On the Process of Space Quantization

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The effect of a rapidly varying weak magnetic field on an oriented atom possessing nuclear spin is discussed. The results are applied to an experiment by Frisch and Segrè. It is shown that by methods embodying these principles one can measure nuclear spin even when the h.f.s. separation is very small. A possible further application is discussed whereby it is possible to measure the sign of nuclear magnetic moment vector with respect to the spin vector.

IN two papers[1] under the equivalent of the title carried by this paper Phipps and Stern and later Frisch and Segrè in the same laboratory have studied experimentally the following problem; a beam of neutral potassium atoms on traversing an inhomogeneous magnetic field is split up into two beams (Stern-Gerlach experiment). One of these beams is selected by means of a slit and permitted to pass through a region in which the field changes rapidly both in direction and magnitude. The resultant beam is then analyzed by means of a third field similar to the first. The question is what will be the resulting space quantization with respect to the field. This question has been discussed theoretically by Güttinger[2] and a complete solution for a particular type of field which is very simple to realize experimentally has been given by Majorana.[3] It is this latter type of field which was used by Frisch and Segrè.

However in evaluating their experimental results these authors neglected to consider the effect of nuclear spin. It is the purpose of this paper to complete the interpretation of this interesting experiment. It will appear that experiments of this type may be used to measure nuclear spin even in cases where the hyperfine structure (h.f.s.) separation of the levels is extremely small. Further it will be shown that by using these nonadiabatic processes it is possible to measure the sign of the nuclear magnetic moment, i.e., to discover whether the nuclear moment is parallel or antiparallel to the angular momentum vector. Such information is of particular interest with regard to the proton and deuteron moments, and cannot at present be obtained in any other way.

THEORY OF THE EXPERIMENT

An atom moving with constant velocity through a magnetic field varying in strength and direction along its path is equivalent, for these questions, to an atom at rest subject to a field varying in time in the same manner. The results of the theory may be summarized in the statement that if the angular velocity of rotation of the field is small compared with the Larmor frequency $\omega=2\pi g\mu_0 H/h$ the atom will remain space quantized with respect to the field direction with the same component m of its total angular momentum F (adiabatic transformability); if the angular velocity is of the same order of magnitude as the Larmor frequency there will be nonadiabatic transitions to states $m'\neq m$.

For the exact theory including nuclear spin we will consider:

(a) The magnetic levels in an external magnetic field

The discussion will be limited to atoms in a normal $^2S_{\frac{1}{2}}$ state, since the normal states of H, D and of the alkalis fall under this head. An atom with nuclear spin I will have two energy states in zero external magnetic field with angular momentum (in units of $h/2\pi$) given by $F_1=I+\frac{1}{2}$ and $F_2=I-\frac{1}{2}$. The energy difference between these states due to the interaction of the nuclear and electronic spins is $\Delta W=hc\Delta\nu$. If the magnetic moment of the nucleus is positive the state with $F=I+\frac{1}{2}$ will have the higher energy level and if the nuclear magnetic moment is negative the opposite will hold. In an external magnetic field each of these levels splits into its

[1] T. E. Phipps and O. Stern, Zeits. f. Physik **73**, 183 (1931); R. Frisch and E. Segrè, Zeits. f. Physik **80**, 610 (1933).

[2] P. Güttinger, Zeits. f. Physik **73**, 169 (1931).

[3] E. Majorana, Nuovo. Cim. **9**, 43 (1932).

