Absolute Values of the Electron Mobility in Hydrogen

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(Received November 22, 1935)

A new method is described for the measurement of electron mobilities in gases. An electrical shutter method is employed in which the shutters take the form of two fine wire grids, alternate wires of which are connected to a high frequency alternating potential. Electrons pass through the grids only when the potential between adjacent grid wires is zero, and only electrons which cross the gas space in one half-cycle are received at the collecting electrode. A sharp maximum is thus observed in the electrometer current when the drift velocity of the electron multiplied by the time of one half-cycle is equal to the distance between the grids. The theoretical shape of the current curve is compared with experiment and good agreement observed. Measurements have been made between X/p of 0.03 and 20, and the values obtained are compared with the Compton mobility equation.

EARLY in the study of phenomena associated with the conduction of electricity in gases, it was observed that under certain conditions and in certain gases, the drift velocity of the negative carriers reached extremely high values. These velocities were of the order of a thousand times those which could be ascribed to carriers of molecular dimensions, and were correctly explained by Franck and Townsend as a characteristic of free electrons moving through a gas. Since these observations various measurements have been made of the drift velocity or mobility of electrons. In general, however, the results of different observers, while of the same order of magnitude, have not been in particularly good agreement. Furthermore the results have not been sufficiently accurate to permit any theoretical test to be made with confidence.

The importance of determining absolute values of electron mobilities in gases is twofold. Their magnitude is intimately associated with many phenomena involved in the electrical breakdown of gases, and a knowledge of its value is essential for the application of theory to experiment. In addition the drift velocity of an electron through a gas under the influence of an electrical field furnishes an excellent test of theories of electron scattering, collision, and energy transfer. Since most direct experimental observations of these quantities become difficult or impossible at very low electron energies, the observations of electron mobility form a most useful method of studying these properties for electron energies between 0 and 10 volts.

It is the purpose of this paper to describe a new and accurate method for the absolute determination of electron mobilities and to present the initial results on the mobilities of electrons in hydrogen.

EXPERIMENTAL METHOD

The principle of the method lies in the application of the electron-ion filter designed by Loeb and developed by Lusk and Cravath[1, 2] to the electrical shutter method of measuring ion mobilities first used by Van de Graaff.[3] A diagram of the apparatus is given in Fig. 1. Electrons which are emitted from the lower zinc plate, P, by ultraviolet light pass upward through the gas under the influence of a uniform field. Between alternate wires of the grids G and G' is applied a high frequency alternating potential whose mean value is the value of the uniform field at that point. The frequency and magnitude of this field may be varied within wide limits. In general, electrons which reach G will be swept out to the grid wires (if the potential between them is high enough) and no current will be transmitted. If, however, the magnitude of the potential between adjacent grid wires is reduced, an intermittent electron current will eventually pass through, consisting, obviously, of those electrons which have reached the grid at a time when the alternating field was zero or nearly so. Under these circumstances a series of pulses of electrons will pass through the grid and be carried by the uniform field towards the grid G'. This grid is subjected to an alternating field of the same frequency, magnitude, and phase as the grid G.

[1] A. M. Cravath, Phys. Rev. **33**, 605 (1929).
[2] N. E. Bradbury, Phys. Rev. **44**, 883 (1933).
[3] R. J. Van de Graaff, Phil. Mag. **6**, 210 (1929).

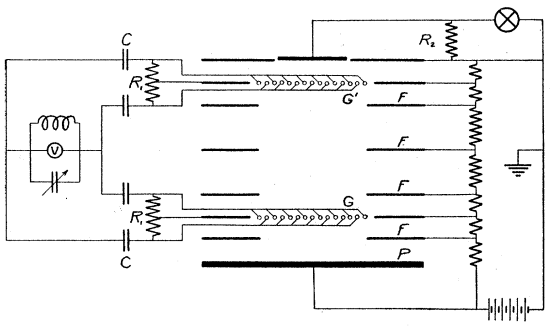

FIG. 1. Schematic diagram of apparatus. $R_1 = 10^6\Omega$ and $R_2 = 10^9\Omega$. $C = 0.01$ mf.

