Note on Electron-Neutron Interaction

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Assuming a possible short-range interaction between electrons and neutrons of the form $V=K\delta(\mathbf{r}_N-\mathbf{r}_e)$ where δ is the Dirac delta function and $\mathbf{r}_N$ and $\mathbf{r}_e$ are the vector positions of electron and neutron, respectively, it is shown that probably $|K|<30mc^2(e^2/mc^2)^3$ from consideration of the effect of the interaction on slow neutron scattering cross sections. If K is positive and a little less than this upper limit this interaction could be responsible for the observed isotope displacement of spectral energy levels.

IN most theoretical speculations of nuclear physics it is usual to omit consideration of terms in the Hamiltonian corresponding to short range forces between electrons and neutrons. In this note the situation is examined more closely.

Dee[1] showed that fast neutrons do not produce more than one ion pair in three meters of air path at normal conditions by interaction with the electrons. As there are 15 electrons in an air molecule and 2.7×10^{19} molecules/cm^3 this result says only that the cross section between electrons and neutrons (of about 5 MEV energy) is less than 8.3×10^{-24} cm^2. But this is not a very small upper limit being in fact larger than the cross section for scattering of fast neutrons by protons.

It is generally felt that the actual cross section must be very much smaller than this. If we suppose the interaction between the neutron and electron to be represented by a potential well (*or* wall) of depth (*or* height) V and radius of action a then the wave function of an electron's motion relative to the neutron is given by

$$\Delta\psi+(2m/\hbar^2)[W-V(r)]\psi=0$$

and the approximate solution due to Born for the scattering is

$$\psi=e^{ikz}-\frac{1}{4\pi}\frac{2m}{\hbar^2}\int\exp\,(i\Delta\mathbf{k}\cdot\mathbf{r})\,V(r)dv\cdot e^{ikr}/r$$

where $\hbar\Delta\mathbf{k}$ is the vectorial change in momentum of the electron in the scattering. The second term becomes $(\pm2ma^3V/3\hbar^2)e^{ikr}/r$ so in the usual way this gives for the total cross section

$$\sigma=4\pi(2ma^3V/3\hbar^2)^2$$

on the assumption that the electron wave-length associated with the change of momentum is large compared to the range of the interaction forces. A "reasonable" value for V is mc^2 and for a is (e^2/mc^2) so on this basis the expected "reasonable" cross section for scattering of electrons by neutrons is $\sigma=(16/9)\alpha^4\pi a^2$, where $\alpha=e^2/\hbar c$ or numerically, $\sigma=1.24\times10^{-32}$ cm^2. To get a cross section as large as Dee's upper limit one would need to have V equal to $2.6\times10^4mc^2$ if one keeps to the same range. This is "unreasonable" nor would it be "reasonable" to get the larger cross section by retaining mc^2 for V and increasing the range of action of the forces. Of course this calculation is really for a free electron rather than an electron bound in an atom but since for a 5 MEV neutron the energy of relative motion of electron and neutron (2500 volts) is large compared to the binding energy it is probable that the order of the result is not much different from the result of a calculation which does not neglect the binding.

Such considerations have been more or less generally known for some time. They make it desirable to have another approach which gives a smaller upper limit to the possible interaction. This is provided by consideration of the possible effect of electron-neutron interaction on the scattering of slow neutrons.

When the neutron energy is small, of the order of thermal energy, there is no possibility of excitation of the atom by the neutron impact so we are dealing with an elastic collision. The usual molecular considerations apply here to indicate that the slow moving neutron will

[1] Dee, Proc. Roy. Soc. **A136**, 727 (1932).

move in an effective potential field given by considering the normal electronic level of the "molecule" of atom+neutron. If we assume a force law of negligible range (alongside of atomic dimensions) we may write $V=K\delta(\mathbf{r}_N-\mathbf{r}_e)$ where K is the $4\pi Va^3/3$ of the square potential well, and the other is the Dirac δ function in three dimensions normalized so its volume integral is unity.

