and thus should increase with atomic number. As is seen from the values in the table, this increase is nearly linear and corresponds to the nearly constant and quite reasonable value

$$(\overline{1/r})\sim 3.5\times 10^{12}\ \mathrm{cm}^{-1}.$$

The high energy of the electron emitters Li^8, B^{12}, N^{16} and F^{20} can be understood on the basis of their disintegration into the tightly packed nuclei Be^8, C^{12}, O^{16}, and Ne^{20}. The nuclei Be^{10}, C^{14}, and O^{18} can be classed together as representing stable or at most slightly unstable configurations.

In conclusion we wish to express our appreciation to Dr. Robert Serber and Dr. J. R. Oppenheimer for discussions of the theoretical aspects of these investigations, to Kurie, Richardson and Paxton for sending us a copy of their manuscript before publication, to Dean Wooldridge for preparation of the carbon targets containing an increased percentage of C^{13}, and to the Seeley W. Mudd fund for financial support.

APRIL 15, 1936 PHYSICAL REVIEW VOLUME 49

A Theoretical Discussion of the Deviation of High Energy Charged Particles in Passing Through Magnetized Iron

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(Received February 24, 1936)

The paper attempts a critical discussion of the situation pertaining to the magnetic field in a piece of magnetized iron, particularly in relation to the deflecting force which it produces on high speed charged particles passing through it. On the Lorentzian theory the magnetic induction, B, is the average value of the true magnetic field H, averaged throughout the magnetic material. In this average, regions inside the magnetic entities responsible for the polarization make contributions which determine the whole difference between B and the ordinary macroscopically defined field, h, equal to $B-4\pi I$. A study is made of the special case where the entities are rotating electrically charged spheres. If the entities are very small in volume the chance of a point electron missing all of them in its passage through a reasonably small length of the magnetized material is considerable. It appears that for such electrons as miss the entities the effective deflecting force is determined on the average by $h+2\pi I$. The true average for all electrons passing through the material is determined by $B=h+4\pi I$; but, that average is contributed to in appreciable amount by very few electrons which experience deflections much in excess of those determined by B. These considerations have important consequences in relation to the interpretation of experimental results.

Introduction

THE interest of the problem cited in the above title arose, primarily in connection with experiments on the deviation of charged particle cosmic rays in passing through magnetized iron. A simple application of the Lorentzian theory seemed to require that the magnetic induction B was the vector concerned in the deviation while the first experiments performed[1] seemed to lead to the conclusion that the magnetic intensity h was the vector operative. Subsequent work[2] has given rise to conclusion that a vector being between B and h is the vector really involved. It is not our purpose to discuss here these experiments, or the validity of their interpretation. It will suffice to say that the results to date are such as to suggest that, in such experiments, the elements operative involve features of greater complexity than would be suggested by the most naïve view of the matter which regarded the iron as the magnetic equivalent of a bundle of continuous tubes of induction. The purpose of the present paper is to look into certain of the theoretical elements concerned in some degree of detail. In this task we meet at the outset certain difficulties which must be faced.

In the first place while recognizing that the problem should be discussed ultimately in terms

[1] B. Rossi, Accad. Lincei, Atti **11**, 478 (1930); B. Rossi, Nature **128**, 300 (1931); L. M. Mott-Smith, Phys. Rev. **39**, 403 (1932).

[2] L. Alvarez, Phys. Rev. **45**, 225 (1934); W. F. G. Swann and W. E. Danforth, Phys. Rev. **45**, 565 (1934).