If a given electron pulse reaches this grid at any time other than when the voltage wave is again passing through zero, it will be swept out to the grid wires, and no current will reach the collecting electrode. However, if the drift velocity of the electrons is such that they traverse the distance between the grids and reach G' in exactly one half-cycle (or a multiple thereof) the voltage between the grid wires will be zero, and a current will pass through to the collecting electrode. Thus, varying either the speed of the electrons or the frequency of the alternating potential will give a sharp maximum in the electrometer current at that combination of frequency and speed which permits the electrons to cross the space between the grids in exactly a half-cycle. Since the time for this passage is given directly by the frequency, and the distance between the grids is known, the drift velocity and mobility may be immediately calculated.

In the experimental tube the distance between the grids G and G' was 5.93 cm. The grids themselves were of No. 40 copper wires mounted 1.0 mm apart on mica frames. They were connected as shown in the figure by short symmetrical leads to a tank circuit which was in turn coupled to a radiofrequency oscillator. The frequency of the latter could be varied between 10^4 and 10^7 cycles/sec., and the potential applied to the grids between 0 and 200 volts. Guard rings, $F \cdots F$, maintained a uniform field in the space through which the electrons moved. The apparatus was sealed in a Pyrex tube with a graded quartz seal for the admission of ultraviolet light from a quartz mercury arc. The apparatus could be baked out at a moderate temperature and pumped down prior to the admission of gas to less than 10^{-7} mm Hg.

Since the current which is passed by a grid is a rapidly varying function of the voltage between adjacent grid wires, it was found easier in practice to fix the oscillator at a constant frequency and voltage and vary the speed of the electrons by varying the total potential applied across the tube. This gives a current voltage curve which has a maximum at a value of X/p (field strength in volts per cm and pressure in mm Hg) to which the electron velocity, calculated from the frequency, corresponds. It is necessary to make a slight correction for current magnitude to the experimental readings since the total current leaving the lower plate is a function of the potential across the tube, but this is quickly determined and applied.

EXPERIMENTAL RESULTS

The hydrogen used in these experiments was obtained from a commercial tank of the gas and purified by passing over hot copper gauze and through a series of traps immersed in liquid air. Experiments in which the liquid air was replaced by solid CO_2 gave identical results. No negative ions could be detected at the pressures used (1.5 mm to 50 mm), even after the gas had been in the tube for several days. Frequencies were measured by a vacuum tube wave meter which had been calibrated against broadcast frequencies and their harmonics. The potentials applied to the grid were measured by a Compton electrometer with a phosphor bronze suspension. One pair of quadrants was removed from the instrument which was used idiostatically.

A typical experimental curve is shown in Fig. 2. This was taken at a pressure of 6.8 mm and a frequency of 2.205×10^5 cycles/sec. No special care was taken to obtain sharp peaks in this curve since it was desired to show the three peaks on the same scale. Maxima are observed at voltages of 390, 119, and 62 corresponding to values of X/p of 5.73, 1.75, and 0.91. Peak A is due to electrons which cross in one half-cycle, B in one cycle, and C in three halves of a cycle. The velocities which correspond to these orders are evidently 26.5, 13.2, and 8.84×10^5 cm/sec., respectively.

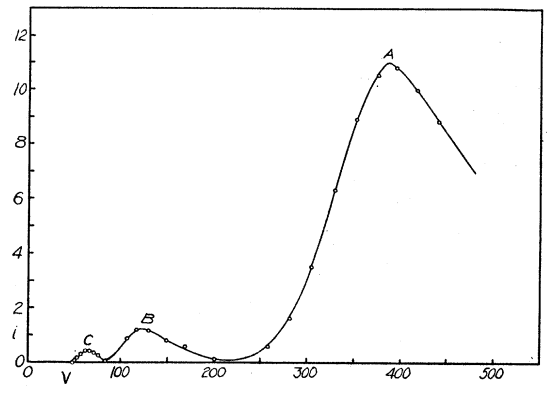

FIG. 2. Typical experimental curve showing peaks due to electrons which cross space between grids in $\frac{1}{2}$, 1, and 3/2 cycle.