Then if $\rho(r_e)$ is the electron density in the atom and the neutron interaction is supposed weak enough not to affect it the potential energy of the system atom+neutron differs by $V(r)=K\rho(r)$ from the energy with the neutron absent. The scattering of slow neutrons by the atom will be governed by the wave equation

$$\Delta\psi+(2M/\hbar^2)[W-K\rho(r)]\psi=0,$$

in which M is the reduced mass of neutron and atom and W is the energy of the internal or relative motion.

The elastic scattering is given by an expression of the same form as before, with these differences, that now we are dealing with a long range force determined by the distribution in space of the bound electrons and the mass is here M instead of the electron mass m. The wave-length is now comparable with the range of the forces so the integral is a little more complicated. However it is the same function of electron density as the structure factor used in x-ray scattering work. Calling the coefficient of e^{ikr}/r in the scattered wave $f(\theta)$ we have

$$f(\theta)=-(4\pi)^{-1}\int \exp ik(\mathbf{n}_0-\mathbf{n})\cdot\mathbf{r}K\rho(r)(2M/\hbar^2)\,d\tau$$

$$=(2M/4\hbar^2)\int_0^\infty \frac{\sin \kappa r}{\kappa r}K\rho(r)r^2dr,$$

where $\kappa=(4\pi \sin \theta/2)/\lambda$ and λ is the de Broglie wave-length of the relative motion, $h/(2MW)^{\frac{1}{2}}$. By introducing the structure factor $F(\kappa)$ as defined on p. 140 of Compton and Allison, *X-Rays and Electrons*, this is

$$f(\theta)=-(2M/\hbar^2)K(4\pi)^{-1}ZF(\kappa)$$

so, as usual the differential cross section for scattering into $d\omega$ is

$$|f(\theta)|^2d\omega=(K^2M^2Z^2/4\pi^2\hbar^4)F^2(\kappa)d\omega.$$

The total cross section is then

$$\sigma=\sigma_0\frac{2}{\kappa^2}\int_0^\kappa F^2(\kappa)\kappa d\kappa$$

with $\kappa=4\pi/\lambda$ and $\sigma_0=4\pi(2MZK/\hbar^2 4\pi)^2$. Since $F^2(\kappa)\rightarrow 1$ for $\kappa\rightarrow 0$ it follows that σ_0 is the limiting value of the cross section.

The value of σ_0 in this case is larger than for the scattering of fast, free electrons by the factor $Z^2(M/m)^2$. Thus for scattering of slow neutrons by atomic hydrogen one need have K only 1/920 as great to get the same cross section as for the previous work.

We have next to consider the value of the integral over the form factor occurring in the cross section expression. For a first orientation the electron distribution is satisfactorily given by the Fermi-Thomas statistical method and for this distribution the form factor may be found in Compton and Allison, p. 148. From this the integral was calculated as a function of J where J is a complicated expression which reduces to $J=6.6V^{\frac{1}{2}}Z^{-\frac{1}{3}}$ where V is the value of the relative energy in electron volts. We find for $G(J)$ where $\sigma=\sigma_0 G(J)$

J	$G(J)$	J	$G(J)$	J	$G(J)$	J	$G(J)$
0.0	1.00	0.6	0.18	0.3	0.38	0.9	0.11
.1	.74	.7	.15	.4	.30	1.0	.09
.2	.53	.8	.13	.5	.23	1.5	.05

The average relative energy Ve is equal to $\frac{1}{2}Mv^2$ where v is the average relative velocity of the neutron and the atom, if the atoms are at rest and the neutrons have room temperature thermal energy, $V\sim 1/40$. For hydrogen then $J\sim 1$ so the correction owing to the structure factor amounts to a factor of about 0.1. In addition there is another factor which undoubtedly plays a role here—the zero-point energy of oscillation of the atoms in a molecule.[2] For hydrogen this may amount to $V\sim\frac{1}{4}$ volt or ten times the thermal energy. This tends to cut down the cross section still more. In comparing hydrogen and deuterium the zero-point energy

