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The Polarization of X-Rays from Thin Targets

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By the use of Ross' method of balanced foils and a modification of the 90° scattering method, the polarization of the portion of the continuous x-ray spectrum between the K limits of tantalum and tungsten has been measured for targets of aluminum foil (0.7μ) and silver foil (0.17μ), at voltages from the quantum limit up to 120 kv. For both elements, the polarization is complete at the quantum limit, and decreases as the tube voltage is raised above the quantum voltage for the spectrum band. The decrease is more rapid for silver than for aluminum, and in the case of the latter the decrease is greater than that predicted by Sommerfeld. A method is described whereby the finite thickness of the target may be taken into account. The resulting correction, however, is too small to account for the discrepancy between theory and experiment.

INTRODUCTION

THE problem under consideration is to determine the form of the polarization isochromats for the continuous x-rays produced from a thin target. Previous experiments in this field have been very scarce. Duane[1] made measurements on the polarization of x-rays produced when a stream of mercury vapor was bombarded with low voltage electrons, and Dasannacharya[2] made measurements on the polarization of x-rays from targets of aluminum foil, which, though thin, were not thin enough to be called thin targets for the purpose of this experiment. In neither of these experiments was any attempt made to isolate a given wave-length, or band of wave-lengths, but measurements were made on the entire spectrum. Kulenkampff,[3] using targets of aluminum 0.6 micron thick and making use of Ross' method of balanced foils[4] to isolate a narrow band of wave-lengths, found the polarization to be complete at the quantum limit, becoming less as the voltage was raised. Kulenkampff's data were taken for two wave-length bands (Ag$-$Cd and Cd$-$Sn) and at three different voltages, from near the quantum limit up to 37.8 kv.

For the purposes of experiments on polarization, a thin target should be one in which the electrons in the cathode-ray beam are not deflected from their original direction before they produce x-rays. The present experiment was done with aluminum foil 0.7 micron thick, but with a wave-length band (Ta$-$W) for which the quantum limit is in the neighborhood of 70 kv, so that the approximation to thin target conditions should be much better than in Kulenkampff's work. Results were also obtained with targets of silver foil 0.17 micron thick.

THEORY

So far no theory has been devised which is strictly applicable to the conditions of this

[1] Duane, Proc. Nat. Acad. Sci. **15**, 805 (1929).
[2] Dasannacharya, Phys. Rev. **35**, 129 (1930); also **36**, 1675 (1930).
[3] Kulenkampff, Physik. Zeits. **30**, 514 (1929).
[4] Ross, J. Opt. Soc. Am. **16**, 433 (1928).

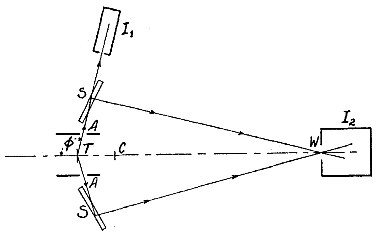

FIG. 1. Diagram of the experimental arrangement.

experiment. Sommerfeld[5] has worked out a quantum-mechanical theory of the continuous spectrum, and gets the curve in Fig. 2 for the variation of polarization with wave-length, for a voltage equal to the quantum voltage of one of the bands used by Kulenkampff. This curve is constructed for the case in which $(V_K/V)^{\frac{1}{2}}$ approaches zero, and takes no account of relativity. Sommerfeld's theory also indicates that the polarization at the quantum limit should be very nearly complete when the target is of low atomic number and the voltage is high. For targets of high atomic number and low voltages, the polarization at the quantum limit, it is indicated, could be as low as 60 percent. Y. Sugiura[6] has also worked out a quantum-mechanical theory of polarization according to which the polarization can never exceed 85 percent.

Experimental Arrangement

An x-ray tube was designed and built which was thought to be especially suited to the demands of the experiment. Fig. 1 shows the principal features of the arrangement. The target T is in the center of a square brass box which has long windows of 0.005 cm aluminum on two sides. These windows allow two beams of x-rays to emerge and strike the scatterer S, which is a surface of revolution coaxial with the target and cathode of the tube. The two long windows allow the use of an unusually wide beam, which is very desirable on account of the low intensity. Rays scattered from S at approximately 90° enter the ionization chamber I_2, and a small portion of the direct beam goes through a hole in the scatterer into the ionization chamber I_1. If the scatterer were a zone of the sphere having T

[5] Sommerfeld, Ann. d. Physik **11**, 257 (1931).
[6] Sugiura, Sci. Pap. Inst. Phys. Chem. Res. Tokyo **17**, 89 (1931).