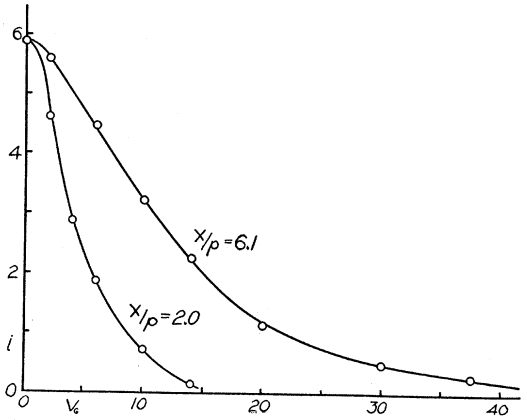

FIG. 3A. Current transmitted by a single grid when a d.c. potential is applied between adjacent grid wires. Pressure = 6.8 mm.

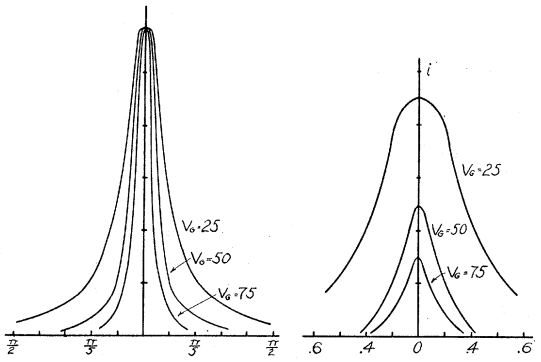

FIG. 3B (left). Form of current pulse transmitted by a single grid for different peak a.c. voltages.

FIG. 3C (right). Calculated form of current peaks received by collecting electrode as a function of phase of arrival of electrons at second grid.

Obviously, it is desirable to have the experimental peaks as sharp as possible, and it is also of interest to examine the shape and magnitude of the experimental curves as compared with the theoretical. Accordingly an investigation of these points was carried out. The theoretical efficiency of the grids can be calculated from the electrostatics of such a grating provided the electrons follow the lines of force. At these pressures, however, there is considerable diffusion and in consequence the actual efficiency is less than that calculated in this manner. Hence it was found advisable to determine the grid efficiency experimentally. This was accomplished by applying a fixed d.c. potential between the grid wires in place of the usual alternating potential. The fraction of current transmitted by a single grid was then measured as a function of this voltage. Such curves are shown in Fig. 3A for different electron velocities. With these curves it is easy to calculate the shape of an electron pulse given by a sinusoidal wave form, obtaining the curves shown in Fig. 3B. Finally with *two* grids, the resultant current received by the electrometer as a function of the phase of the arrival of the electron pulse at the second grid may be determined and such curves are exhibited in Fig. 3C. It is seen that the breadth at half-maximum decreases as the grid voltage is increased, and hence the peaks may be made as narrow as the sensitivity of the current-measuring device will permit. In practice a Compton electrometer of sensitivity 10,000 mm/volt gave ample accuracy in determination of the peak.

An example of a first-order peak under the conditions given in the caption is shown in Fig. 3D. The calculated curve, now as a function of voltage, is shown in the dotted line. The agreement in form of the two curves is most gratifying. Actually, there should be another small correction applied to the experimental curves to take account of the variation in grid efficiency with the velocity of the electron. This, however, is a very small effect for a peak of ordinary breadth and makes no measurable change in the position of the maximum. In addition the position of the peaks was found to be independent of the grid voltage, and measurements in either the first or second orders, at the same X/p, gave identical values.

