where

$$\eta = (a^2 - r^2)^{\frac{1}{2}} \text{ if } r < a;\ \eta = 0 \text{ if } r > a. \tag{14}$$

We include the case $r > a$ in anticipation of the discussion of the passage of electrons near to but not through the entity. Now

$$\begin{aligned} H_x' &= -p[(x^2+y^2+z^2)^{-3/2} - 3x^2(x^2+y^2+z^2)^{-5/2}] \\ &= -(p/r^3)[(1+z^2/r^2)^{-3/2} - (3x^2/r^2)(1+z^2/r^2)^{-5/2}]. \end{aligned} \tag{15}$$

If we substitute this expression in (13) and put $\tan\theta = z/r$, so that $\sec^2\theta = 1 + z^2/r^2$ and $dz/r = \sec^2\theta d\theta$; we obtain

$$\pi a^2 J_1 = -2p\int_0^a \frac{dr}{r}\int_0^{2\pi}\left[\int_{\theta_0}^{\pi/2}\cos\theta d\theta - \frac{3x^2}{r^2}\int_{\theta_0}^{\pi/2}\cos^3\theta d\theta\right]d\varphi. \tag{16}$$

The limit θ_0 corresponds to η. From (14) we see that

$$\theta_0 = \sin^{-1}(1 - r^2/a^2)^{\frac{1}{2}} \text{ if } r < a;\ \theta_0 = 0 \text{ if } r > a. \tag{17}$$

Thus

$$\pi a^2 J_1 = -2p\int_0^a \frac{dr}{r}\int_0^{2\pi}\left[1 - \frac{2x^2}{r^2} - \left(1 - \frac{3x^2}{r^2}\right)\sin\theta_0 - \frac{x^2}{r^2}\sin^3\theta_0\right]d\varphi. \tag{18}$$

If x were replaced by y, the φ integral in (18) would be the same as before. Hence we may replace x^2 by $(x^2+y^2)/2 = r^2/2$, and the integral will be unaltered. By doing this

$$J_1 = 2\frac{p}{a^2}\int_0^a (\sin^3\theta_0 - \sin\theta_0)\frac{dr}{r}. \tag{19}$$

Using the expressions for θ_0 given above, for $r < a$ we find

$$J_1 = -2p/3a^2. \tag{20}$$

Now with regard to the integral J_2, we observe that the field inside the sphere, and resulting from the current, is $2p/a^3$. Thus

$$\begin{aligned} \pi a^2 J_2 &= \frac{2p}{a^3}\int_0^a dr\int_0^{2\pi} r d\varphi\int_{-\eta}^{\eta} dz = \frac{8\pi p}{a^3}\int_0^a (a^2 - r^2)^{\frac{1}{2}} r dr, \\ J_2 &= 8p/3a^2. \end{aligned} \tag{21}$$

Eqs. (15) and (16) establish the results quoted above, and lead to $J = 2p/a^2$.

Problem 3. Calculation of the average contributions of an individual entity to the line integral of H_x along a path which does not thread it

This case corresponds to $r > a$. The calculation proceeds exactly as for J_1 except that, in (18) and the following equations, the integral with respect to r extended from 0 to ∞, and the limit θ_0 is zero, since $r > a$. Thus, (19) becomes replaced by

$$J_2 = \frac{2p}{a^2}\int_0^\infty (\sin^3\theta_0 - \sin\theta_0)\frac{dr}{r}$$

where θ_0 is zero. Hence $J_2 = 0$.

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The Deflection of Cosmic-Ray Charged Particles in Passing Through Magnetized Iron

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Experiments are described in which the deflections suffered by cosmic-ray electrons in passing through the saturated iron core of an electromagnet are detected by means of Geiger counters. The observed effects are compared with the results of calculations in which we have used the energy distribution as found by Anderson and Neddermeyer with the cloud chamber. In this way we have found it possible to set limits to the effective magnetic vector within the iron. One of our experiments indicates a value lying between the induction B and $B/2$. The other points to the limits $3B/4$ and $B/4$. A theoretical discussion is included in which it is pointed out that all electrons of the same energy will not necessarily experience the same deflection but will show a statistical distribution of deflections with an arithmetic average corresponding to the induction B. The present type of experiment, however, does not give a true arithmetic average and would be expected to indicate, for the effective deflecting vector, a quantity less than B to an extent dependent upon the particular geometrical arrangement.

Introduction

The negative results obtained by B. Rossi[1] and by L. M. Mott-Smith[2] in their attempts to realize deflection of cosmic-ray electrons in magnetized iron has excited speculation as to the correctness of using the induction B rather than the magnetic intensity h as the vector determining the force which such an electron experiences. Indeed, Rossi's first experiment gave magnetic deflections no more than comparable with the experimental error. His second experiment[3] did result in a small effect, which, however,

[1] B. Rossi Accad. Lincei, Atti **11**, 478 (1930).