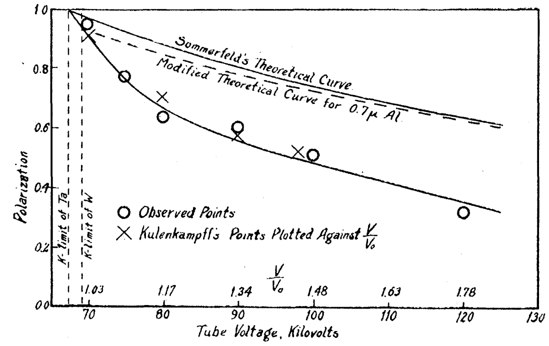

FIG. 2. Polarization of Ta — W band from thin aluminum.

and the window of I_2 at the ends of its diameter, then all rays entering I_2 would be the products of 90° scattering. Since it is necessary, however, to use finite sized windows and focal spots, and a finite amount of scattering material, the scatterer was made in the shape of a frustum of a cone; with this arrangement it turned out that about 1000 cubic centimeters of scattering material could be used, while the scattering angle ranged between 87° and 93°.

The ionization chamber I_1 is a cylinder 5 cm in diameter and 20 cm long, with a window of thin steel in one end. The collector is a brass rod which is connected to a Compton electrometer. The ionization chamber I_2 is a cylinder 18 cm in diameter and 20 cm long, and has a thin bronze window 6 cm×12 cm. The collector is a thin brass rod which runs along the axis of the chamber, and connects by a very short lead with the needle of a Hoffmann electrometer. The dimensions of the collector and lead are kept as small as possible in order to make their capacitance small. Both ionization chambers are filled with methyl iodide at a pressure of about 15 cm of mercury, and are equipped with electromagnetic shutters. The readings taken were the ballistic deflections of the electrometers when the shutters were open for a definite time, which was controlled by a master clock and photoelectric relay.

The ionization chamber I_1 receives the direct, or unscattered, radiation, and hence measures the intensity of the x-ray beam. The chamber I_2 receives only that component which is polarized with its electric vector perpendicular to the cathode-ray beam. Since for completely polarized x-rays the electric vector is in the plane of the cathode-ray beam, this is a null method for complete polarization. For incomplete polarization, the system has to be calibrated. This was done by substituting a thick target for the thin ones and making use of the results obtained by Cheng[7] for the polarization of thick-target radiation. If we call X the intensity of the component whose electric vector is in a plane containing the cathode rays, and Y the intensity of the component whose electric vector is perpendicular thereto, the polarization is

$$P=(X-Y)/(X-Y)=1-2Y/(X+Y). \quad (1)$$

Since Y is measured by I_2, and $X+Y$ by I_1, this may be written

$$P=1-mR_2/R_1, \quad (2)$$

in which R_1 and R_2 are the readings for I_1 and I_2, respectively. The factor m was evaluated by making measurements on radiation of known polarization,[7] namely, the Ta—W band from thick aluminum at 90 kv.

Geometric Corrections

The ionization produced in I_2 is not an exact measure of the intensity of the component of the x-ray beam polarized perpendicularly to the cathode-ray stream because of the finite sizes of target and window and because of the finite thickness of the scatterer. The error introduced thereby may be easily calculated.

If the angle of scattering differs from 90° by the small angle α, then a portion of the component X will be scattered with intensity proportioned to $X \sin^2 \alpha \sim X\alpha^2$. From the dimensions of the apparatus the maximum value of α is $\alpha_{\max}=0.06$, so that

$$\alpha_{\max}{}^2=0.0036, \quad \text{or} \quad \overline{\alpha^2}\sim 0.002. \quad (3)$$

Hence the lack of geometric perfection introduces into the measurement of Y an error of 0.2 percent of the value of X.

Another correction of a geometric nature is introduced by the fact that the observations were all made on a beam which makes an angle of 105° with the cathode-ray beam, whereas the theoretical results are given for an angle of 90°. The effect of this change in the angle of observation may be approximately calculated from classical electromagnetic considerations. Table I shows the result of such a calculation.

Table I. *Effect on the polarization of a change in angle from 90° to 105°.*

$P_{90°}$	1.000	0.800	0.600	0.400	0.200	0.000
$P_{105°}$	1.000	0.789	0.583	0.383	0.187	0.000

It happens that both of these corrections are small compared to the experimental errors.

Experiments on Thin Targets

The x-ray tube was excited by a 200 kv d.c. outfit which was controlled manually by an assistant whenever readings were to be taken. This outfit, and the voltmeter used with it, have been described previously.[8] Data on polarization were obtained by taking readings with the balanced foils alternating at several voltages from the quantum limit up to 120 kv. Several readings were taken with each foil at each voltage. This was especially necessary in the case of the scattered radiation because of the relatively large statistical fluctuations resulting from the extremely low intensity. Table II shows the observed values of the polarization for 0.7-micron aluminum and Table III shows the results for 0.17-micron silver.

