Scattering of High Energy Protons in Hydrogen

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7340 photographs of high energy proton tracks in hydrogen were taken and the observed frequency of scattering was compared with Mott's wave-mechanical treatment. Strong anomalies were found when the energy of the incident proton exceeded 600 kv. The anomalies consisted chiefly in finding about ten times too much scattering in the angular range 40°–45°, where this angle is the angle of scattering of the longer tine as measured in the observer's coordinate system. The angular distribution for energies less than 600 kv was found to be in accord with Mott's expression to within the rather large statistical fluctuations. Although only 74 collisions with an energy greater than 600 kv were observed it seems very improbable that the distribution can be entirely accounted for by statistical fluctuations. Since Mott's formula is based upon the assumption of Coulomb forces the experimental discrepancies indicate a departure from the inverse square law when the energy of the incident proton exceeds about 600 kv. This means that for a classical distance of closest approach of about 6×10^{-13} the two protons can no longer be treated as classical point charges, whereas on purely classical grounds there should be no such difficulties down to 1/1846 the electron radius or about 10^{-16} cm.

INTRODUCTION

THE scattering of high energy protons in hydrogen is of especial interest to theoretical physicists because of the supposed fundamental nature of the proton. As is well known the scattering depends upon the character of the forces between the colliding particles and in the case of Coulomb forces the classical and wave-mechanical solutions are exactly the same except where identical particles are involved. Mott[1] has calculated the Coulomb wave-mechanical scattering taking into account the spin and identity of the particles. Blackett and Champion[2] have shown that in the case of alpha-particle helium impacts Mott's formula is probably correct provided the energy is low enough to insure the applicability of Coulomb's law. It was already known that the scattering of low energy alpha-particles by most nuclei was in accord with the Rutherford formula (based on the inverse square field), but that large anomalies arose when the struck nucleus was light and the alpha-particle energy fairly high. The explanation was advanced that the law of force between two particles is only Coulombian at distances greater than about 8×10^{-13} cm since a more intimate approach than this might give rise to polarization forces which would alter the law of force. Many attempts were made to formulate the scattering deviations in this way but the outcome was not entirely successful. More recently Taylor[3] has provided a more complete explanation of the scattering of high energy alpha-particles by helium and hydrogen by assuming that the potential energy of the system may be described in the manner of Gamow, Condon and Gurney. Of course the ultimate nature of the forces necessary to give such a potential function is still not understood. It is to be hoped that some new information may be gained by scattering high energy protons in hydrogen for such collisions are presumably more easily discussed from a theoretical standpoint. On purely classical grounds one would expect proton-proton scattering to be characteristic of Coulomb forces down to distances of 1/1846 the electron radius or 10^{-16} cm. To attain such intimate impacts would require energies not now available in the laboratory, but in view of the many striking results of nuclear disintegration experiments it is well worth while to see if the proton does behave like a classical point charge or whether it too shows scattering anomalies when the distance of closest approach is of the order of the electron radius.

Low energy collisions of protons in hydrogen were investigated by Gerthsen,[4] but the energy

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[1] Mott and Massey, *Theory of Atomic Collisions* (Oxford University Press), p. 73.

[2] P. M. S. Blackett and F. C. Champion, Proc. Roy. Soc. **A130**, 380 (1931).

[3] H. M. Taylor, Proc. Roy. Soc. **A134**, 103 (1932); **A136**, 605 (1932).

[4] C. Gerthsen, Ann. d. Physik **9**, 769 (1931).

available was only 30 kv so it was to be expected that no deviations from Coulomb's law would be found, and indeed his results agreed with Mott's formula. Wells[5] has observed a few swift protons scattered in a hydrogen filled cloud chamber. His results did not differ much from inverse square expectations, but the number of collisions was too few to admit of any strict comparison. In a preliminary report the author[6] presented data which indicated strong deviations when the bombarding energy exceeded 600 kv. The present report confirms and somewhat amplifies the previous findings.

Experimental Method

The apparatus for accelerating the protons to high energies was the magnetic resonance accelerator developed by Lawrence and Livingston.[7] The original apparatus was altered to give a somewhat more energetic beam than was obtained by the author in his earlier experiments on the disintegration of lithium and boron. In addition it was necessary to devise some means of getting the protons out of the accelerating chamber and into a Wilson cloud chamber. The high velocity of the protons and the necessary presence of a high magnetic field for acceleration made it somewhat diffcult to deflect electrostatically the beam away from its spiralling path. Although the problem will eventually find a solution a subterfuge was resorted to in the present case. A thin aluminum foil of 1.5-mm air equivalence was inserted in the beam near the end of its spiralling journey with the result that a few of the protons, on passing through the foil, picked up an electron and became neutral and being no longer restrained by the magnetic field were free to shoot off on a tangent into the cloud chamber. Professor Oppenheimer kindly calculated for me the probability of capture of high velocity electrons under the above conditions and found that about 10^{-4} of the main beam should become neutralized. This result was roughly verified experimentally.