A summary of the data taken at a temperature

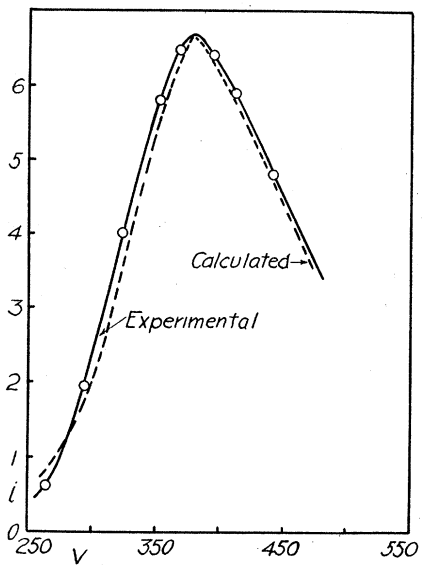

FIG. 3D. Comparison of calculated and observed peaks in first order with a pressure of 6.8 mm and a frequency of 2.2×10^5 cycles/sec.

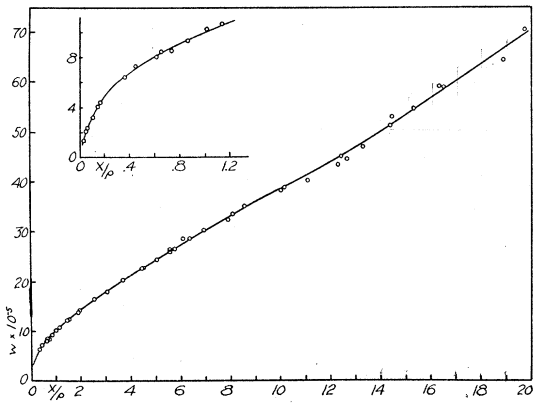

FIG. 4. Drift velocity of electrons in hydrogen as a function of X/p.

of 20°C between values of $X/p=0.03$ and 20 is shown in Fig. 4. The actual drift velocity, w, of the electrons is plotted instead of the mobility, inasmuch as this is the measured and generally more useful quantity. The mobility constant, $K_{20°}$, may be obtained immediately by the equation

$$K=w[(X/p)760]^{-1}. \quad (1)$$

The agreement of successive experimental points taken at different pressures and in different orders is very good. At the highest values of X/p, the oscillator voltage could not be raised high enough to obtain sharp peaks, and the difficulty of measuring closely pressures of the order of 2 mm cause some of the highest values to be slightly less accurate.

The values of the drift velocity obtained in this experiment lie between those of Townsend and Bailey[4] (magnetic deflection method) and those of Loeb[5] (Rutherford a.c. method). They agree better with the former at high X/p, where a consideration of the magnetic method would lead one to expect the greatest accuracy, and lie fairly close to Loeb's at low X/p. Hence it is believed that the present values represent quite accurately the value of the drift velocity of electrons in hydrogen over the range of X/p studied.

DISCUSSION

A theoretical derivation of an equation for electron mobilities has been given by Compton.[6] In this derivation it is assumed that the collision of an electron and a molecule is that of elastic spheres, and hence that scattering is equally probable in all directions. Furthermore, the mean free path under such a definition has a real meaning and is taken as $(N\pi\sigma^2)^{-1}$ where N is the number of molecules per unit volume, and σ the molecular collision radius. Each collision is then assumed to be completely elastic and the fraction of electron energy transferred from electron to molecule is $4m/M(\sin^2\theta)$, where m and M are the masses of the electron and molecule, and θ is the angle through which the relative velocity of the particles is turned at a collision. Equating the energy gain per mean free path from the field to the energy loss by momentum transfer at collision, an equation for the drift velocity, w, as a function of X/p is obtained which is of the form

$$w=\frac{1.115\times10^{-10}(X/p)(1/\sigma^2)T^{\frac{1}{2}}}{[1+(1+1.265\times10^{-32}(X/p)^2 1/\sigma^4 f)^{\frac{1}{2}}]^{\frac{1}{2}}}, \quad (2)$$

[4] Townsend and Bailey, Phil. Mag. **47**, 873 (1921).

[5] L. B. Loeb, Phys. Rev. **19**, 24 (1922).