[2] L. M. Mott-Smith, Phys. Rev. **39**, 403 (1932).

[3] B. Rossi, Nature **128**, 300 (1931).

he interpreted as insignificant compared with results which would have been obtained had B been fully operative.

In February 1934, L. Alvarez[4] published results of an attempt to deflect β-particles as they passed through a thin (0.36 mm) sheet of magnetized iron. The deflections which he observed were hardly larger than would be produced by a field one-tenth as large as the induction B.

Now that the cloud chamber has provided us with data concerning cosmic-ray energies one should be able to realize a nonambiguous solution of the problem. If it should be true that the appropriate vector were not B but something smaller, a thorough reexamination of molecular magnetic theory would be in order.

We reported the result of one experiment at the New York meeting of the American Physical Society (Feb. 23–24, 1934). The effect observed was rather smaller than was calculated using the vector B and the cloud chamber energy determinations made up to that time by C. D. Anderson[5] and P. Kunze.[6] These energy determinations were measurements made on photographs taken at random. The more recent work of Anderson and Neddermeyer[7] with a Geiger counter controlled cloud chamber is more applicable to our purpose since it does not include shower particles. The purpose of this paper is, therefore, to describe our magnetic deflection experiments and to analyze their results in terms of the most recent energy distribution with the intention of thereby determining (or at least setting limits to) the effective magnetic field within iron. In its simplest form the problem is one in which all rays of the same energy experience the same deflection. On such a view of the problem there yet remain many geometrical and instrumental considerations which become involved in the interpretation of the relation between the data and the magnitude of the magnetic vector involved, but with these matters taken into account it is possible to calculate a magnitude for the vector in question. The results of the investigation will first be presented with this end in view, in Section A, as it will then be possible to compare them with the results of others.

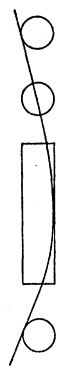
The theoretical considerations pertaining to the passage of electrons through magnetized iron have recently been examined by one of us[8] and are published in this issue. These considerations are digested in Section B of the present paper. It appears that the difference between the magnetic induction B and the ordinary macroscopically defined magnetic field h arises by the contribution of the fields inside the necessary vector entities responsible for the polarization. On account of this fact, it results that, in the case of magnetic entities of small size, situations may arise in which, even for a fixed electron energy, a distribution of deflections may result. Such a distribution alters the interpretation of the final data, and leads to a condition in which the realization of a vector smaller than B by the more naïve calculations is not necessarily inconsistent with strict obedience of the experiment to the electromagnetic theory which demands that the true force on an electron at any point shall be determined by a vector (the true field at a point, submacroscopically considered), whose average value throughout the macroscopic element of volume is the ordinary induction B.

Section A. Apparatus and Experimental Results

The apparatus is represented schematically in Fig. 1. Three Geiger-Müller counters shown with their length perpendicular to the paper, are arranged in a vertical plane and with vertical line of centers. Between the lower two is placed a slab of iron which may be magnetized in a direction normal to the plane of the diagram. One measures the frequency of coincident discharges in all three counters. Magnetization of the iron should, by deflecting the rays before they strike the bottom counter, diminish this counting rate. The magnitude and significance of the diminution thus obtained is the object of study of this paper.

To increase the counting rate we operated four 3-counter units simultaneously (Fig. 2). Two closed cores provided the four saturated iron sections for the four counter units. The mag-

[4] L. Alvarez, Phys. Rev. **45**, 225 (1934).
[5] C. D. Anderson, Phys. Rev. **44**, 406 (1933).
[6] P. Kunze, Zeits. f. Physik **80**, 559 (1933).
[7] C. D. Anderson and S. H. Neddermeyer, Int. Conf. Physics, London, Oct. 1934.
[8] W. F. G. Swann.