of wave-mechanical principles, we shall in the present investigation confine our attention to an approach founded purely upon classical electrodynamics, in the belief that the essential elements which the discussion will reveal are of such a nature as to have their representatives in wave-mechanical story. Even in confining ourselves to the classical treatment, however, certain elements of fundamental difficulty present themselves. A piece of magnetized iron owes its state of magnetic polarization to the presence of a number of polarized entities. In the older views, these entities were represented by the revolutions of electrons around the atomic nuclei. Each atom was the statistical equivalent of a number of amperian whirls, which, in turn, were the equivalents of magnetic doublets at distant external points. A more modern picture associates the magnetic entities with the spins of the electrons. Wave mechanically, this spin is something which, for each electron, has symmetry with respect to the center of coordinates of the atom to which it belongs. In our treatment we shall idealize the problem by supposing that the entity is a uniformly rotating sphere of radius a, carrying a uniform surface charge density. Such an entity gives a uniform field for its interior and a field the equivalent of that of a magnetic doublet at all external points. Then, even in the absence of a *resultant orientation*, the entities will exert forces upon moving charges passing near them, so that there will be a magnetic scattering action in the absence of magnetic orientation. In our ideal problem we shall have to discard this feature in the belief that, to a first approximation, the *alterations* of the deviation produced by orienting the magnetic entities is calculable from the ideal problem of the passage of electrons through the medium with its entities completely oriented and adjusted in strength to produce the resulting state of magnetization under consideration. Again, if the magnetic entities were to be regarded as having a nature in any way similar to rotating electrons of classical dimensions, the problem of what happens when another electron passes through one of them becomes of serious moment. The question arises as to whether forces exist which would prevent such penetration. The classical equation of motion for an electron gives the force per unit charge on it as $E+[wH]/c$, when E is the electric field, w is the velocity of the electron, H is the true magnetic field at a point, and c the velocity of light. The significance of this quantity must be viewed with many misgivings when the vectors E and H vary to an extent comparable with the whole of their values over the volume of the electrons whose motion is under discussion, and where the electron and the rotating shell actually penetrate each other. The possibility of the electron penetrating the magnetic entity must not be dismissed as an event whose probability of occurrence is negligibly small; for, as will presently be seen, the magnetic fields in the interior of the entities plays an important part not only in providing for the difference between h and B but in determining the average deflection of the electron in passing through the magnetized material.

To sum up, therefore, the present investigation is founded upon the consideration of an ideal problem in which a definite assumption is made as to the nature of the magnetic entity, in which all the magnetic entities are oriented alike in the magnetized state, and in which the charged particle whose deflection is studied is regarded as a point which traverses the magnetic entities as well as the space between them and experiences everywhere a force given by the Lorentzian theory in the form $[wH]/c$.

1. Review of Certain General Matters Pertaining to Polarized Media

For the benefit of those who are not specialists in electrodynamics, it will be of advantage to review certain matters which are well known but whose relationship to one another is not always evident. Those to whom these matters are well known may desire to omit this section (Section 1).

Definitions

In what follows, we shall use P, u, E, H, for the electric density, velocity, electric field and magnetic field at an ideal point in the medium, inside or outside an atom, or even inside an electron. c is the velocity of light. We shall denote the electronic velocity as a whole by w.

When passing to a material medium, it will become necessary to consider vectors macroscopically defined. The only ones which concern us are e, h, P, D, B and I, which are, respectively, the electric field intensity, the magnetic field intensity, the dielectric polarization, the dielectric displacement, the magnetic induction, and the intensity of magnetization. The definition of these quantities varies according to the theoretical avenue of approach from the submacroscopic to the macroscopic case.

The primitive electrostatic and magnetostatic macroscopic theory

The elementary theory of dielectric and magnetic media made prior to the Lorentzian and allied theories still figures prominently in the thoughts of most physicists so that a word is necessary concerning it. The theories for the dielectric and magnetic cases are exactly the same. The magnetic medium, for example, is represented by a distribution of polarization, or magnetization, arising from magnetic doublets. It is a mathematical fact that such a distribution of doublets produces, at points external to the medium, a potential equal to that which would be produced by a fictitious distribution of magnetism of volume density ρ_f and surface density σ_f given by

$$\rho_f = -\text{div } I, \quad (1)$$

$$\sigma_f = I_n. \quad (2)$$

where the subscript n refers to the normal component.

If we now *define*, as the magnetic intensity *in the medium*, a quantity h, which is to be understood as the field calculable from magnets or currents outside the medium *plus* the field *calculable by the law of inverse squares* at the point in the medium by use of the fictitious magnetic "charges" aforesaid,[3] this definition of h endows it with the characteristic given by

$$4\pi\rho_f = -\text{div } h, \quad (3)$$

which, by combination with (1) tells us that

$$\text{div } (h + 4\pi I) = 0. \quad (4)$$

The vector $h + 4\pi I$ thus has an important property, and so it constitutes a new vector valuable in our discussions, which vector we call the magnetic induction B, so that

$$B = h + 4\pi I. \quad (5)$$

The electrostatic case proceeds in the same manner, except that h and I are replaced by e and P, and (4) has as its analog

$$\text{div } (e + 4\pi P) = \rho \quad (6)$$

the right-hand side being now no longer zero since the possibility of the presence of real volume density ρ of electricity must be admitted. The definition of D then follows as

$$D = e + 4\pi P \quad (7)$$

and Eq. (6) becomes $\quad \text{div } D = \rho. \quad (8)$

It is of interest to observe that, in the development of the theory, the order of appearance of the vector is $I \rightarrow h \rightarrow B$ for magnetism, and in exact correspondence, $P \rightarrow e \rightarrow D$ for dielectric theory.