[6] K. T. Compton, Rev. Mod. Phys. **2**, 123 (1930); L. B. Loeb, *Kinetic Theory of Gases* (McGraw-Hill, New York, 1934), p. 600; Morse, Allis and Lamar Phys. Rev. **48**, 412 (1935)) have derived an expression for the electron drift velocity which is essentially the same as that of Compton if the thermal energy of the molecules is neglected. There is a slight (10 percent) difference in the numerical constant. Davydov (Physik. Zeits. Sowjetunion **8**, 59 (1935)) has made similar calculations and reached the same result although his final equation has a numerical error.

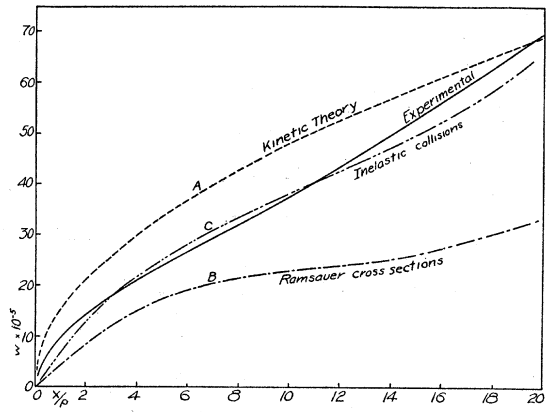

FIG. 5. Comparison of experimental and calculated curves based on the Compton electron mobility equation. Curve *A* is calculated with spherical scattering and cross section equal to the kinetic theory value: Curve *B*, spherical scattering, but Normand's values of the hydrogen molecule cross section; and Curve *C* assumes inelastic collisions with Normand's cross sections.

in which f is the fraction of its energy which an electron loses at a collision.

Assuming for the moment that the collisions are those of elastic spheres, the average energy loss becomes $2m/M$. Taking the kinetic theory value of $\sigma=1.045\times10^{-8}$ cm, and substituting M for H_2, one obtains curve A in Fig. 5. It is seen that in general the theory predicts a drift velocity of the proper order of magnitude, but that there are rather serious discrepancies. These pertain to both the character and trend of the curve, and the actual magnitude of the values predicted. However, it is a well-known fact that the scattering cross section of molecules varies greatly with the energy of the colliding electron. Townsend[7] has given some values of the electron energy in hydrogen as a function of X/p which were obtained by measuring the lateral diffusion of a beam of electrons passing through a gas. For the range of X/p in which we are interested, these energies lie between 0 and 3 electron volts; electrons of one electron volt average energy occurring at an X/p of approximately five. With the help of this information, one may calculate w from (2) using the data of Normand[8] on Ramsauer cross sections in H_2. This gives curve B of Fig. 6. This again fails to agree in magnitude with the experimental curve, but an improvement is seen in the general trend. This arises from the fact that Ramsauer cross sections in H_2 decrease rapidly with increasing energy, thus increasing w.

The discrepancy in magnitude, which is far greater than can be accounted for by possible experimental inaccuracies in the observation of Ramsauer cross sections, must still be considered. The above calculations have been based on the assumption that the scattering is spherically symmetrical. Actually the experiments of Ramsauer and Kollath[9] have shown that in the case of hydrogen and helium there is a pronounced tendency towards backwards scattering at very low electron energies (0–10 volts). This will tend to increase f since the energy loss depends upon $\sin 2\theta$. No great benefit results, though, even if it is assumed that *all* the electrons are scattered backwards, since this will merely double f and increase w by a factor of approximately $2\frac{1}{4}$. It seems inescapable that the average fraction of its energy which an electron can transmit to a molecule must be greater than that indicated by classical momentum transfer. In diatomic molecules, the comparatively close spacing of rotational and vibrational levels make such transfers energetically possible, and it has been suggested that such energy losses do occur, even when the colliding electron does not have sufficient energy to raise the molecule to a higher electronic state. The mechanism accomplishing this must have its origin in a perturbation of the nuclear binding forces by the presence of the slow colliding electron resulting in the possible excitation of a vibrational level according to the Franck-Condon principle. Such inelastic impacts have not infrequently been observed.[10] Ramien[11] has investigated the energy loss of slow electrons in hydrogen and found that on the average a 4.15-volt electron loses 0.02 volt energy per collision. Hence it seems most likely that the main cause of the lack of agreement between the experimental and the theoretical curve lies in the choice of the factor f. If it be assumed that the factor f is sixteen times the classical elastic value at $X/p=20$ (a not unreasonable value in view of Ramien's results) and also that