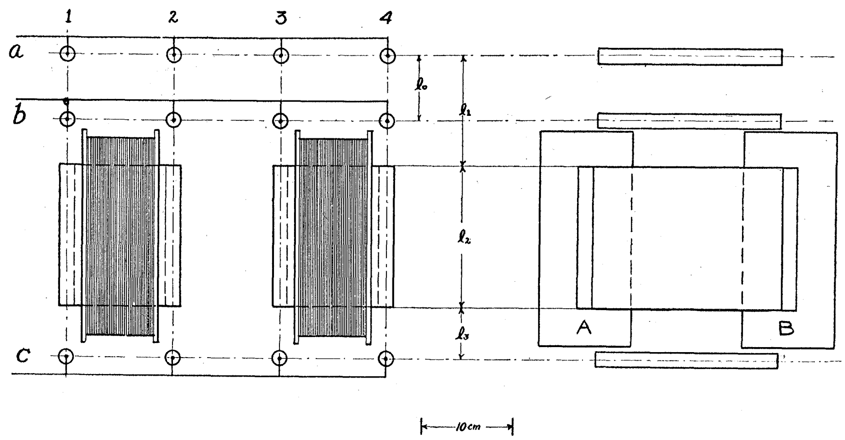


FIG. 1 (left). Schematic arrangement of counting tubes and iron.
FIG. 2 (right). On the left are shown the two rectangular closed cores, the planes of the rectangular openings being perpendicular to the plane of the paper. For each core is shown one of the coils, *viz.* that wound on the front wall of the frame. Another coil wound on the back wall completes the magnetizing system. The circles represent end views of the cylindrical counting tubes. On the right is a side view of the apparatus with the coil spools shown at A and B.

netizing windings were placed on those core sections not in the planes of the counters. Variation of induction along the unwound sections (deflecting sections) was found not to exceed two percent. We used an induction of 14,800 gauss, measured in a turn of wire around the core. Residual magnetism, amounting to 5800 gauss, was removed by consecutive diminishing current reversals when it was desired to count coincidences with no deflecting field.

We measured the percent diminution in counting rate produced by the field for two different sets of values of the distances l_0, l_1, l_2, l_3. Table I gives these distances in centimeters.

TABLE I. *Values of the distances in centimeters for the two experiments.*

	l_0	l_1	l_2	l_3
Exp. 1	7.9	12.0	15.2	5.5
Exp. 2	15.4	20.6	15.2	46.1

Counters 1a, 2a, 3a and 4a were connected to the same amplifier, as were 1b, 2b, 3b and 4b, and 1c, 2c, 3c, and 4c. A count would be recorded when three counters, one from each of the three groups a, b, and c, discharged simultaneously. The spacing of the units was such, however, that no single ray could be recorded unless it passed through three counters all of the same unit. Thus all recorded single rays must have traversed the iron. Showers, however, could produce counts without traversing the iron, and, a correction for such counts was made. The possibility exists also of a ray, magnetically eliminated from one unit, being deflected into the bottom counter of another unit and so still producing a recorded count. Since only a small fraction of that area into which a ray may be deflected is occupied by another counter, this source of error is probably negligible. It would, however, be more serious in Experiment 2 than in Experiment 1. In any future experiment of this type it would be well to have the units record independently in order to avoid any possibility of this "cross deflection." Such independence would also greatly diminish the effect of showers.

Values of counting rate were recorded over periods of length 24 hours or less in the following sequence: on—off—off—on—on—off and so forth. Each counting rate quoted below is the average of about 30 short period values. Taking values in the above sequence would, if the ratios between adjacent readings were computed independently and then averaged, eliminate any slow charge of sensitivity that might occur. The r.m.s. deviation ("standard deviation") of the actual values agreed so well with the assumption of constant sensitivity, however, that it was thought unnecessary to compute the

TABLE II. *Effect of the magnetic field on triple coincidence counting rates in the two experiments.*

	Field	No. of Counts	Time (min.)	Counting Rate (counts/min.)	% Diminutions in Counting Rate
Exp. 1.	off	11659	12305	0.945 ± 0.009	8.3 ± 1.3
	on	10037	11603	$0.865 \pm .009$	
Exp. 2.	off	1925	9190	$0.210 \pm .005$	12.6 ± 3.1
	on	1683	9210	$0.183 \pm .005$	

separate ratios. Each quoted counting rate is, therefore, a total number of counts divided by a total time. During a run with field *on*, heat was dissipated which raised the temperature of the middle counters by perhaps as much as 10°C, and that of the top counters by a smaller amount. A comparison of the double coincidence counting rate between the upper pair with the magnet current on and off enables us to set an upper limit for any temperature effect at one percent.

Table II shows the effect of the magnetic field on the triple-coincidence counting rates of the two experiments.

The sources of error referred to above cause these values of percent diminutions to be regarded as lower limits to the actual percentage elimination resulting from the magnetic field. The correction to the data on account of showers will now be discussed.

Correction of results for effects of showers

The magnitude and frequency of the showers is unknown; but, since showers affect the data in ways other than through the magnetic diminution ratio, it is possible by measuring their effect in such connections to establish the amounts which they contribute to the current recorded by a system. Thus in the absence of showers it is possible to calculate the ratio of the counting rates for Experiments 1 and 2, with magnetic field absent. The ratio is found different from that obtained experimentally. It is then possible to estimate the counts contributed by showers as the number necessary to bring the two ratios concerned with harmony.