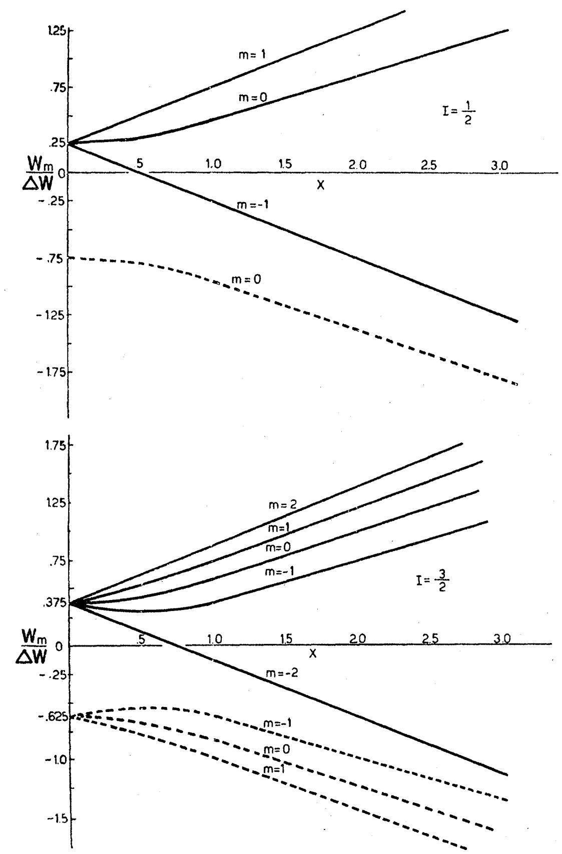

FIG. 1a. Variation of the energy with the magnetic field. Nuclear moment assumed positive. The dotted lines are the magnetic levels arising from the $F=I-\frac{1}{2}$ state.

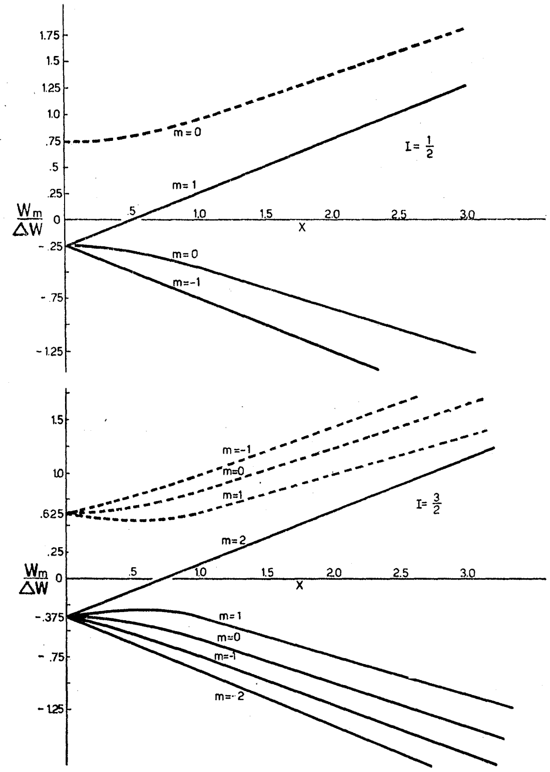

FIG. 1b. Same as 1a but with nuclear moment negative.

$2F+1$ magnetic levels. The energy shift of each level measured with respect to energy of the state unperturbed by either external magnetic field or nuclear and electronic-spin interaction is[4]

$$W_m=\frac{-\Delta W}{2(2I+1)}\pm\frac{\Delta W}{2}\left(1+\frac{4m}{2I+1}x+x^2\right)^{\frac{1}{2}} \quad (1)$$

$$x=2\mu_0 H/\Delta W \quad (2)$$

if the nuclear moment is positive and

$$W_m=\frac{\Delta W}{2(2I+1)}\mp\frac{\Delta W}{2}\left(1-\frac{4m}{2I+1}x+x^2\right)^{\frac{1}{2}} \quad (3)$$

if negative. H stands for the external magnetic field, μ_0 for the Bohr magneton. The upper sign in Eqs. (1) and (3) is taken for the levels arising from $F_1=I+\frac{1}{2}$ and the lower for $F_2=I-\frac{1}{2}$. All states with a given m are double except when $m=\pm(I+\frac{1}{2})$.

In Fig. 1 the energy, together with the assignment of each level to the magnetic quantum number and F from which it arises, is plotted as a function of the magnetic field in terms of x for $I=\frac{1}{2}$ and for $I=3/2$. The diagrams are similar to those giving the transition from the anomalous Zeeman effect to the Paschen-Back effect, but simpler because the nuclear moment may be neglected compared with the electronic moment.

In deflection experiments the effective magnetic moment of the atom is the important quantity. The effective moment for each state is proportional to the slope of the energy curves and is given by

$$\mu_m=-\partial W_m/\partial H,$$

$$\mu_m=\mp\frac{(2m/2I+1)+x}{(1+(4m/(2I+1))x+x^2)^{\frac{1}{2}}}\mu_0 \quad (4)$$

[4] Breit and Rabi, Phys. Rev. **38**, 2082 (1931).