The protons were led into the cloud chamber by a suitable flexible tubing terminated by a thin lacquer window of 1.5-mm air equivalent. In order that the window could withstand a pressure of more than an atmosphere it was cemented to a copper disk which was pierced by five small holes 2.0 mm apart and 0.3 mm in diameter. Such an arrangement served not only to support the window but also to separate the protons into more easily photographed groups. A main beam current of 10^{-14} amp. was sufficient to give several hundred tracks per expansion. However it is not possible to use more than about 50 tracks per expansion because the overlapping of the tracks becomes too severe if this number is exceeded.

It was found experimentally that after an expansion was made the cloud chamber was sensitive for less than 1/60 sec. It is essential that the protons enter the chamber during this interval because only then is the stopping power of the gas known. It is true that the stopping power is known before the expansion is started, but if the particles are allowed to enter then, the general diffusion of the tracks impairs the photography. This diffusion, which is so pronounced in hydrogen, can be reduced by the addition of about 10 percent of oxygen, but doing so shortens the range of the protons by a very appreciable amount with a resulting loss in accuracy and number of collisions. Of course the range of the particles could be increased again by operating at a lower total pressure, but still the frequency of collisions would be smaller than in pure hydrogen. It was found possible to avoid entirely the use of oxygen by proper adjustment of the cloud chamber and timing devices described below.

Ordinarily the construction of a shutter that would admit protons only upon the completion of the expansion would present no difficulties, but the use of a rubber diaphragm type chamber ruled out the simple mechanical schemes. Furthermore the action of the shutter had to be very fast in order that the particles might enter just at the end of the expansion. It was found that a delay of even a few thousandths of a second caused dead spots to appear in the chamber, spots where the tracks were not well developed. The following method proved to be very satisfactory. An electrical switch was built into the bottom of the cloud chamber so that the rubber diaphragm

[5] W. H. Wells, Phys. Rev. **47**, 591 (1935).
[6] M. G. White, Phys. Rev. **47**, 573 (1935).
[7] E. O. Lawrence and M. S. Livingston, Phys. Rev. **40**, 19 (1932).

would strike and close it upon completing an expansion. The closing of the switch ignited a small thyraton which in turn shorted out the high grid bias which had been placed on the oscillator to suppress oscillations. Provided the anode was positive the tube was now free to oscillate and produce a proton beam. The operation of this cycle of events was of course very rapid. The problem of synchronizing the expansion with the proton beam was further complicated by the pulsating nature of the beam. The high frequency oscillator anode voltage was supplied by a 12,000 volt, 60-cycle transformer and as a consequence oscillations take place only during the positive half of the cycle. In addition the acceleration of the protons is efficient only when the anode supply voltage is a maximum; so the net result is a proton beam that arrives in bursts every 1/60 sec. and is of duration not more than 3×10^{-3} sec. It is clear that if expansions are made at random the average time delay between the end of the expansion and the arrival of the protons will be 1/120 sec. The appearance of the tracks obtained under these conditions was quite unsatisfactory. The obvious solution, rectified current for the oscillator anode, is rather expensive so a thyratron timing circuit was devised to cause the cloud chamber to start expanding at a definite time with respect to the 60-cycle voltage applied to oscillator. A complete description is of little interest, but in essence it consisted in activating the cloud chamber valve magnet by a large thyratron which was in turn controlled by the 60-cycle mains.

The cloud chamber was adapted from the recent design of C. T. R. Wilson in which use is made of a rubber diaphragm instead of the customary piston or sylphon. The active volume was 17 cm in diameter and 4 cm deep, a depth sufficient to accommodate any scattered particle. The chamber was filled with ordinary tank hydrogen and water vapor.

A stereoscopic camera designed by Dr. F. N. D. Kurie[8] and already described by him, was mounted over the cloud chamber. The essential features of this camera are its compactness and the ease with which the finished film may be replaced in the camera and tracks measured up by reprojection.

The energy of the particles was determined by measuring the range and reading off the corresponding velocity from Blackett's[9] experimental range velocity curve. A small correction must be made because of the presence of water vapor.