A further step of less fundamental significance is taken in seeking the actual field at the center of a spherical cavity in the interior of the medium. This field is h', where

$$h' = h + 4\pi I/3 \quad (9)$$

for magnetism, with a corresponding expression $e' = e + 4\pi P/3$ for the dielectric case. The customary derivations then proceed to show that when the spherical hole is refilled with the material which may be imagined to have been removed from it, the field h'(or e') may be regarded as the actual field at the point occupied by one of the doublets, and due to all of the other doublets in the medium, and to external influences, provided that the doublets are fortuitously distributed in space or are distributed with cubical symmetry. The writer feels that very little fundamental significance can be attached to the quantity h' (or e') as above provided for. For the case of cubical symmetry it has significance only at the center of one of the entities, and has no immediate relation to the forces on an electron traveling between the entities, for example. For the case of fortuitous distribution of the doublets, with respect to each other, and so with respect to

[3] The definition is preferred in contrast to the usual definition in terms of the actual magnetic field in an elongated cavity with axis parallel to I, to which definition it is, however, equivalent. It will be observed that h, calculated as it is in part from the smothered out fictitious distributions ρ_f and σ_f is itself mathematically continuous in the body of the medium.

points in space, the quantities h' (or e') would not represent the actual field at one of the doublets due to all causes other than itself, but only the average of such fields. In any particular case there would be large fluctuations from the average, depending upon the degree of proximity of the nearest doublets. With regard to the actual field at a point between the doublets, h' (or e') again represents an average for the case of fortuitous distribution, but there are large fluctuations from this average in any particular case. The significance of h' (or e') will appear in much less distorted form as a result of considerations to be presented later.

The Lorentzian theory of polarized and magnetized media

In the hands of Lorentz, the starting point is the assumption that, on a sufficiently fine grained scale, there are no absolute discontinuities of field, and the real fields E and H are controlled by the equations for free space, *viz.*,

$$(1/c)(4\pi\rho u+(\partial E/\partial t))=\text{curl } H, \tag{10}$$

$$4\pi\rho=\text{div } E, \tag{11}$$

$$-(1/c)(\partial H/\partial t)=\text{curl } E, \tag{12}$$

$$0=\text{div } H. \tag{13}$$

The vector B at a point now becomes defined macroscopically, as the true volume average value of H, in the vicinity of the point. The vector I at a point becomes defined macroscopically, as the true volume average in the vicinity of the point of a certain quantity having to do with the velocity of the electricity at a point and its displacement from a standard position. There are independent contributions from the positive and the negative electricity. The vector h now becomes defined as $B-4\pi I$.[4] On the other hand, P at a point becomes defined in terms of the displacement of the positive and negative electricity from a standard position. e at a point becomes defined macroscopically as the true volume average of E in the vicinity of the point, and D becomes defined as $e+4\pi P$. It will thus be observed that while, in the primitive macroscopic theory, the orders of definition are completely analogous for the magnetic and electric quantities, being, in fact, $I\rightarrow h\rightarrow B$ and $P\rightarrow e\rightarrow D$, respectively, in the case of the Lorentzian theory, the orders are $B\rightarrow I\rightarrow h$ and $P\rightarrow e\rightarrow D$, respectively. However, these matters are not pertinent to the main part of our discussion, and are included only for completeness of the picture. The primary fact which concerns us here is that B is defined macroscopically at a point as the true volume average of H in the vicinity of the point.

Visualization of the significance of B on the Lorentzian theory, for a special case

Following the line of procedure indicated in the Introduction, we shall limit our discussion to the case where the magnetic entities are spherical shells of radius a, with uniform surface charge, and rotating with uniform angular velocity. It is easy to show, and is well known, that such a sphere gives at a point for which the radius vector r from the center is greater than a, a field which is the exact equivalent of a doublet of moment p depending upon the surface density, radius, and angular velocity. Moreover the field within the sphere is uniform, parallel to the axis of rotation and equal to $2p/a^3$. Hand in hand with the above entity, we shall study another one, an entity of radius a with a surface distribution of positive magnetism on one side of the equator and of negative magnetism on the other side; so chosen as to cause the entity to act as a doublet of moment p for points outside the sphere,[5] i.e., points for which r is greater than a. We shall call this a polar entity in contrast to the rotating shell which we shall call a current entity. In the case of the polar entity the field is also uniform within the entity, but it is in the opposite direction to that of the corresponding current entity of moment p, and is equal to $-p/a^3$.