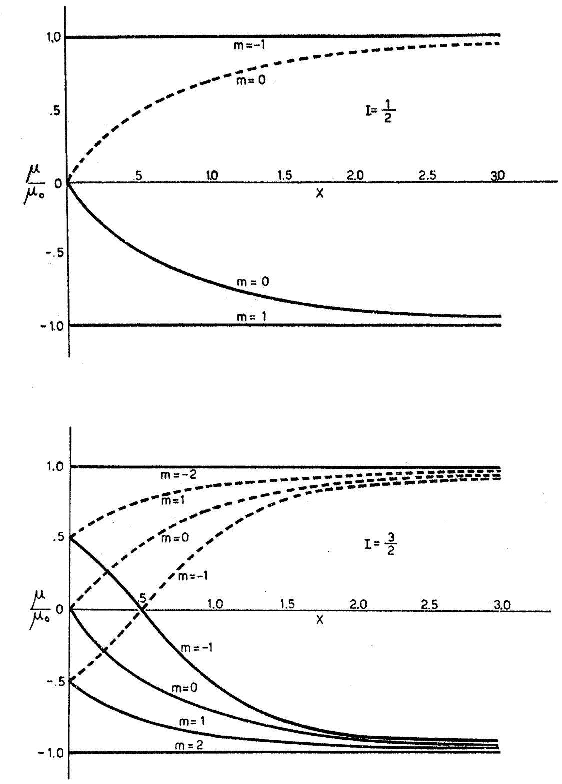

FIG. 2a. Variation of moment with magnetic field. The dotted lines are the moments of the magnetic levels arising from the $F=I-\frac{1}{2}$ state. Nuclear moment assumed positive.

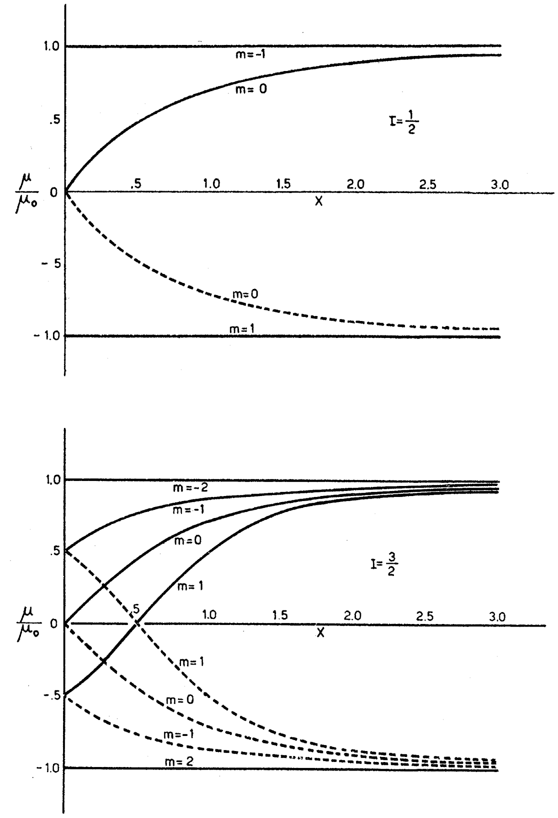

FIG. 2b. Same as Fig. 2a, but with nuclear moment assumed negative.

for nuclear moment positive, and

$$\mu_m = \pm \frac{x-2m/2I+1}{(1-(4m/(2I+1))x+x^2)^{\frac{1}{2}}}\mu_0 \qquad (5)$$

for nuclear moment negative. The same sign convention holds as for the expressions for the energy. The upper sign is for states arising from $F=I+\frac{1}{2}$ and the lower from $F=I-\frac{1}{2}$. The values of the moments are plotted as functions of x in Fig. 2. The dotted curves represent the moments of the $F=I-\frac{1}{2}$ levels. At high fields these moments all become $\pm\mu_0$ to a very close approximation.

