equipotential lines are circular, and remain quite uniform from one element to the next. Since a conductor is sheathed by an equipotential surface when in an electrostatic field, one of the equipotentials may be replaced by the physical surfaces of grid wires, as is common practice.

For computation the expression

$$1/\Gamma(Z) = Z + C_2 Z^2 + C_3 Z^3 + \cdots$$

is useful. L. Bourguet<sup>5</sup> has given twenty-three

coefficients of this series to sixteen places. The gamma-function of a complex argument has been tabulated by H. T. Davis.<sup>6</sup> Several other tables of the gamma and polygamma-functions are also in existence.

The author's best thanks are extended to Dr. M. E. Haller for checking the work here given.

<sup>5</sup> L. Bourguet, Acta Mathematica 2, 289 (1883).

<sup>6</sup> H. T. Davis, Tables of the Higher Mathematical Functions.

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# III. <sup>2</sup>S<sup>2</sup>P Multiplets in Strong Fields

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> The Paschen-Back effect of  ${}^2S^2P$  multiplets has been discussed and the theory verified in detail. In particular, the  ${}^{2}S^{2}P$  multiplet of Li,  $\lambda 6708$ , has been resolved in reversal and shows the fine structure predicted but not heretofore observed.

**`**HE effect of strong magnetic fields on  ${}^{2}S^{2}P$ multiplets was first investigated theoretically by Voigt<sup>1</sup> on the basis of the classical theory. His predictions and formulae agreed in most essentials with those developed later by Heisenberg and Jordan<sup>2</sup> and by Darwin<sup>3</sup> according to the quantum mechanics.

We usually hear the statement that "for very strong magnetic fields, a multiplet exhibits the normal Zeeman effect." This statement is not accurate, in that the word "strong" is not sufficiently definite. So far, it has not been possible to produce magnetic fields that would be strong enough to bring about this condition. We should require fields so strong that the LScoupling would be completely broken down. For field strengths ordinarily obtainable in the laboratory this condition is not even approximately approached, except for LS structure constants that cannot be resolved with our best attainable resolving power, and then the argument loses its potency.

An examination of the formulae for the positions of Zeeman components shows that in strong fields where the normal Lorentz separation is very large compared with the LS separation these positions are given by the difference between two terms like

$$W = \omega H(m_l + 2m_s) + am_l m_s$$

where  $\omega = eh/4\pi m_0 c$  and a is the fine-structure constant. Thus, in very strong fields we should, indeed, have a normal Zeeman pattern but instead of having single lines for the components each of these is split into a number of components equal to the multiplicity of the electronic levels.

This statement has been tacitly accepted for a long time but has not yet been subjected to experimental verification. To be sure, for hyperfine structure the experimental status of the Paschen-Back effect is definitive and has yielded many fruitful results. But there is this difference between the technique in the two cases. Some heavy atoms show small hyperfine structure separations, and the Doppler effect is sufficiently small so that the resulting components are quite sharp; while for fine structure it is necessary to use very light atoms in order to have small enough fine-

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<sup>&</sup>lt;sup>1</sup> Voigt, Ann. d. Physik **40**, 368 (1913); **41**, 403 (1913); **42**, 210 (1913). <sup>2</sup> Heisenberg and Jordan, Zeits. f. Physik **37**, 263 (1926).

<sup>&</sup>lt;sup>3</sup> Darwin, Proc. Roy. Soc. A115, 1 (1927).

structure constants, and then the Doppler effect causes the lines to broaden out and the fine structure is unresolvable.

A study of Kent's work<sup>4</sup> on the red lithium line  $\lambda 6708$  brings out this last point quite clearly. For field strengths up to 18,000 gauss, a Geissler discharge was used and studied with an echelon grating. The experimental results were in very good accord with the theory.<sup>5</sup> Higher field strengths, using a vacuum arc, showed the lines to be quite broad. At the time, the peculiar behavior of this line in a magnetic field caused considerable discussion, but its importance has disappeared in the limbo of rapid progress in spectroscopy and the theoretical results have been tacitly accepted. It had not been thought worth while to pursue the point. Indeed, the matter has not been touched experimentally for more than twenty years, although the theory has since been put on a sounder quantum-mechanical basis. For the sake of completeness, we have endeavored to satisfy this one point.

The work of Back<sup>6</sup> on emission lines at strong fields and of Hansen<sup>7</sup> on the inverse effect (absorption) showed that there was little chance of success of resolving the perpendicular components in any of the ordinary ways, even though Zeeman in a private communication to Voigt<sup>1</sup> indicated that "these components were broader than the central component (parallel polarization) and appeared sometimes to be double, although on this last point, he could not be sure."

With the use of a vacuum chamber of design very similar to that of Back,<sup>8</sup> we studied the Li line in the first order (dispersion 1.32 A/mm) of the 21-foot concave grating. First trials, with LiCl and LiO deposited on the anode strip (brass), yielded only the fuzzy components found by Back and by Kent. But one exposure, run at a somewhat higher current, indicated signs of reversal. This suggested the idea that if we could have large quantities of vapor present, we might be able to get the effect for which we were look-



FIG. 1. (left) Cu doublet  ${}^{2}S^{2}P$ ,  $\lambda\lambda 3247$ , 3274,  $\omega \doteq a/100$ , FIG. 2. (center) Be II doublet  ${}^{2}S^{2}P$ ,  $\lambda\lambda 3131$ ,  $\omega \doteq a/2$ . FIG. 3. (right) Li doublet  ${}^{2}S^{2}P$ ,  $\lambda 6708$ ,  $\omega \doteq 8a$ , Paschen-Back effect in reversal.

ing, but in reversal. We therefore used lithium metal packed into slots in the brass anode and ran the arc with a tungsten cathode at about 3 amperes average current, about twice the normal operating value. Eastman Panatomic film was used, because the separation sought was only 0.075 mm (approaching the resolving power of the instrument), and our first attempt met with success. Each of the perpendicular components showed two absorption lines against the fuzzy background, while the parallel component showed only one. The short wave-length component showed this more clearly than the long wave-length component, and the cause of this was at first not quite clear. But an accurate calculation of the separations of these two doublets shows that even at the field strength used,  $\omega \doteq 8a$ , the doublet on the short side should be ten percent wider than that on the long side, instead of both of them being equal to 2/3 a, the value for extremely large fields. (See Fig. 3.)