[4] Sometimes another term depending upon the velocity of the medium and the polarization at a point is included in the quantity subtracted from B to produce h. The details of many of these matters are irrelevant to the main purpose of this paper. Some mention of them must be made to provide a complete story; but, no detailed discussion of them will be attempted here. The reader will find reference to them in H. A. Lorentz, Enzyk. der Math. Wiss. **2**, 200–209; also in E. Cunningham, *The Principle of Relativity*, in J. H. Van Vleck, *Electric and Magnetic Susceptibilities* (Clarendon Press), and in W. F. G. Swann, *The Fundamentals of Electrodynamics*, pp. 5–74 (1922); a part of Bulletin No. 24 of the National Research Council.

[5] The elements pertaining to these matters are established in the Appendix to this paper, Problem 1.

Let us start with a picture in which the entities are current entities, and let there be n of them per cc. The vector B represents the average value of H where the regions inside and outside the entities are included. Suppose now we change to the corresponding polar case, and take the volume average of the true field H. It will differ from the former average on account of the change of the fields inside the entities from $2p/a^3$ to $-p/a^3$. It will, in fact, be $B-(3p/a^3)(4\pi a^3/3)n$. Since $np=I$, this amounts to $B-4\pi I$. In other words the average of the true H for the polar case is the quantity h as defined in the Lorentzian theory. This is entirely in harmony with our expectations; for, h is recognized as a vector whose line integral between two points is the line integral of the true H for the polar case.

Suppose that now on taking the average we simply omit the field inside the entities altogether. We obtain

$$B-(2p/a^3)(4\pi a^3/3)n=B-8\pi I/3=h+4\pi I/3.$$

In the case where the entities are very small in volume, the omission of the contributions of the fields inside them to the average is the equivalent of evaluating the average for the regions outside. This average is then $h+4\pi I/3$; and, it would appear that, obtained in this way, the status and meaning of this vector is clearer than when evaluated according to the principles cited in connection with the primitive electrostatic and magnetostatic theory.

2. The Passage of Electrons Through the Magnetized Medium

We shall confine our discussion to the case where the magnetic entities are rotating charged spheres of the type already referred to. We shall consider an electron constrained to move in a linear path parallel to the z axis which is perpendicular to the vector I which shall be parallel to the x axis. The time integral of the force on, i.e., the momentum imparted to, the electron parallel to the y axis is

$$F_y=\int (v_z/c)H_x dz.$$

For all particles of cosmic-ray energy we may put $v=c$ in the above expression. For lower energies, v is at least constant to a first approximation; and, indeed, would be absolutely so for the case of a path constrained to be always along the z axis. For the case $v_z=c$, we have

$$F_y=\int_0^L H_x dz,$$

where L is the path length.

Consider a length M parallel to the y axis; and, perpendicular to it, draw a number of the above paths parallel to the plane y, z. All of the above integrals have the same value if the path L is long enough. Now the flux of H_x through the plane of areas LM is

$$\int_0^M\int_0^L H_x dydz=\int_0^M dy\int_0^L H_x dz=M\int_0^L H_x dz.$$

Since the integral with regard to z is independent of y as above stated. Thus if $\bar{H}_x$ is the average, value of H_x along the path L, the above flux through the plane of area LM is $ML\bar{H}_x$. Let there be planes of the above kind drawn perpendicular to the axis of x for a length N thereof. Then, the volume average of H_x, which is B_x, is

$$B_x=(LMN)^{-1}\int_0^N\int_0^M\int_0^L H_x dxdydz$$
$$=(LMN)^{-1}\int_0^N dx\int_0^M dy\int_0^L H_x dz.$$

Now if the path L is long enough, the integral with regard to z is independent of x and y, and is $\bar{H}_x L$, where $\bar{H}_x$ is the average value of H_x along the path L, and is a quantity independent of y and z. Thus

$$B_x=(LMN)^{-1}NML\bar{H}_x$$

so that $\bar{H}_x=B_x$

as was, of course, expected to be the case.