Defining the "admittance" of a counter pair as that number by which the intensity must be multiplied in order to equal the counting rate we have[9]

[9] J. C. Street and R. W. Woodward, Phys. Rev. **46**, 1029 (1934).

TABLE III. *Comparison of counting rates and theoretical admittances in the two experiments.*

	Admittance A	Counting Rate (counts/min.)
Exp. 1	3.404	0.945
Exp. 2	0.592	0.210
Ratio of admittances A_1/A_2	5.76	
Ratio of counting rates N_1/N_2	4.50	

$$A=\frac{a^2}{4}\left\{\frac{b^2}{b^2+L^2}+\frac{3b}{L}\tan^{-1}\frac{b}{L}\right\},$$

where A is the admittance of the (vertical) counter pair, a the width (diameter) of counter, b the length of counter, and L the distance between counters.

In the absence of showers, and of magnetic field, the counting rates for Experiments 1 and 2 should be in the ratio of the corresponding admittances. The results of a comparison of the experimental and theoretical ratios are shown in Table III. One sees that the decrease in counting rate from the first to the second experiment is less than the decrease in the admittance. Hence, a component (or components) must be present which does not decrease as rapidly as does the calculated admittance when the distance between counters is increased.

Representing the intensity of single rays (in number per unit solid angle per square cm per second) by j, the number of valid counts (due to rays which traverse the iron) is jA_1 in the first experiment and jA_2 in the second. With the field *on* these rates become respectively $j\alpha_1A_1$ and $j\alpha_2A_2$ where the α's are positive quantities less than unity indicating that fraction of the radiation still capable, despite the deflecting field, of reaching the bottom counters. In addition to counts produced by the intensity j we have components, for example showers, which are not changed by the magnetic field but which will probably be different in the two experiments although not in proportion to the admittances. We thus have four equations

$$N_1=jA_1+s, \tag{1}$$

$$N_2'=j\alpha_1A_1+s, \tag{2}$$

$$N_2=jA_2+\eta s, \tag{3}$$

$$N_2'=j\alpha_2A_2+\eta s, \tag{4}$$

where the N's represent observed counting rates and s denotes that spurious component in Experiment 1 which is unchanged by the magnetic field. The factor η permits this component to be different in Experiment 2. The subscripts indicate first and second experiments and the primes, values with the field on. We have five unknown quantities, j, α_1, α_2, s and η, in these four equations, so that we cannot solve for all of them without making some assumption with regard to one of them. We shall concentrate our attention for the moment upon η. It follows immediately from (1) and (3) that

$$s=(N_2A_1-N_1A_2)/(\eta A_1-A_2). \qquad (5)$$

The experimental data make $N_2A_1>N_1A_2$ so that the denominator must be positive. We thus see that the values of s calculated by assumption of a value of η increase with decrease of the value of η assigned. By multiplying (5) by η, it is immediately obvious that ηs also increases with decrease of the value of η assigned.

Again from Eqs. (1) and (2) we obtain

$$\alpha_1=(N_1'-s)/(N_1-s) \qquad (6)$$

and from (3) and (4) we obtain

$$\alpha_2=(N_2'-\eta s)/(N_2-\eta s). \qquad (7)$$

The conclusions derived immediately above therefore tell us that the values α_1 and α_2 calculated both increase with increase of the values of η assigned. If therefore we can determine upper and lower limiting values of η, we can calculate from them the corresponding upper and lower values of α_1 and α_2. Now all reasonable views as to the effect of showers would make the contribution for case (1) greater than, or at least equal to, that for case (2), where the counter distances involved are greater. Hence we conclude that $\eta\leqq 1$. We can calculate a lower limit for η by visualizing that particular shower mechanism which would make the shower counts vary most rapidly with the distances concerned. The simple picture is one where we have a shower of particles extending over the area of the apparatus, and one ray at least passes through each counter. In this process there would be very little change of shower counting rate with change of position of the counters, unless the changes are so great as to extend beyond the limits defined by the shower beam. A shower mechanism which is particularly sensitive to change of counter position is one in which a single ray passes through the upper two counters, and a second ray passes through the lower counter. The frequency of such shower counts is roughly proportional to the inverse square of the distance of separation of the upper two counters, and exactly proportional to the measured counting rate of this pair when considered as a double coincidence set. While, it is admitted that the arguments are not absolutely unassailable, it is felt that a reasonable assumption is one which regards the variation of shower counting rate with separation of the upper counters as no greater than that defined by the process above stated. On this basis, and taking the measured frequency of the coincident counts of the upper pair of counters for cases 1 and 2, we find for η a value equal to 0.59, so that considering what has been stated earlier we may conclude that $1\geqq\eta\geqq 0.59$. In line with conclusions as to the dependence of the α's upon the assumed values of η, and utilizing the data contained in Table III, we find $0.910\geqq\alpha_1\geqq 0.905$ and $0.826\geqq\alpha_2\geqq 0.818$. The corrected percentage diminutions in counting rate produced by the magnetic field are $100\ (1-\alpha_1)$, and $100\ (1-\alpha_2)$, respectively. Further correction of these values to allow for statistical fluctuations gives us as the percentage diminutions for the two experiments:

Experiment 1: Between 7.5 and 10.8 percent,
Experiment 2: Between 12.8 and 23.0 percent.