The effect of increasing the field strength on a  ${}^{2}S^{2}P$  multiplet is shown by means of the above

<sup>&</sup>lt;sup>4</sup> Kent, Astrophys. J. 40, 337 (1914); Physik. Zeits. 15, 383 (1914).

<sup>&</sup>lt;sup>5</sup> For a complete discussion of this, see Condon and Shortley, Atomic Spectra (Cambridge Univ. Press). <sup>6</sup> Back, Ann. d. Physik **39**, 929 (1912).

<sup>&</sup>lt;sup>7</sup> Hansen, Ann. d. Physik 43, 252 (1914).

<sup>&</sup>lt;sup>8</sup> Back, Ann. d. Physik 70, 333 (1923).

three cuts. Instead of using the same multiplet and varying the field strength, the same field strength (about 38,000 g) was used and the multiplet separation was varied. Thus Fig. 1 represents the Cu multiplet  $\lambda\lambda 3247$ , 3274,  $\omega \doteq 1/100a$ . The comparison with the theoretical intensities shows that the pattern is normal in spacing and intensity. (The parallel polarized components appear weaker than the perpendicular, although they should be stronger. This is caused in large measure by the slit and rulings of the grating, which are set perpendicular to the magnetic field and show a preference for polarizations in that direction. No method has yet been devised for the comparison of intensities of differently polarized lines using the ordinary concave grating set-up.)

Fig. 2 shows the Be II doublet  $\lambda$ 3131,  $\omega \doteq 1/2a$ ,

first studied by Popow<sup>9</sup> and classified correctly by him. Asymmetries in intensity and position begin to be apparent.

Fig. 3 shows the Li line  $\lambda 6708$ ,  $\omega \doteq 8a$ . The effect is shown in reversal; all previous symmetry is lost and the line has reverted to the "normal" triplet with fine structure, as predicted by the theory.

In all three figures the microphotometer trace is shown at the bottom. Then the calculated pattern, and finally, an enlargement of the original negative. The enlargement of Fig. 3 shows a faint line on the red side of the parallel component. This was noted by Kent<sup>4</sup> and has been attributed by Schüler and Wurm<sup>10</sup> to Li<sup>6</sup>, the weaker isotope of lithium.

<sup>9</sup> Popow, Physik. Zeits. 15, 756 (1914).
<sup>10</sup> Schüler and Wurm, Naturwiss. 15, 971 (1927).

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### IV. Mutual Spin-Orbit Interaction in Two-Electron Spectra

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The effect of a strong magnetic field on the Be triplet  $\lambda\lambda 3321$ ,  ${}^{3}P^{3}S$  has been studied. The experimental patterns have been compared with calculations which take account of the interaction between the spin of one electron and the orbit of the other. The experimental results are in excellent agreement with the theory, but in some disagreement with Back's earlier work on this group of lines.

\*HE Paschen-Back effect in two-electron spectra, one electron in an s state, has been discussed and verified for the cases of LS coupling,<sup>1</sup> calculated by Darwin's<sup>2</sup> method, and *jj* coupling,<sup>3</sup> calculated from Houston's<sup>4</sup> method. Darwin's calculations are essentially the extension of a single-electron calculation assuming the Landé interval rule, while Houston's calculations consider only the effects of electrostatic interaction between the electrons and of the magnetic

<sup>&</sup>lt;sup>1</sup> Green and Gray, Phys. Rev. 45, 273 (1934).

 <sup>&</sup>lt;sup>2</sup> Darwin, Proc. Roy. Soc. A115, 1 (1927).
<sup>3</sup> Green and Loring, Phys. Rev. 46, 888 (1934).
<sup>4</sup> Houston, Phys. Rev. 33, 297 (1929).

interaction between the spin and orbit of each electron. The magnetic interaction of the spin of one electron with the orbit of the other was neglected, as being of order 1/Z, compared with the latter interaction term. It has been pointed out in an earlier paper<sup>5</sup> that this term could not be neglected for the lighter elements where Z is small, nor for higher values of l (where  $Z_{eff}$  is small). An attack on the problem to include this other interaction has been made by Wolfe.6 Because of the difficulty of calculation of the

<sup>&</sup>lt;sup>5</sup> Green and Loring, Phys. Rev. 38, 1289 (1931).

<sup>&</sup>lt;sup>6</sup> Wolfe, Phys. Rev. 41, 443 (1932).



FIG. 1. (left) Cu doublet  ${}^{2}S^{2}P$ ,  $\lambda\lambda 3247$ , 3274,  $\omega \doteq a/100$ . FIG. 2. (center) Be II doublet  ${}^{2}S^{2}P$ ,  $\lambda\lambda 3131$ ,  $\omega \doteq a/2$ . FIG. 3. (right) Li doublet  ${}^{2}S^{2}P$ ,  $\lambda 6708$ ,  $\omega \doteq 8a$ , Paschen-Back effect in reversal.