(b) The effect of the varying field

As previously stated, the influence of the varying field will in general cause transitions from the state m to other states m'. These transition probabilities are given by Majorana as

$$W(\alpha, m, m') = (\cos \alpha/2)^{4F} \times (F+m)!(F+m')!(F-m)!(F-m')! \cdot \left[\sum_{\nu=0}^{2F} \frac{(-1)^\nu (\tan(\alpha/2))^{2\nu-m+m'}}{\nu!(\nu-m+m')!(F+m-\nu)!(F-m'-\nu)!}\right]^2. \qquad (6)$$

The value of α which occurs in this expression is obtained from the dynamical theory of the process which also shows that it is the same for all m.

The field which is considered is the field in the neighborhood of a point where the field vanishes. The value of the field is to a sufficient approximation proportional to the distance from this point. If d is the distance of closest approach of the path of the atom to this point, the minimum value of the field is $A=bd$. If we define $C=bv$ we have from Majorana's calculation

$$\alpha = 2 \text{ arc sin } e^{-\pi k/4}, \qquad (7)$$

$$k = (2\pi/h)g\mu_0(A^2/C). \qquad (8)$$

The physical significance of this expression can be seen by rewriting $k=((2\pi/h)g\mu_0 A)(v/d)^{-1}$. The first factor is the Larmor frequency at the minimum field in radians per second and the second represents the angular velocity of rotation of the field at this point. If the first is large compared with the second k is large and α small (adiabatic case). When the reverse holds we have the nonadiabatic case as expected.

If the atom has a nuclear spin I, we have two varieties of atoms with $F=I+\frac{1}{2}$ and $F=I-\frac{1}{2}$. In the weak rotating magnetic field which causes the nonadiabatic transition the two states are independent since the energy interval between them is large compared with the energy of the states in the field. We may consider each state separately as if we had a mixture of gases. The magnetic moment for the first species ($F=I+\frac{1}{2}$) is μ_0 and the g value is therefore $1/(I+\frac{1}{2})$. The magnetic moment of the second species ($F=I-\frac{1}{2}$) by Eq. (4) is $(I-\frac{1}{2})/(I+\frac{1}{2})\mu_0$ and the g value is therefore also $1/(I+\frac{1}{2})$. The angle α is thus the same for each state.

The Experiment of Frisch and Segrè

In this experiment both the deflecting and analyzing fields were so strong that the Paschen-Back effect for the decoupling of the nuclear and electronic spins was complete.

On the other hand the Majorana field was so weak that the spins were practically completely coupled. To analyze the experiment from the data one must consider four possibilities; the nuclear moment may be either positive or negative and the selecting slit may have been placed on the side toward the stronger field which selects positive over-all moments of the atom or on the other side which selects negative over-all moments.

From Fig. 2 it is seen that for case I (nuclear moment +, and over-all moment +) the selected beam consists of the $m=-(I+\frac{1}{2})$ magnetic level of the $F=I+\frac{1}{2}$ state and the $2I$ levels of the $F=I-\frac{1}{2}$ state. All the $2I+1$ levels are equally populated. Transitions between the $2I$ levels of the $F=I-\frac{1}{2}$ state do not lead to any change in the high field moment as is evident from Fig. 2. However the transitions which the atoms in the $m=-(I+\frac{1}{2})$ level may make all result in a change in the sign of the high field moment. If all the atoms go over ($\alpha=\pi$) the subsequent analyzing field will split the beam into two separate beams with a maximum intensity ratio of $1/2I$.

In case II (nuclear moment +, over-all moment −) we have selected $2I+1$ out of the $2I+2$ levels of the $F=I+\frac{1}{2}$ state. Obviously transition from one of these levels to another does not lead to a change of sign of the over-all strong field moment. Transition to the $m=-(I+\frac{1}{2})$ moment does change the sign. The fraction of the total number of atoms which make such a transition is

$$\frac{1}{2I+1}\sum_{m=-(I-\frac{1}{2})}^{I+\frac{1}{2}} a^{m}{}_{-(I+\frac{1}{2})}$$

and since $a^{m}{}_{-(I+\frac{1}{2})}=a_m{}^{-(I+\frac{1}{2})}$ this is equal to

$$\frac{1}{2I+1}\sum_{m=-(I+\frac{1}{2})}^{I+\frac{1}{2}} a_m{}^{-(I+\frac{1}{2})}$$

and is thus the same as the fraction which in case 1 made a transition resulting in a change in sign. It is thus evident that no matter which side of the beam is selected the maximum ratio of intensities of the two beams resulting from the analyzing field is $1/2I$. The other two cases give the same result on analysis. The intermediate ratios can be obtained from Eq. (6). The maximum intensity ratio is thus seen as a direct measure of the nuclear spin. Since the Majorana field is very weak the method will apply even when the h.f.s. separation of the states is very small.