Numerical values for the relative intensities of showers involved in the above corrections may be of incidental interest. Assuming that the shower counting rate is the same in both experiments ($\eta=1$) we find it amounts to six percent in the first experiment and 26 percent in the second. The assumption that the shower rate is proportional to the double rate in the upper pair leads to the values 11 percent in the first experiment and 30 percent in the second. We ascribe these high percentages to the following causes: (1) The extreme counters were rather far apart (34 cm and 81 cm) and the true counting rate should diminish more rapidly than the shower rate as the distance between counters increases. (2) The arrangement of four separate sets all operating in parallel on the same amplifier would be expected to receive about four times as many showers as if the sets were on independent amplifiers.

We have, also, results from two experiments to substantiate the order of magnitude of the above percentages. The three counter groups (a), (b), and (c) were laid flat on a table, out of line. In this position a counting rate, for showers of at least three rays, of 0.00040 ± 0.00008 was measured. Our correction assuming $\eta = 1$ gives the value 0.00095.

The arrangement shown in Fig. 3 might be expected to give a counting rate of about a quarter of the total shower component. The distances involved were the same as those in the second experiment. The result indicated that the shower component in the second experiment was between 25 and 40 percent of the total.

While the foregoing considerations are not affected by absorption in the iron, it is of interest to note that in the presence of the iron, the values of the intensity (j), come out 0.0040 and 0.0042 (rays per unit solid angle per square centimeter per second) with the two different assumptions concerning showers. The decrease due to the iron was found to be 20 percent giving values 0.0050 and 0.0052 without iron. The fact that the experiment was performed under two floors of reinforced concrete is a contributing factor to the low single-ray intensity and high shower percentage.

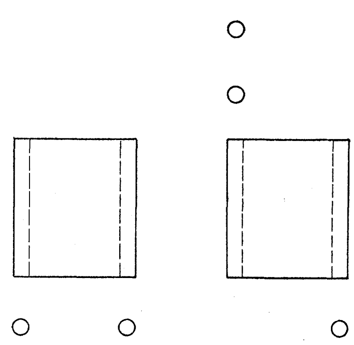


FIG. 3. The arrangement here is the same as that represented in Fig. 2, the difference being that certain counters have been removed, as indicated.

Interpretation of results in terms of an energy distribution

In a magnetic deflection experiment of this type all rays below a certain energy are prevented from reaching the last counter. Rays of higher energy, however, will also be eliminated to a certain extent. For a given field strength and geometry one can compute a function of energy $\epsilon(V)$ which represents the fraction of rays eliminated. This function is equal to unity for energies below a certain value V_1 and approaches the asymptote zero monotonically as V increases. If $p(V)dV$ represents the probability that a ray will have energy between V and $V+dV$, the fraction of all rays initially present eliminated by the field will be

$$E = \int_0^\infty p(V)\epsilon(V)dV.$$

We have performed two experiments giving two values E_1 and E_2 corresponding to different dimensions with the same field strength. We have available the energy distribution as determined by Anderson and Neddermeyer.[7] Our purpose is now to compute the values of E_1 and E_2 which one would theoretically expect on the basis of present knowledge regarding the energy distribution.

Besides the geometrical dimensions, and the magnetic field strength, the analysis must also include the loss of energy per centimeter as the particle traverses the iron. This energy loss produces a continual decrease in radius of curvature along the path of the particle and so may be said to assist the magnetic field in the deflection.

The general plan of attack will be to calculate the fractional diminutions E_1 and E_2 as functions both of (1) a vector which we shall call F and which is the vector representative of B or h or whatever intermediate quantity is really effective in producing force on the moving electron, and (2) the energy loss per cm (v). It will then be our purpose to see what values of F and v are consistent with the observations.

Before proceeding we must examine data regarding the probable values of v. Using a cloud chamber in a magnetic field Anderson and Neddermeyer measured the curvature of particle tracks before and after the tracks passed through a thickness of material. Using lead as the absorber they obtain as a mean of a number of observations 57×10^6 volts per centimeter. Assuming proportionality to number of extra nuclear electrons this reduces to 49×10^6 for iron. A similar measurement (based on fewer observations) for carbon gives 5×10^6 or 17×10^6 for iron. Other considerations, however, lead them to believe 13×10^6 as a more probable value for carbon which gives 45×10^6 for iron.