[7] J. S. Townsend, J. Frank. Inst. **200**, 563 (1925).
[8] C. E. Normand, Phys. Rev. **35**, 1217 (1930).
[9] Ramsauer and Kollath, Ann. d. Physik **12**, 524 (1932).
[10] H. Baerwald, Ann. d. Physik **76**, 829 (1925); W. Harries, Zeits. f. Physik **42**, 26 (1927); also Loeb, reference 5, p. 603.
[11] H. Ramien, Zeits. f. Physik **70**, 353 (1931).

f increases linearly with electron energy, one obtains the surprisingly good curve C shown in Fig. 6. This agreement is probably partially fortuitous, and a more rigorous development must eventually involve allowances for the effects of varying cross section, scattering angle, and inelasticity of impact in detail. The general character of the problem, however, seems to be indicated by the above considerations, and furnishes further evidence for the inelasticity of electron impact in molecular gases.

These experiments are being continued to determine electron mobilities in other gases. The authors desire to thank the Research Committee of Stanford University for a grant which has made these investigations possible.

MARCH 1, 1936 PHYSICAL REVIEW VOLUME 49

Note on the Quantum-Mechanical Theory of Measurement

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(Received November 12, 1935)

In recent notes by Einstein, Podolsky and Rosen and by Bohr, attention has been called to the fact that certain results of quantum mechanics are not to be reconciled with the assumption that a system has independently real properties as soon as it is free from mechanical interference. We here investigate in general, and in abstract terms, the extent of this disagreement. When suitably formulated, such an assumption gives to certain types of questions the same answers as does quantum mechanics; this is true of the formulas usually given in discussions of the theory of measurement. There exists, however, a general class of cases in which contradictions occur. That such contradictions are not restricted to the abstract mathematical theory, but can be realized in the commonest physical terms, is shown by the working out of an example. .

1. INTRODUCTION

SOME time ago there appeared a paper by Einstein, Podolsky and Rosen[1] entitled "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" The writers concluded that the answer must be negative, on the ground that quantum mechanics forbids simultaneous measurement of two noncommuting variables even when both variables simultaneously possess "physical reality," in the sense that either might be measured "without in any way disturbing the system." Recently Bohr[2] has upheld the view that quantum-mechanical description of nature can be considered complete, by demonstrating how the restrictions on simultaneous measurement which it imposes are inherent in the character and use of the measuring instruments. These measuring instruments must always be included as part of the physical situation from which our experience is obtained, and by doing this one sees that quantum mechanics provides a complete and peculiarly apt interpretation of experience.

Bohr has again clearly called attention to this circumstance, and has remarked that one must be careful not to suppose that a system is an independent seat of "real" attributes simply because it has ceased to interact dynámically with other systems. The paper of Einstein, Podolsky and Rosen has shown the sort of situations in which this characteristic of quantum mechanics may become especially prominent. This indicates an extension of the usual discussions of the theory of measurement.[3] In the present note a discussion more comprehensive in this respect will be summarized, and some further consideration will be given to the possibility of illustrating the point in question in concrete physical terms.

We shall have to make use of the concepts and results presented in von Neumann's rigorous and

[1] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935). Referred to as EPR.

[2] N. Bohr, Phys. Rev. **48**, 696 (1935).

[3] Cf. W. Heisenberg, *The Physical Principles of the Quantum Theory*, particularly pp. 55ff.; J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Chap. VI; W. Pauli, *Handbuch der Physik*, Vol. 24, No. 1, pp. 143ff., p. 165.