In the particular case of potassium which was used by Frisch and Segrè we know from Millman's experiments[5] that for K^{39}, I is 3/2. The ratio of the two peaks of Frisch and Segrè should be $\frac{1}{3}$. To within their experimental error this is the value which they obtained rather than the complete disappearance of the original peak which they expected. If one uses the correct value $g=\frac{1}{2}$ rather than $g=2$ used by the authors, one obtains good agreement with their experimental results as given in Fig. 4 of their paper. This is not the case when nuclear spin is neglected.

[5] Millman, Fox and Rabi, Phys. Rev. **46**, 320 (1934); Millman, Phys. Rev. **47**, 739 (1935).

The Sign of the Nuclear Magnetic Moment

From the spectroscopic methods of measuring nuclear spin the sign of the nuclear moment is obtained by noting which of the F states has the higher energy. To obtain this information with atomic beam methods we must know whether we are dealing with Fig. 2a or 2b. Owing to their symmetry it is impossible to obtain this information by deflection experiments alone. However if deflection is combined with the nonadiabatic transitions, sufficient data can be gathered to deduce the sign of the nuclear moment. However, in a two-field system the arrangement must utilize at least one weak field deflection. The experiment of Frisch and Segrè which used two strong field deflections does not yield sufficient data.

Example 1. Hydrogen

If we first deflect the hydrogen atoms in a weak field corresponding to about $x=0.4$ the deflection pattern has two components $m=\pm 1$, $F=1$ which are deflected more than the components $m=0$, $F=0$ and $m=0$, $F=1$. We can by means of a slit select atoms which are in one of these inner states. We select the atoms which are deflected toward the stronger field (positive over-all moment), pass them through the rotating field, and follow this by a strong analyzing field. If the state we have selected is the $F=0$ state then no transitions are possible and we obtain only one component. If the state is $F=1$, $m=0$, there will be transitions to the other levels with the nonadiabatic field properly chosen. Some of these levels have moments of opposite sign which the analyzing field can resolve into two components. If one component is obtained the proton moment is positive; if two are observed the moment is negative. A check is obtained by a similar observation of the side deflected into the weaker field, where the opposite situation prevails. The angle α should be $\pi/2$ for these transitions as is evident from Eq. (6). Many modifications of this procedure, but dependent on the same principle will readily suggest themselves.

Example 2. Potassium

Although the pattern as given in Fig. 2 for $I=3/2$ is more complicated than for $I=\frac{1}{2}$, a similar procedure can be applied. With a weak deflecting field we select the atoms from one of the levels by means of a slit on the strong field side and subject them to the rotating field. In the subsequent strong field analysis the line will be either single or double. If single the nuclear moment is positive, if double, negative. This is so because transition amongst the $F=I-\frac{1}{2}$ levels does not affect the strong field moment, but the $F=I+\frac{1}{2}$ levels always have the possibility of making transition which will result in a strong field moment of opposite sign. The treatment of any other case is along similar lines.

In conclusion the writer wishes to express his indebtedness to Professor E. Segrè for discussions on the details of the Frisch and Segrè experiment and to Messrs. Clark, Heller and Motz of the theoretical physics seminar for discussions on the details of the interpretation of Majorana's paper.

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Does the Alpha-Particle Possess Excited States?

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The kinetic energy operator for the internal motion of the alpha-particle can be expressed without cross derivatives in terms of a suitable set of internal coordinates. The usual methods when applied to a restricted class of Hamiltonian operators then yield sum rules from which are deduced upper limits to the excitation energies of the $2p$ levels. These upper limits involve certain diagonal matrix elements which are easily evaluated by using an approximate normal state wave function. Computations on three different nuclear models indicate definitely the existence of a singlet $2p$ level in the discrete eigenvalue spectrum of the alpha-particle if the range of the intranuclear forces exceeds 2.0×10^{-13} cm. A simple variation calculation gives excitation energies which fall slightly below the upper limit fixed by the sum rules.