Now in spite of the fact that the average magnetic vector $\bar{H}_x$ responsible for deflecting the electron is B_x, this average is secured in a manner which invites comment and which leads to ultimate results different from those to which a naïve consideration of the theory would lead. It may be remarked here that the problem of visualization of the details of the elements involved is one which is apt to invite doubts founded upon intuitive consideration of special cases. However, the whole story is perfectly consistent in the long run. We shall later discuss one of these special cases, but for the moment we shall confine attention to a fortuitous distribution of entities. For such a distribution, all paths parallel to a fixed direction have the same history in the long run. The contribution to $\bar{H}_x$ along a path is made up from contributions from entities which it does not thread, and from contributions by entities which it does thread. The contribution from the latter source is quite a large proportion of the whole contribution by the magnetization of the iron, even in the case of entities of very small dimensions, when the event of threading is very rare. As a matter of fact, it turns out, as will presently appear, that the contribution in question is independent of the size of the entity for an assigned value of I_x.

In order to avoid interrupting the line of thought, we shall quote here a result proved in Appendix (Problem 2), to the effect that the average contribution by an entity to the line integral along a path which threads it is $8p/3a^2$ from the inside and $-2p/3a^2$ from the outside, making $2p/a^2$ where a is the radius of the entity.

Consider a tube of length l along whose axis a particle flies. It passes through the entities whose centers are contained within a cylinder of radius a. If n is the number of entities per cc, the number in the cylinder is $\pi a^2 nl$, and the contribution to the above-named line integral is $2\pi pa^2nl/a^2 = 2\pi I_x l$, since $pn = I_x$. It will be observed that this result is independent of the size of the entity for a given value of I_x. Small size entities correspond to very infrequent contributions of very large amounts to the line integral.

The contribution per unit length to the line integral is $2\pi I_x$, and this represents the contribution of the entities threaded on a path to the average magnetic vector along that path which is responsible for producing deflection in an electron traversing it. The effect of any entity threading a path in the fortuitous distribution is confined to a length of the path which is small compared to the distance between it and the next entity which threads the path, for small entities. *Thus, if the thickness of the material traversed is small compared with what we may call the mean free path between the entities,*[6] *the effective field along the path will be* $\bar{H}_x - 2\pi I_x = B_x - 2\pi I_x = h_x + 2\pi I_x$.

Now it is admitted that however thin may be the material which is traversed by the electrons, a parallel beam containing many point electrons traversing it will experience a deflecting force determined on the average by B_x; for, the many short paths taken in different places through the material average in their characteristics to those of a long path through the material. However, while the average deflection is determined by B_x, that average will be produced, in the case of a thin slab of small entities, by a process in which nearly all of the electrons experience a deflection determined by $B_x - 2\pi I_x$, while a very few, which pass through entities will have very large deflections which will bring the average deflection up to that calculable from B_x. As a matter of fact, the complete story may be formulated according to the usual statistical methods, leading to a law of distribution of scattering; and, in this story the large deflections experienced by electrons passing *near* but not through entities would figure. Such a formulation would probably have but little meaning on the basis of the present analysis in view of the restricted nature of the simplifying fundamental assumptions involved. The essential fact is that, in the case of a sufficiently thin piece of matter traversed by a beam of point electrons, any practical experiment would reveal $B_x - 2\pi I_x$, or in other words, $h_x + 2\pi I_x$, as the deflecting vector for the case of magnetic entities of very small size. In such experiments as those of Alvarez,[7] the number of electrons experiencing the large deflections necessary to bring the average deflecting vector up to B_x would be too small to observe. In the case of the experiments of Danforth and the writer, published in 1934[8] and now in detail in the present issue of the *Physical*

[6] To fix our ideas, suppose that every orbital electron in a piece of iron figured as an entity, and suppose, taking a cubic centimeter of iron lying on a sheet of paper we could precipitate all of those electrons onto the piece of paper. The total area which they would occupy on the paper would be $28\pi a^2 N$, where N is the number of atoms of iron per cc, a is the radius of the electron, and the 28 refers to the number of electrons in an atom of iron. Now $28N$ is 23×10^{23}. If we should take for the classical radius 2×10^{-13}, the area covered would be only 0.3 of the square centimeter. In other words, a point particle shot through a centimeter of iron would probably miss all the entities.

[7] L. Alvarez, Phys. Rev. **45**, 225 (1934).

[8] W. F. G. Swann and W. E. Danforth, Phys. Rev. **45**, 565 (1934).

Review, the measurements involved finding the number of rays which were deviated by more than a certain assigned amount by the magnetic field. In such experiments, the small number of electrons deviated through large angles (in the case of small entities) are unable to exhibit their potency in contributing to the average; and, the results are to all intents and purposes determined by the group whose deflection is determined by $B_x - 2\pi I_x$.