We observed that 15.2 cm of iron absorb (or scatter) 20 percent of the radiation. If one assumes that the energy loss is proportional to the thickness and is independent of energy one could use this value in conjunction with the energy distribution to obtain a value for v. Doing this we get 61×10^6 volts, somewhat higher than the other data indicate. Measurements of absorption in iron by Street, Woodward, and Stevenson[10] give 18 percent as the absorption in 15.2 cm, which also points to a rather high value for v. The fact that absorption measurements give a high value for v is probably due, as pointed out by the above authors, to the presence of more low energy rays in the absorption experiments than are indicated by Anderson's data. Also, it may be that with a greater thickness of material, more rays are eliminated by scattering than in the relatively small thickness used in Anderson's experiments. If this phenomena is important the decrease of true energy loss per cm with energy would be even more rapid than Street, Woodward and Stevenson observed. These authors assume that 57×10^6 volts for lead is correct for low energies (3×10^8 volts) and adjust the energy distribution by adding low energy rays until it agrees with their absorption data using small thickness. Then using this corrected energy distribution they measure absorption in larger thickness and so obtain v as a function of energy. Their values range from 45×10^6 for energies $0\text{–}6.8\times10^8$ to 20×10^6 for energies $(19\text{–}25)\times10^8$. They ascribe the decrease to fewer nuclear encounters involving radiative losses. Such encounters also doubtless alter the direction of the ray and would be expected to eliminate a certain fraction through multiple scattering.

Our computations of expected magnetic diminutions are carried out for two values of v, 30×10^6 and 60×10^6 volts per cm in iron. We assume that v is independent of energy and write as the differential equation of the path.

$$\frac{d^2y}{dx^2}=\frac{Fc}{V}\frac{1}{1-vx/V},$$

where y and x refer, respectively, to horizontal and vertical directions in Fig. 1, and F is perpendicular to the plane of the figure. This expression is derived on the assumption that the rest mass of the particles may be neglected, and involves the approximation that the angle between the path and the vertical direction is small.

The details of this analysis are published elsewhere.[11] For rays of a given energy an element of area of the top counter looking along the trajectories of the particles, can "see," with the field *on*, only a certain fraction of the bottom counter. Integrating this fraction for all elements of the top counter and subtracting the result from unity, gives one the fraction eliminated $\epsilon(V)$.

Since this is a rather laborious graphical process it was carried out for only two values of v and three values of F. In Table IV, E_1 is the calculated fraction eliminated in Experiment 1, and E_2 that in Experiment 2. The vector F is given in terms of the measured induction B which amounted to 14,800 gauss. The data in Table IV are plotted in Fig. 4. The coordinates of the solid curves represent, for the two values of v, the calculated percent diminution as a function of the vector F. Although increasing v

TABLE IV. *Calculated fraction of the coincidence counting rate eliminated with the field on in Experiments 1 and 2.* E_1 (obs) lies between 7.5 and 10.8 percent; E_2 (obs) lies between 12.8 and 23.0 percent

F	$v\times10^{-6}$	E_1 (calc) %	E_2 (calc) %
$B/3$	30	4.4	15.0
$2B/3$	30	9.6	30.1
B	30	15.2	40.5
$B/3$	60	2.2	10.9
$2B/3$	60	6.3	24.5
B	60	9.9 ± 2.5	35.0 ± 5.0

[10] J. C. Street, R. W. Woodward and E. C. Stevenson, Phys. Rev. **47**, 891 (1935).

[11] W. E. Danforth, J. Frank. Inst. **220**, 377 (1935).

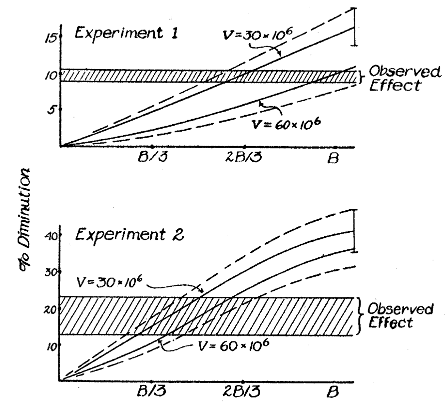


FIG. 4. Plot of the data given in Table IV.

tends to assist in the deflection by softening the radiation, it also tends to reduce the observed effect by absorption of the softer components. The net effect of increasing v is to reduce the calculated effect. The shaded areas bounded by horizontal lines give the range in which the actual diminution may be expected to lie, on the basis of the considerations developed earlier in the paper. The dotted curves indicate an estimate of the probable error in the calculated effect arising from statistical uncertainties which it seems reasonable to suppose probably exist in the energy distribution.