Energy Distribution

The previously reported work was based upon 2340 photographs containing an average of 106 tracks per picture. This series of observations will be referred to as Series A. In order to arrive at the total number of tracks and the distribution of energy among them several photographs in each run of three hundred expansions were examined. Fig. 2A is a histogram based upon several hundred tracks and the dotted curve is the integral energy distribution curve. Such a curve simply shows the number of entering protons which had an energy in excess of the amount indicated by the abscissa. A glance at the typical Series A photograph shown (Fig. 1) indicates the difficulty of actually counting the number of tracks. The histogram previously mentioned was obtained from only those pictures in which it was possible to see each track. The distribution of energy in the more densely filled photographs should be the same since the method of reducing the intensity of the beam does not affect the energy. The determination of the number of tracks in the more dense photographs was carried out in two ways, (1) by actually attempting to estimate the number by comparison with the less densely populated pictures, and (2) by counting the total number of large angle oxygen deflected protons. Since there was a large number of such collisions it was possible to obtain a fair estimate of the total number of protons by comparison with the number of such collisions in the sparsely filled photographs. Both methods are admittedly quite rough but should be accurate to 25 percent. Method 1 gave a result of 200,000 tracks while Method 2 was somewhat higher with 250,000. The larger number was taken as more reliable since the overlapping of tracks would tend to make the directly counted number too low.

Although the overlapping of tracks made impossible the observation of low energy and small

[8] F. N. D. Kurie, Rev. Sci. Inst. 3, 655 (1932).

[9] P. M. S. Blackett, Proc. Roy. Soc. **A135**, 132 (1932).

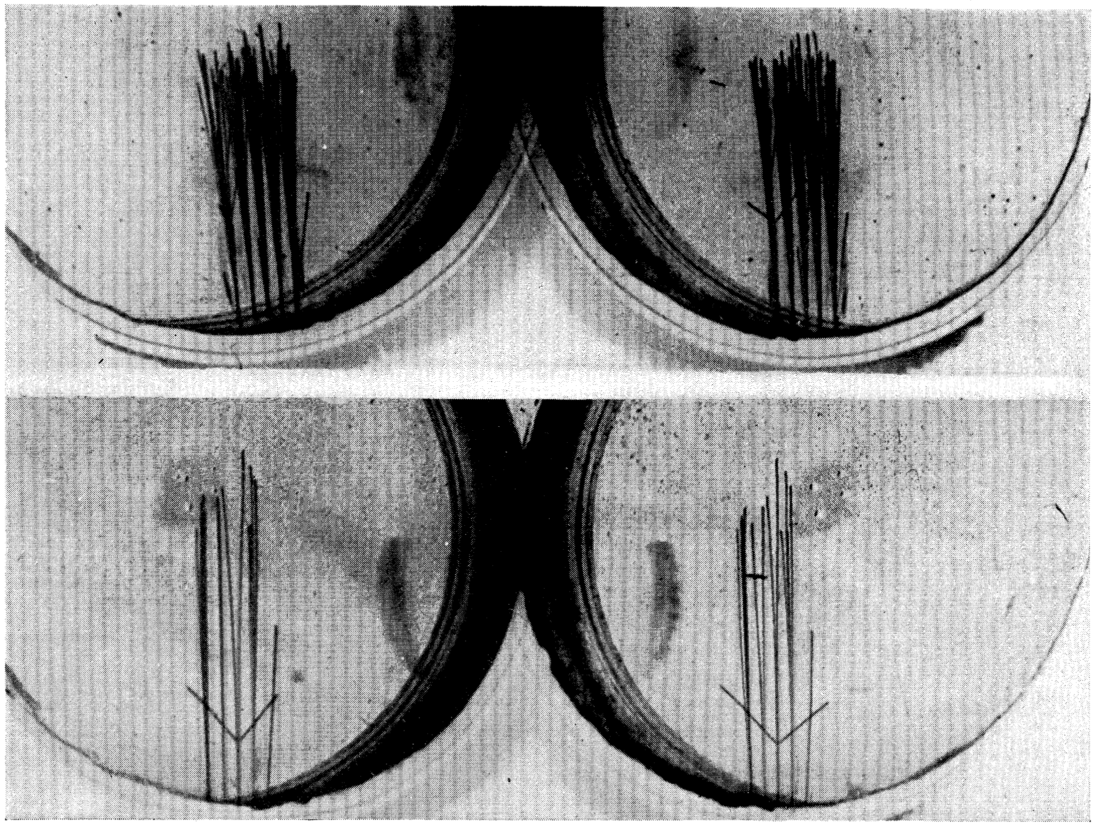

FIG. 1. Stereoscopic photographs of high energy proton collisions in hydrogen. Series A typical photograph above; Series B below.

angle scattering it did not seriously affect the high energy scattering, except at very small angles, because all such events must occur near the beginning of the track. The Series A photograph shows clearly that near the wall of the chamber the five entrant beams are quite well collimated and that the chance of observing a small angle recoiling proton is quite good. A more definite statement regarding the minimum observable angle of scattering will be given when the scattering data are discussed. The above confusion was tolerated because the primary goal of this research was the study of high energy, large angle collisions since such impacts imply the closest distance of approach.