Of course, it is only when the magnetic entity is of very small size or the material is very thin that the effective vector becomes $B_x - 2\pi I_x$. In general, for larger sized entities, the effective vector may lie anywhere between $B_x - 2\pi I_x$ and B_x. A complete set of experimental data should be capable of revealing the effective size of the entity, although here, again, the imperfections of the fundamental assumptions would probably render such a determination of little significance.

The present problem has some of the features of the alpha-particle scattering problem. It is rather curious that in that problem interest has been centered on the single scattering and has concerned itself but little with the small deviations produced by the fields between the scattering nuclei, while in the magnetic problem interest has centered on the effect of the average magnetic vector in the medium and has concerned itself but little with the scattering produced by close encounters.

It may be thought that, in the foregoing discussion, the concentration of attention on electrons which actually go *through* the entities is misleading, since an electron passing very near to an entity experiences forces comparable with those which it will experience in passing through it. The point is that, as will be shown in the Appendix (Problem 2) the average contribution to $\overline{H}_x$ by paths passing near but not through entities is zero, so that while such paths result in large electronic deflections, they do not disturb the conclusion that, in the case of entities of very small size an overwhelmingly large proportion of the paths experience a magnetic vector determined by $B_x - 2\pi I_x$. On the other hand the paths which thread the entities do make a significant contribution to the true average $\overline{H}_x$, raising it from the practically measured $B_x - 2\pi I_x$, to the ideal value B_x.

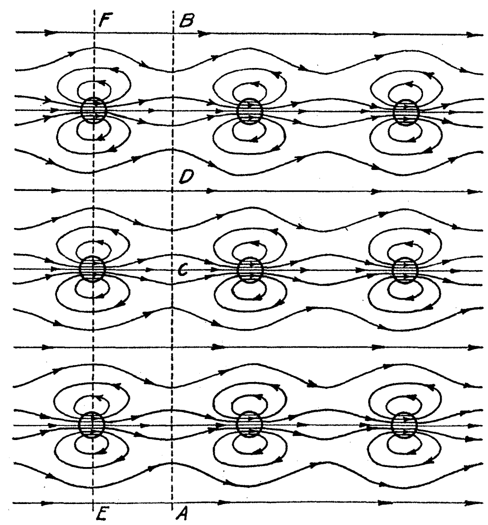

Fig. 1.

A special case

The consideration of certain special cases may raise questions in the mind as to the importance of the part played by the interiors of the entities in contributing to B. A particularly drastic special case is one in which the entities are arranged at the corners of cubes, in a cubical array. Suppose, for example, the sides of the cubes are parallel to the directions we have taken for the x, y and z axes. Suppose Fig. 1 represents a plan of the lattice seen as one looks in a direction parallel to the axis of z. The entities are shown by the circles. Consider a plane perpendicular to the paper, and containing the dotted line AB. It is obvious that the average flux across the plane is B_x. An electron moving in this plane will never pass through an entity, yet, the average deflecting force experienced by such electrons will be determined by B_x. Of course, two electrons traveling perpendicular to the paper so as to pass through D and C, respectively, will not experience the same deflection, for the field is by no means uniform over the plane AB.[9]

Suppose now we consider a plane perpendicular

[9] It is of interest to note that the contribution of the nearest entities to the field is very considerable. Thus the field f due to an entity at a distance r from its center is of the order $2p/r^3$. If l is the distance between the entities, the number n of entities per cc is $1/l^3$. Thus $f = 2pnl^3/r^3$. Now $pn = I$, so that if $r = l/2$, we have, for the contribution of a single entity to the field at a distance from it equal to half the distance between entities the value $4I$. This is comparable with the whole vector $h + 4\pi I$.

to the paper and containing the line EF. The average flux through this plane is again B_x, but, the flux is very unequally distributed. If l is the distance between the entities, there are nl entities per square centimeter; and, the part of the flux which is contributed by the entities themselves and *which passes through their own small cross section is* $(2pnl/a^3)\pi a^2$, which amounts to $2\pi Il/a$; and, for small entities would amount to far more than $h+4\pi I$, so that the discrepancy is taken care of by a reversed field outside the entities as shown. Thus, while electrons traveling perpendicular to the paper in the plane defined by EF would experience, on the average, a deflecting force determined by B_x, there would be an enormous variation around that average, extending even to an important range of negative values. An electron passing downward through a line of entities, would pass through $1/l$ entities per centimeter of path. This would be an enormous quantity compared with the mean free path calculated for the fortuitous distribution of entities, referred to above, and exemplified in magnitude by the calculation in foot-note 6. An electron traveling down through a line of entities as above would experience an enormous deflecting momentum as compared with the average experienced by the electrons. Of course, it would in actuality be deflected out of the line at the first encounter in question.