The results of our observations and estimates of various errors may therefore be epitomized as follows. On the assumption that the upper and lower limits of the energy loss are those given, the field producing the deflection appears to lie between 0.50 B and 1.03 B in Experiment 1 and between 0.23 B and 0.75 B in Experiment 2.

Taking the upper limit of the observed effect as correct and assuming that the calculated effects are too high by an amount equal to the probable error, one sees that Experiment 1 is consistent with the values $v=45\times10^6$ volts/cm and $F=B$. On the basis of these values, however, Experiment 2 should have yielded an effect of at least 32 percent whereas our observed upper limit is 23 percent.

In other words, by stretching all probable errors to the limit the results of Experiment 1 can be brought into agreement with the assumption that the effective deflecting force is equal to the measured induction, but the observed effect in Experiment 2 appears to be too small.

Therefore, although the effective force cannot, with much conviction, be said to differ from the induction, there does seem to be a tendency for the observed deflections to be smaller than one would expect. Reasons for such a discrepancy may be classified as either errors of interpretation or real phenomena. What is certain is that the deflections are much larger than would be calculated from the ordinarily defined magnetic field h in the iron.

One possible error of interpretation which might account for the discrepancy is the "cross deflection" mentioned in Section B. This would, indeed, produce a larger error in Experiment 2 than in Experiment 1, as would be required to explain the results.

Another explanation which, in view of reference 10 seems rather promising, is the rapid decrease of energy loss with increasing energy. This would result in both observed diminutions being smaller than the calculated values, and since Experiment 2 involves larger energies it would suffer the larger discrepancy. Although it happens that assigning a smaller constant v gives a larger value of calculated effect, the assignment of a v decreasing with energy could have the opposite effect. For if the reduction of v applies only to those energies capable of penetrating the iron (and therefore does not affect the energy distribution) the only effect it has is to decrease the magnetic deflections.

The possibility that our energy distribution differs from that observed by Anderson must also be reckoned with. One could of course modify the distribution assumed in such a way as to remove the discrepancies. The *statistical* error which we computed from Anderson's data does not, of course, include possibilities of *real* differences, such as would result from different amounts and kinds of material above the apparatus.

Multiple scattering has not been included in our calculations. Its net influence on our results is difficult to determine. It, too, would probably be of more importance in Experiment 2 because of the greater distances involved.

Counts due to secondaries produced in the iron, by rays which traversed the upper counter but not the bottom ones, would not be proportional to the admittance, and hence are, in effect, taken care of by the "shower correction."

The presence of protons rather than positive electrons cannot be invoked as an explanation because as far as we are concerned the "energy" is merely a number enabling us to compute the same *deflectability* observed in the cloud chamber.

Most of the possible errors of interpretation seem, therefore to be in the sense of reconciling the discrepancy and making it likely that the induction B is really the correct vector. Apart from consideration to be presented in Section B, the only way in which our results can be reconciled with those of Alvarez is by assuming an energy distribution different, to a very unlikely extent, from that of Anderson.

A real phenomenon which might account for the result is the fact which Anderson has observed, *viz.*, that an electron which enters a plate of lead may fail to reappear even though its energy be more than sufficient to penetrate the material. The inference is that the particle is transformed into a photon. This photon may travel in the same direction as the original electron and may, at some point farther along, produce another electron which may actuate the counter. On this view an electron which traverses the iron may only exist, as an electron, over a fraction of its total path, and would therefore suffer a smaller deflection than one would expect.

Finally we come to the question which motivated this work. Is it possible that the effective force is really other than the induction B?

Section B. Theoretical Considerations

In its most naïve form, the problem of the passage of electrons through magnetized iron is one beset with no ambiguities in regard to the classical Lorentzian theory. On that theory it is the magnetic induction B which determines the deflection produced. A more sophisticated view of the matter brings out the fact that, in the case of passage through reasonably small thickness of iron, there should be a statistical distribution of deflections. This distribution averages, it is true, to the value determined by B in the simple solution; but, in such a manner as to cause experiments whose data have been interpreted in terms of the customary naïve theory to lead to a vector smaller than B as the apparent quantity which determines the deflection observed.