The new data were obtained from about 5000 additional photographs (Series B) in which the average number of tracks per picture was 21 (see Fig. 1). It was possible to determine accurately that 100,000 protons entered the chamber with a distribution in energy as shown in Fig. 2B. Of course the efficiency of observation of small angle and low energy collisions was much increased.

Conservation of Energy and Momentum

Although a collision between swiftly moving protons involves large forces it is not to be expected that electromagnetic energy will be appreciably radiated because of the large masses involved and consequent small acceleration. Assuming that any energy radiated will not alter the momentum of the system one may calculate in terms of the observed angles the percentage change in kinetic energy during the collision.

$$\frac{\Delta E}{E}=\frac{\cos^2(\vartheta+\varphi)-\cos(\vartheta+\varphi)\cos(\vartheta-\varphi)}{\sin^2(\vartheta+\varphi)},$$

where ΔE is the energy change and E is the energy of the incident proton (see Fig. 6). If

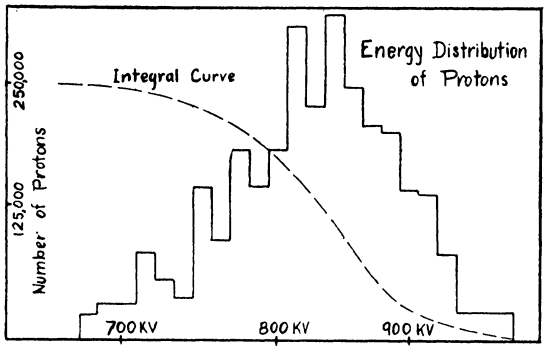

Fig.2A

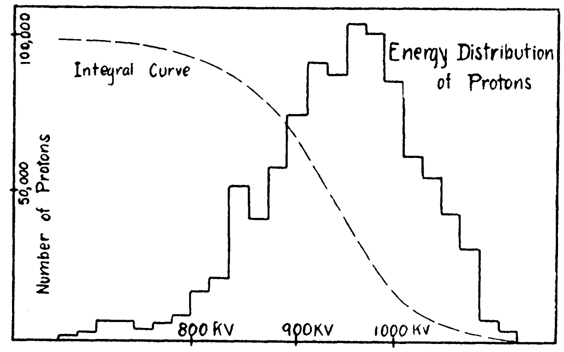

Fig.2B

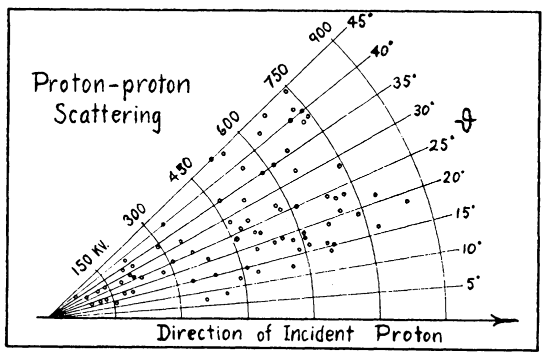

Fig.3A

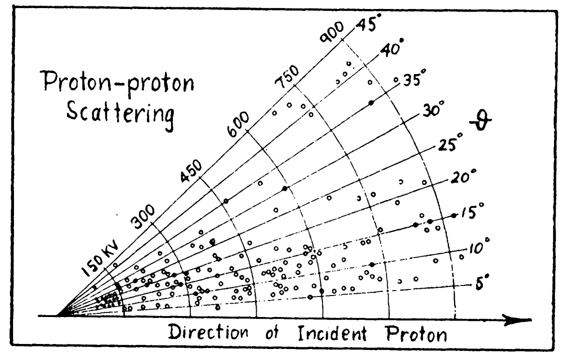

Fig. 3B

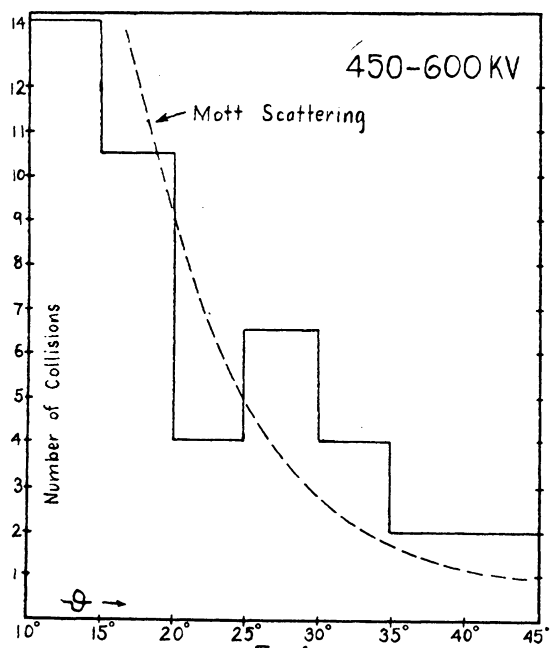

Fig 4

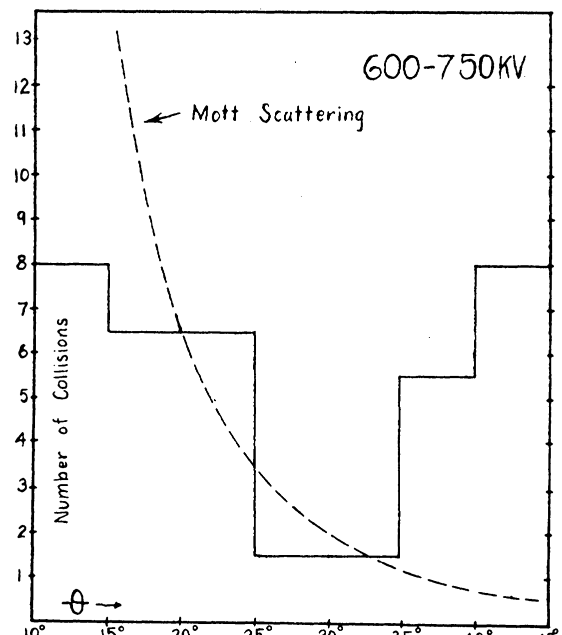

Fig.5

FIG. 2. Histogram of energy distribution of protons entering cloud chamber.

FIG. 3. Angle and energy distribution for proton-proton scattering. A, for Series A photographs; B, for Series B photographs.

FIGS. 4 and 5. Experimentally determined angular distribution of proton scattering compared with Mott's theoretical curves.

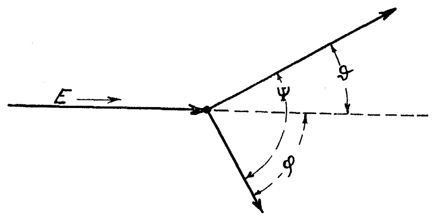

FIG. 6.

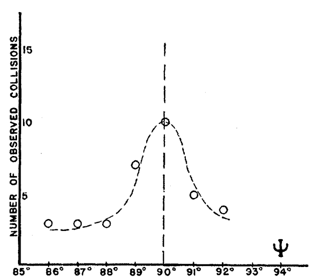