The symmetry of arrangements of fields introduced by the cubical arrangement of entities results in a cooperation of influence which prevent our concluding immediately that, in the case of a combination of small entities and small thickness of material, an overwhelming proportion of paths would experience a deflecting force determined by $B_x—2\pi I_x$.

Appendix

Problem 1. Magnetic field of a uniformly charged shell rotating with uniform angular velocity

Maxwell* shows that the magnetic potential of unit circular current is given by Ω', where

$$\Omega'=2\pi(1-\nu^2)\sum_{n=1}^{\infty}\frac{1}{n+1}\left(\frac{a}{r}\right)^{n+1}P_n(\mu)\frac{dP_n(\nu)}{d\nu}\quad \text{for } r>a$$

$$\Omega'=-2\pi\left\{1-\nu+\sum_{n=1}^{\infty}\frac{1-\nu^2}{n}\left(\frac{r}{a}\right)^{n}P_n(\mu)\frac{dP_n(\nu)}{d\nu}\right\}\quad \text{for } r<a$$

where the symbols have the following meaning. The origin is the center of a sphere of radius a. The potential Ω' applies to the point r, θ, in space. The potential is caused by a circular current of unit amount, flowing in an infinitely thin wire lying on the sphere, symmetrically with the axis of symmetry and subtending an angle 2α at the origin. μ and ν are written for $\cos\theta$ and $\cos\alpha$, respectively, and the P's are the ordinary Legendre polynomials.

As applied to our problem, the potential $d\Omega$ due to the portion of the rotating sphere contained between α and $\alpha+d\alpha$, rotating with angular velocity ω and charged to a surface density σ electromagnetic units is obtained by multiplying each of the above expressions for Ω' by $\sigma\omega a\sin\alpha d\alpha$, by $-\sigma\omega a d\nu$. The complete potential is then obtained by integrating from $\nu=1$ to -1. The only terms surviving the integrations are the terms for which $n=1$, and the term $1-\nu$ for the case $r<a$. We thus, obtain for Ω, the magnetic potential

$$\Omega=-(4/3)\pi\sigma\omega(a^3/r^2)\cos\theta\quad \text{for } r>a$$

and

$$\Omega=2\pi\sigma\omega a(2+(4r/3a)\cos\theta)\quad \text{for } r<a.$$

The external field is consequently that produced by a doublet of moment $p=4\pi\sigma\omega a^3/3$, and the internal field is uniform, parallel to the axis of symmetry and equal to $2p/a^3$.

The polar case is solved in J. J. Thomson's *Elements of Electricity and Magnetism*,† from which it will be seen that a suitable distribution of positive and negative magnetism on the sphere will cause the sphere to act like a doublet of moment p at external points, and to have a field uniform and equal to $-p/a^3$ inside.

Problem 2. Calculation of the average contribution of an individual entity to the line integral of H_x along a path which treads the entity

Following the scheme already used, let the axes of x, and of z, be parallel, respectively, to the polarization and to the line of flight of the particle. Let $r^2=x^2+y^2$, and let φ be measured in the x, y planes. Let there be a uniform density of flux of electrons, and let J be the average contribution of an entity to the line integral in question. J is composed of 2 parts, a part J_1 arising from the contribution to the line integral outside the entity, and a part J_2 arising from the contribution inside the entity. We have

$$\pi a^2 J_1=2\int_0^a dr\int_0^{2\pi} r d\varphi\int_\eta^{\infty} H_x' dz, \qquad (13)$$

where H_x' represents the contribution to H_x by the entity in question, the 2 is introduced to take account of the two symmetrical contributions before entry and after exit, and

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* Maxwell, *Electricity and Magnetism*, Vol. 2, third edition, p. 333.