[2] This idea that the zero-point oscillations of the hydrogen in a molecule may affect the slowing of neutrons by matter was suggested by Professor Rabi in a conversation last fall. See also Halban and Preiswerk, Nature **136**, 951 (1935) for experimental indications of such an effect.

will be less in the latter so this would tend to make the deuterium cross section greater. Also the difference in the reduced masses relative to the neutron introduces a factor of 16/9 in the ratio of the cross sections. Both work however to make the hydrogen cross section less than that for deuterium whereas Dunning[3] finds 35×10^{-24} cm^2 for hydrogen and 4×10^{-24} cm^2 for deuterium.

For heavier atoms the zero-point energy is negligible and owing to the appearance of $Z^{-\frac{1}{3}}$ in J the form factor average $G(J)$ does not change by more than a factor of two. So the electron-neutron interaction would vary smoothly as Z^2. As there is no trace of such a "Moseley law" for the slow neutron cross sections we must conclude that this possible manifestation of the interaction contributes less than 10^{-24} cm^2 to the slow neutron scattering cross section of hydrogen.

If this be so one can set the limit

$$|K| < 30mc^2(e^2/mc^2)^3,$$

which is much lower than the limit

$$|K| < 2.6\times10^4 mc^2(e^2/mc^2)^3,$$

which one sets from Dee's study of the upper limit of the ionization produced by fast neutrons.

Another way in which electron-neutron interaction would manifest itself is in the isotope displacement of spectral terms. If the interaction energy is of very short range type as we have been supposing then the energy difference between a heavier and a lighter isotope due to this interaction will be K times the total electron density at the origin for each neutron added to the nucleus. The particle density at the origin is zero except for s electrons, and the only contribution that will be observable in the shift of frequency of a spectrum line will be that from the particular s electron that is involved directly in the optical transition which produces the line.

Accordingly the energy is raised by $K\psi^2(0)$ for each neutron if the electron-neutron interaction is repulsive, where $\psi^2(0)$ is the particle density at the origin for a valence electron. Breit[4] has discussed the observed isotope shifts from the standpoint of the effect of departure from the Coulomb law associated with a change in nuclear radius for the different isotopes. The formulas occurring in that discussion also require a knowledge of $\psi^2(0)$ so we may use the values that he calculates to estimate how large K must be to explain the entire effect as due to electron-neutron interaction. Here the levels are observed to be raised in going to heavier isotopes indicating electron-neutron *repulsion* if this interaction is mainly responsible for the isotope shift. The sign of K is not determined by the scattering considerations so this is a new bit of information.

In the $6s$ term of Hg II there is a difference of 0.52 cm^{-1} between Hg^{204} and Hg^{202}, that is, 0.26 cm^{-1} per neutron assuming the effect on the lines is all due to a shift of the s term. Breit calculates $\psi^2(0)=1.45\times10^{-26}$ cm^{-3} for the $6s$ electron. Writing $K=kmc^2(e^2/mc^2)^3$, the whole shift is accounted for if $k=20$, which is consistent with the upper limit obtained from consideration of the slow neutron cross sections. The same value is obtained for Tl I $7s$ using $\psi^2(0)=0.17\times10^{-26}$ cm^{-3} and an observed $\Delta\nu=0.03$ cm^{-1} per neutron. The data for Hg I $7s$ indicate $k=5.5$ so that there is no quantitative consistency except as to order of magnitude.

In conclusion, the foregoing discussion shows that electron-neutron interaction if of the form $K\delta(\mathbf{r}_N-\mathbf{r}_e)$ must have $K<30mc^2(e^2/mc^2)^3$ to be in accord with slow neutron scattering data; and if K is about of this magnitude, say from $\frac{1}{6}$ to $\frac{2}{3}$ of this upper limit, and is positive, such interaction could be the main source of the isotope displacement of spectral lines.

[3] Dunning, Pegram, Fink and Mitchell, Phys. Rev. **48**, 265 (1935).

[4] Breit, Phys. Rev. **42**, 348 (1932).