FIG. 7. Number of proton-proton collisions showing angle ψ between tines.

$\Delta E=0$, then $\vartheta+\varphi=\pi/2$. In the special case where ϑ and φ are nearly equal (and hence the energy exchange is large) one may write, putting $\vartheta+\varphi=\psi$,

$$\Delta E/E \cong \cos\psi/(1+\cos\psi).$$

Fig. 7 is a plot of the number of collisions found with an angle ψ between the recoiling tines. The forks chosen for the analysis were ones in which the incident energy was high and the angle of scattering large. It is evident that radiation of energy would cause ψ to collapse while a gain in kinetic energy would increase ψ. Since it hardly seems likely that energy is gained in the collision one must interpret a ψ greater than 90° as an observational error. The similar appearance of the curve below 90° indicates that these points are also largely observational errors. It is certain that in these collisions ψ does not actually deviate more than one degree from 90° and one may conclude that no energy in excess of 20 kv is radiated. Of course the foregoing has no bearing on the possible emission of an almost massless particle or quantum if the energy is derived from internal sources, i.e., a mass change. It should be remarked that if a low ionizing particle, such as a positron, were emitted it would probably not be photographed because the light source was just sufficient for the proper exposure of heavy particle tracks. If the unobserved particle carried much momentum then the observed tracks would not be coplanar. Although occasionally a fork was found which was as much as 3° from being coplanar it was also noticed that usually the incident stem was curved downwards, a phenomenon caused by too great a delay in the photography.

