **17**, 83 (1933).

## Photoelectric Properties of Sodium Films on Aluminum

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Previous work has shown that thin films of potassium, rubidium, and caesium deposited on a silver surface give a maximum sensitivity when the thickness of the film is of the order of ten molecular layers or less. As the deposit continues the sensitivity decreases before reaching a constant value. Sodium films were deposited on aluminum, and it was found that the maximum sensitivity was not attained until the thickness of the film had reached a value of eighty molecular layers. The sensitivity was then found to remain constant for greater thicknesses. No detectable photo-current was observed until the film reached a thickness of five molecular layers. Spectral distribution curves were obtained and analyzed according to Fowler's theory. This yielded a value of 5150Å for the threshold of the

sodium film. These observations indicated that the threshold was the same for all thicknesses of film. Current-voltage curves were obtained for different thicknesses of film, and these were analyzed according to DuBridge's theory. These results also indicate that the work function of the various films is the same. It was concluded from these data that the work function of the freshly distilled aluminum on which the deposits were made was 4.08 volts. This gives a value of 3020Å for the threshold of aluminum. It was found that the photoelectric current decreased immediately after forming a film, but that the decrease was much less for films greater than fifty molecular layers. It is suggested that this decrease results from the surface migration of the sodium atoms in forming aggregates.



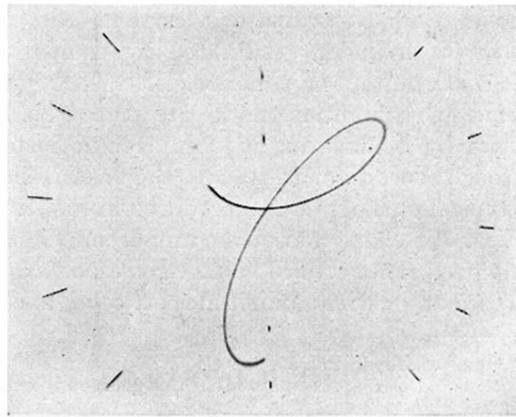


FIG. 3. Polar oscillogram of the piezoelectric oscillator current amplitude.