These matters have been considered by one of us[8] in detail, and an extended discussion of the situation is published in this issue. For the immediate present it must suffice to summarize the conclusions, at any rate in an approximate form, but sufficiently completely to bring out the salient features involved. In so doing, it must be remarked that the discussion is based, in the first instance, upon classical rather than upon wave-mechanical principles, although it is believed that the essential facts will have their representatives in the wave-mechanical story. In the second place, it must be remarked that, even in confining ourselves to the classical treatment, certain elements of fundamental uncertainty present themselves. One must first form some usable idea as to the magnetic entity whose naïve representative in elementary theory is a magnetic doublet. Then, even in the absence of a resulting polarization, magnetic entities will exert deflections upon the electrons passing near them. Moreover, if the magnetic entities are to be thought of as having a nature in any way similar to rotating electrons of a classical kind, the problem of what happens when another electron passes through them becomes of serious moment, both as regards the possibility and nature of the occurrence and as regards the process of interaction occurring in it. Such a drastic event must not be eliminated from consideration with apologies on the basis of probable infrequency of occurrence because, in the case cited, it is the contribution of the large fields in the interior of the rotating shells which is responsible for raising the average field in the medium from the value defined as h to the value B in accordance with the Lorentzian theory. Then there is the part played by interactions of a kind not depending upon the magnetic field and the velocity of the electron, electrostatic interactions and the like. It is necessary to avoid all of these difficulties by studying an ideal problem in which some definite assumption is made with regard to the magnetic entity, and in which the particles traversing the material, and whose deflections are under consideration, may be regarded as points which traverse the magnetic entities and experience everywhere the force given by the Lorentzian theory in the form $v \times H/c$, where H is the true magnetic field at a point, v is the velocity of the deflected particle and c the velocity of light. It is likely that the study of this ideal problem will serve, at any rate, to point out the relevant considerations which pertain to the real problem, and that these factors will have their counterparts even in a more sophisticated theory built upon quantum-mechanical lines.

As will be more completely demonstrated in the publication subsequently to be made and referred to above, the following conclusions result:

(1) In a sufficiently long path, every corpuscle moving perpendicular to B, has the same history on the average, and passes through the same number of magnetic entities per unit of length of its path. The average magnetic vector responsible for deflections along this path is B.

(2) In any one long path of the type cited under (1), the average represented by B is made up out of contributions from the entities actually passed through in the path, and contributions from the other entities not passed through in the path. The former contributions turn out to be $2\pi I$, so that the latter is $B-2\pi I=h+2\pi I$, where h is the ordinary macroscopically defined field in the medium, and is equal to $B-4\pi I$.

(3) It is of interest to note that the $2\pi I$ cited above as contributed by the entities passed through is made up of two contributions, one from the field actually inside the entities, and amounting to $8\pi I/3$, and the other from the fields outside the entities and amounting to $-2\pi I/3$.

(4) The significance of the foregoing considerations depends upon the size of the magnetic entities. If they were as small as classical electrons, and were as numerous as the orbital electrons in iron, then in the passage of an electron through one centimeter thickness of iron there would only be about 3 chances in 10 of the center of the moving electron passing through one of the magnetic entities.

(5) Under such a case as is cited under (4), most of the electrons would experience a deflecting force determined on the average by $h+2\pi I$. The average for all of the electrons would be determined, even in the case of a thin piece of material, by $h+4\pi I$; but, the contribution $2\pi I$, to the average would be made up from a few deflections of very large amount. In an experiment which estimated the effective deflecting vector by counting the number of rays deviated outside of a certain area determined by the geometry of the counters, the few highly deviated rays would not succeed in contributing a proper representation of their status. With regard to the time average, their smallness in number is compensated by largeness of deviation. Once the deviation is so large as to take them out of the counter area, further deviation adds nothing to the measurement, so that as a practical fact, smallness of number receives no compensation as regards the measurements. The highly deviated rays contribute negligibly to the result and the average effective deflecting vector is $h+2\pi I$.

The foregoing statement represents the matter in its most primitive form. Even the deviations experienced by the electrons which do not pass through entities will show statistical fluctuations; and the whole story is bound up with the assumptions concerning the nature of the entities. Our discussion serves, however, to demonstrate that the problem of passage of electrons through magnetized iron is not as simple as the existing line of thought would suggest; and, consistent with the fundamental idea of the Lorentzian theory, there is ample room for the naïve interpretation of the experimental data to lead to the conclusion that the apparent deflecting vector is less than B.

The foregoing considerations have an increased significance for the case where the path traversed in the material is small; and it will be evident why such experiments as those of Alvarez should yield results which, at first sight differ markedly from those to be expected from our own experiments. With a sufficiently small size for the magnetic entity, we should expect, in the Alvarez experiment a deviation of *most* of the electrons centering about a vector considerable smaller than that determined by B.

Erratum: Variation of the Properties of Cosmic Shower Radiation with Altitude

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AND
Darol K. Froman, *McDonald College*
(Phys. Rev. **49**, 473 (1936))

THROUGH error the name of the second author was omitted from the article in the March 15 issue.