SCATTERING DISTRIBUTION

Figs. 3A and 3B represent all the scattered particles found in the Series A and B photographs, respectively. Each circle indicates a collision in which the incident proton energy is represented by the radius vector and the direction of scattering of the longer tine by the angle made by this vector with the base line. The bleak appearance of the small angle region in Fig. 3A is largely instrumental, at least for energies below 600 kv. A 600-kv proton glancing off at an angle of 13° forms a recoiling tine of only 1.3 mm length in the cloud chamber and the general congestion of tracks makes its detection uncertain. It is clear that the Series B photographs suffered much less from this defect. These last results should be fairly trustworthy down to 200 kv and 10°.

COMPARISON WITH COULOMB SCATTERING

Mott's formula for the scattering of protons by protons may be written,

$$Q=\frac{4\pi N_0 n e^4}{m^2}\int_{R_1}^{R_2}\frac{dR}{v^4}\int_{\vartheta_1}^{\vartheta_2}[\csc^4\vartheta+\sec^4\vartheta$$
$$-\csc^2\vartheta\sec^2\vartheta\cos(\alpha\log\tan^2\vartheta)]\sin 2\vartheta d\vartheta,$$
$$\alpha=2\pi e^2/hv,$$

where Q is the number of protons with velocity between v_1 and v_2 which are scattered between ϑ_1 and ϑ_2. R_1 and R_2 are the ranges of protons of these velocities. N_0 is the number of incident protons of mass m and charge e. dR is the distance traveled by a proton in going from a velocity v to $v+dv$ in a gas containing n hydrogen atoms per cubic centimeter. The angle ϑ is the angle of scattering as measured in the observer's coordinates. The number N_0 is obtained from Figs. 2A and 2B. Since there is a distribution in energy one must take the weighted average number of protons which could produce collisions in a given energy range, the weighting coefficient being concerned with the distance the protons can travel before falling out of the energy range in question. The integral involving the range is most easily evaluated by graphical integration of the experimental relation between $1/v^4$ and the range.

The number of scattered particles calculated by this formula must be divided by two because only one-half of the collisions will be oriented in the proper plane for the system of measuring used in this experiment. Or stated more explicitly, it was not possible to recombine and measure a fork whose plane was inclined more than 45° to the plane of the cloud chamber. This correction applies to all energies and scattering angles.

Fig. 4 is the sum of all collisions found in the two series of observations in which the incident energy of the proton was 450–600 kv. One must not be too deeply concerned over the small number of collisions found in 10°–15° region because as was pointed out the Series A photographs were sadly handicapped here. The total number of protons giving rise to these collisions was 348,000. To within the statistical fluctuations the distribution agrees with Mott's formula.

Fig. 5 is the total scattering for energies between 600 and 750 kv. It is apparent that there is more large angle scattering than can be easily accounted for purely by statistical fluctuations. The probable fluctuation in the region 40°–45° is about three and the chance of observing five or more collisions when the expected number is 0.75 is less than 1/1000. This consideration by itself is not conclusive but the fact that the adjoining region is also anomalous and the fact that the higher energy 750–900 kv (Table I) shows similar scattering makes one believe that there is definitely too much large angle scattering. Regarding the small angle scattering little can be asserted since one is not sure of the efficiency of such observations in the Series A photographs. The Series B pictures also indicate too little scattering at 12° and 17° but the data are insufficient to warrant conclusions.

Table I gives again the results expressed in Figs. 4 and 5 and in addition the scattering at lower and higher energies. Q_0 is the number of observed collisions, R_0 is the ratio of Q_0 to the Mott expected value, and N_0 is the number of protons responsible for the observed collisions. The column 300–450 kv was taken only from the Series B photographs since the Series A observations were obviously in error.

TABLE I. *Summary of proton-proton scattering.* Note: When an observation occurs on the boundary between two cells it counts as one-half in each.