† J. J. Thomson, *Elements of Electricity and Magnetism*, third edition, pp. 223–226.

where

$$\eta=(a^2-r^2)^{\frac{1}{2}} \text{ if } r<a;\ \eta=0 \text{ if } r>a. \quad (14)$$

We include the case $r>a$ in anticipation of the discussion of the passage of electrons near to but not through the entity. Now

$$H_x'=-p[(x^2+y^2+z^2)^{-3/2}-3x^2(x^2+y^2+z^2)^{-5/2}]$$
$$=-(p/r^3)[(1+z^2/r^2)^{-3/2}-(3x^2/r^2)(1+z^2/r^2)^{-5/2}]. \quad (15)$$

If we substitute this expression in (13) and put $\tan\theta=z/r$, so that $\sec^2\theta=1+z^2/r^2$ and $dz/r=\sec^2\theta d\theta$; we obtain

$$\pi a^2 J_1=-2p\int_0^a\frac{dr}{r}\int_0^{2\pi}\left[\int_{\theta_0}^{\pi/2}\cos\theta d\theta-\frac{3x^2}{r^2}\int_{\theta_0}^{\pi/2}\cos^3\theta d\theta\right]d\varphi. \quad (16)$$

The limit θ_0 corresponds to η. From (14) we see that

$$\theta_0=\sin^{-1}(1-r^2/a^2)^{\frac{1}{2}} \text{ if } r<a;\ \theta_0=0 \text{ if } r>a. \quad (17)$$

Thus

$$\pi a^2 J_1=-2p\int_0^a\frac{dr}{r}\int_0^{2\pi}\left[1-\frac{2x^2}{r^2}-\left(1-\frac{3x^2}{r^2}\right)\sin\theta_0-\frac{x^2}{r^2}\sin^3\theta_0\right]d\varphi. \quad (18)$$

If x were replaced by y, the φ integral in (18) would be the same as before. Hence we may replace x^2 by $(x^2+y^2)/2=r^2/2$, and the integral will be unaltered. By doing this

$$J_1=2\frac{p}{a^2}\int_0^a(\sin^3\theta_0-\sin\theta_0)\frac{dr}{r}. \quad (19)$$

Using the expressions for θ_0 given above, for $r<a$ we find

$$J_1=-2p/3a^2. \quad (20)$$

Now with regard to the integral J_2, we observe that the field inside the sphere, and resulting from the current, is $2p/a^3$. Thus

$$\pi a^2 J_2=\frac{2p}{a^3}\int_0^a dr\int_0^{2\pi}rd\varphi\int_{-\eta}^{\eta}dz=\frac{8\pi p}{a^3}\int_0^a(a^2-r^2)^{\frac{1}{2}}rdr,$$
$$J_2=8p/3a^2. \quad (21)$$

Eqs. (15) and (16) establish the results quoted above, and lead to $J=2p/a^2$.

Problem 3. Calculation of the average contributions of an individual entity to the line integral of H_x along a path which does not thread it

This case corresponds to $r>a$. The calculation proceeds exactly as for J_1 except that, in (18) and the following equations, the integral with respect to r extended from 0 to ∞, and the limit θ_0 is zero, since $r>a$. Thus, (19) becomes replaced by

$$J_2=\frac{2p}{a^2}\int_0^\infty(\sin^3\theta_0-\sin\theta_0)\frac{dr}{r}$$

where θ_0 is zero. Hence $J_2=0$.

APRIL 15, 1936 PHYSICAL REVIEW VOLUME 49

The Deflection of Cosmic-Ray Charged Particles in Passing Through Magnetized Iron

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(Received February 24, 1936)

Experiments are described in which the deflections suffered by cosmic-ray electrons in passing through the saturated iron core of an electromagnet are detected by means of Geiger counters. The observed effects are compared with the results of calculations in which we have used the energy distribution as found by Anderson and Neddermeyer with the cloud chamber. In this way we have found it possible to set limits to the effective magnetic vector within the iron. One of our experiments indicates a value lying between the induction B and $B/2$. The other points to the limits $3B/4$ and $B/4$. A theoretical discussion is included in which it is pointed out that all electrons of the same energy will not necessarily experience the same deflection but will show a statistical distribution of deflections with an arithmetic average corresponding to the induction B. The present type of experiment, however, does not give a true arithmetic average and would be expected to indicate, for the effective deflecting vector, a quantity less than B to an extent dependent upon the particular geometrical arrangement.

Introduction

The negative results obtained by B. Rossi[1] and by L. M. Mott-Smith[2] in their attempts to realize deflection of cosmic-ray electrons in magnetized iron has excited speculation as to the correctness of using the induction B rather than the magnetic intensity h as the vector determining the force which such an electron experiences. Indeed, Rossi's first experiment gave magnetic deflections no more than comparable with the experimental error. His second experiment[3] did result in a small effect, which, however,

[1] B. Rossi Accad. Lincei, Atti **11**, 478 (1930).

[2] L. M. Mott-Smith, Phys. Rev. **39**, 403 (1932).

[3] B. Rossi, Nature **128**, 300 (1931).