$E=$	300–450 kv		450–600 kv		600–750 kv		750–900 kv	
$N_0=$	1.0×10^5		3.48×10^5		3.0×10^5		1.88×10^5	
ANGLE	Q_0	R_0	Q_0	R_0	Q_0	R_0	Q_0	R_0
10°–15°	7.5	0.3	14.0	0.3	8.0	0.2	3.0	0.2
15°–20°	7.0	0.8	10.5	0.7	6.5	0.6	4.0	0.8
20°–25°	1.5	0.4	4.0	0.6	6.5	1.3	3.0	1.3
25°–30°	2.0	0.9	6.5	1.8	1.5	0.6	0.0	0.0
30°–35°	1.0	0.8	4.0	1.9	1.5	1.0	0.5	0.7
35°–40°	0.0	0.0	2.0	1.4	5.5	5.8	1.5	3.8
40°–45°	0.0	0.0	2.0	1.8	8.0	10.8	3.0	10.0

DISCUSSION

Although the foregoing experimental data do not tell us what the actual form of the scattering distribution is it is nevertheless interesting to see what the theoretical implications are and whether or not it is possible to devise a proton model that will give the observed scattering.

On Dirac's theory of the positron, E. A. Uehling[10] calculated the deviations from a Coulomb potential caused by continual creation and annihilation of electron pairs in the electromagnetic field of the protons. He found that these deviations become of importance at distances of the order of the Compton wave-length (10^{-10} cm) and that the scattering deviations increase with ϑ in the neighborhood of $\vartheta=0$ and then remain essentially constant thereafter. The total deviation is only a few percent and in no way corresponds to the above observations.

Fermi's recent theory of beta-ray emission indicates that perhaps a proton may exist momentarily in the form of a neutron plus a positron plus an antineutrino. If this system lasts an appreciable time then one would expect scattering deviations when two such systems collide. However, at present the theory is much too arbitrary to admit of any conclusions.

It is of interest to treat proton-proton collisions in a manner similar to Taylor's work on alpha-

[10] E. A. Uehling, Phys. Rev. **48**, 55 (1935).

particle helium scattering. Dr. R. Serber has very kindly made these calculations for me. The analysis consisted in finding what phase shifts in the scattered waves are necessary to account roughly for the observed scattering at 700 kv. The scattered waves may be identified with the angular momentum of the colliding particles. Serber found that assuming a 45° shift in the *S* wave and 2.5° shift in the *P* wave would be sufficient for the present accuracy of the experiment. To represent the data exactly would require several more phase shifts, but this would imply long range forces and one is not ready to admit the possibility. Fig. 8 shows the ratio of Serber's modified scattering to Mott's scattering. The experimentally observed ratios for collisions of 600–750 kv are shown as circles with the probable fluctuations indicated by the vertical line segments. It might be mentioned here that there is a one-percent chance of observing a fluctuation of more than twice the probable fluctuation ($\pm Q_0^{\frac{1}{2}}$). Using the method of Breit and Yost,[11] Serber then calculated the depth of the potential hole necessary to give the observed phase shifts. It was necessary to assume a hole radius; so more or less arbitrarily the electron radius was taken, (2.8×10^{-13} cm). It might be remarked here that the closest distance of approach of a 700 kv proton is about 5×10^{-13} cm. The depth of the hole was found to be 17.2 mv. In view of the similar results of Taylor this figure is not unreasonable, but on current theories of the mass defects of the heavy elements such a strong interaction is difficult to understand. Inasmuch as the theory of nuclear mass defects is under a heavy cloud of suspicion there is no real argument on this basis against strong proton-proton forces.

However, if one examines more carefully the effect of Serber's potential on the scattering at lower energies than 600 kv there is found strong disagreement with the observations. The experimental data indicate an almost Coulomb scattering at 450–600 kv while Serber's modified scattering is about 8.5 times the Coulomb expectation at 45°. This slow decrease in the theoretical scattering with energy may be just a peculiarity of the particular potential model chosen, but it does not seem likely. It is indeed very disturbing to find that the potential required to fit the high energy scattering fails so markedly at energies not much less than 600 kv.

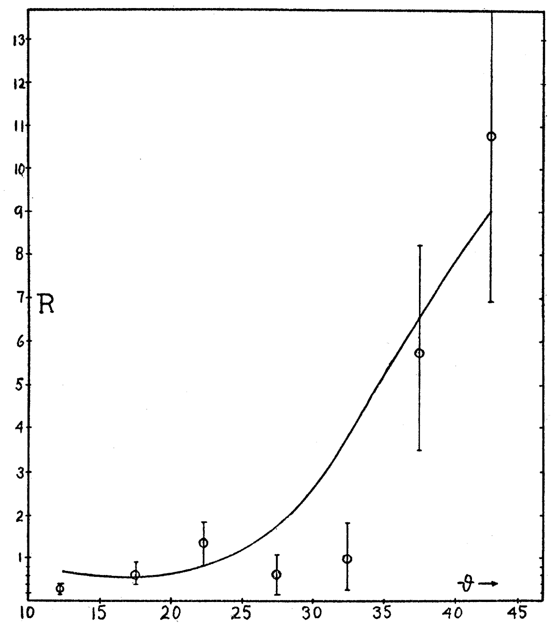

Fig. 8. Ratio of Serber's modified scattering to Mott's scattering at 700 kv.

The theoretical situation may be summarized briefly.[12]

(1) The 650–900 kv scattering can be reasonably fitted by Serber's short range potential by assuming a hole depth of 17.2 mv and a radius of 2.8×10^{-13} cm. The minimum at 30° cannot be fitted closely by any short range potential, but it can be fitted to within twice the probable fluctation.

(2) The energy dependence of the 45° anomaly cannot be accounted for at all by any short range force.

(3) If further data are in substantial agreement with the above observed scattering then present theoretical ideas about intranuclear forces will have to be seriously modified.

It is a pleasure to acknowledge the constant and helpful interest taken in this work by Professors E. O. Lawrence and J. R. Oppenheimer. The author is also grateful to the Charles A. Coffin Foundation of the General Electric Company for the award of a fellowship in 1933 and again in 1934.

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[11] G. Breit and F. L. Yost, Phys. Rev. **48**, 203 (1935).

[12] I am very deeply indebted to Professors Oppenheimer and Condon for illuminating discussions on these matters.

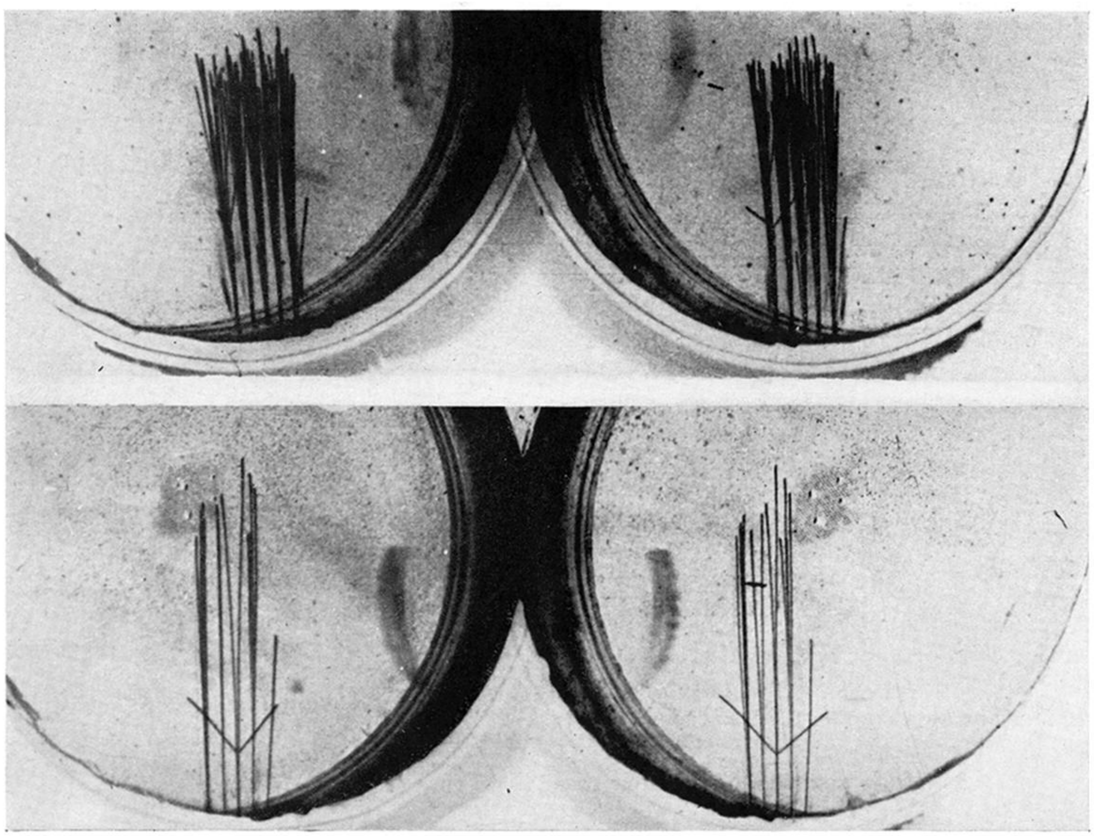

FIG. 1. Stereoscopic photographs of high energy proton collisions in hydrogen. Series A typical photograph above; Series B below.