These results are shown graphically in Figs. 2 and 3. In the case of aluminum (Fig. 2), it will be observed that the polarization decreases with increasing voltage in a way similar to that predicted by Sommerfeld's theory, but more rapidly. This is in accord with the findings of Kulenkampff[3] for longer wave-lengths. In fact, if the polarization is plotted as a function of

Table II. *Observed values of the polarization for aluminum of 0.7 micron in thickness.*

Tube Voltage (kv)	Polarization	Tube Voltage (kv)	Polarization
70	0.95±0.06	90	0.60±0.08
75	.77± .10	100	.51± .07
80	.63± .09	120	.31± .10

Table III. *Observed values of the polarization for silver of 0.17 micron in thickness.*

Tube Voltage (kv)	Polarization	Tube Voltage (kv)	Polarization
69	0.89±0.05	90	0.39±0.06
70	.83± .06	100	.35± .06
75	.61± .06	120	.28± .06
80	.49± .06		

[7] Cheng, Phys. Rev. **46**, 243 (1934).

[8] Webster, Hansen and Duveneck, Phys. Rev. **43**, 839 (1933); H. Clark, Rev. Sci. Inst. **1**, 615 (1930).

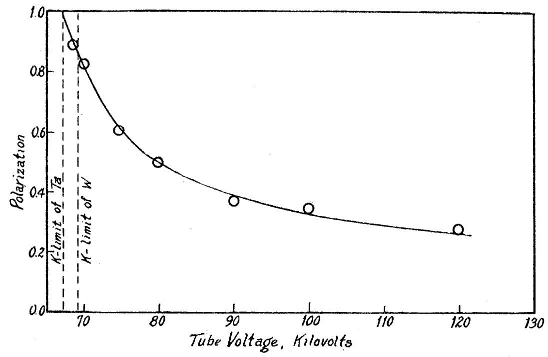

FIG. 3. Polarization of Ta — W band from thin silver.

V/V_0, V being the tube voltage and V_0 the quantum voltage for the wave-length used, there is extremely good agreement between Kulenkampff's experiments and those herein presented.

As Sommerfeld's theory does not show how to find the variation of polarization with voltage for elements of high atomic number, no comparison between experiment and theory can be made in the case of silver. It will be observed, however, that the experiments indicate complete polarization at the quantum limit for both metals. Sommerfeld predicts a polarization at the quantum limit of 0.98 for aluminum and 0.85 for silver.

Effect of Finite Target Thickness

An ideal thin target would be one in which no cathode ray suffered any change in direction before emitting x-rays. In any real target, however, the electrons are diffused so that by the time the radiation is emitted they are going in various directions. The amount of this diffusion may be estimated from Kulenkampff's curves[9] or from a formula given by Bothe.[10] According to Bothe's formula, the most probable deflection for the thin aluminum used in this experiment ranged from 6° to 11°, and for the silver from 12° to 22°. The effect of this diffusion on the polarization may be calculated approximately. Let ψ (Fig. 4) be the average angle which the electric vector in the emitted radiation makes with the direction of the impinging electron, the average being taken in such a way that the depolarization for an infinitely thin target is

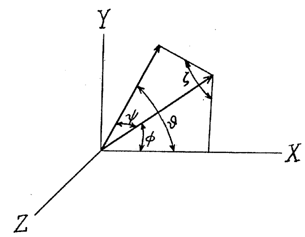

FIG. 4.

$$D_s = \tfrac{1}{2}\tan^2\psi. \tag{4}$$

We now let ϕ be the angle between the direction of motion of the impinging electron and its original direction. Assuming a Gaussian distribution in angle of the diffused electrons, and assuming the most probable deflection (λ) to be given by Bothe's formula,[10] we can integrate over all values and orientations of ϕ and over the thickness of the target to obtain an expression showing how the depolarization is modified by diffusion of electrons within the target. This may be done as follows.

In the diffused beam of electrons, let the number per unit solid angle be given by the two-dimensional form of the Gauss error equation:

$$dN/d\omega = N_0/2\pi\lambda^2 e^{-\phi^2/2\lambda^2}, \tag{5}$$

in which N_0 is the total number of electrons producing radiation, ϕ is the angle of deflection, and λ is the most probable angle of deflection. The total number of electrons deflected through a range of angles between ϕ and $\phi+d\phi$ is then

$$dN_\phi = (N_0/\lambda^2)\sin\phi e^{-\phi^2/2\lambda^2}d\phi. \tag{6}$$

In a very thin film, we may without serious error assume the same fraction of this number to produce radiation in any layer of thickness dx. If t is the thickness of the foil, this number is

$$dN_{\phi,\,x} = (N_0/t\lambda^2)\sin\phi e^{-\phi^2/2\lambda^2}d\phi dx \tag{7}$$

or, remembering that λ is a function of x, as given by Bothe,

$$dN_{\phi,\,\lambda} = (cN_0/t\lambda)\sin\phi e^{-\phi^2/2\lambda^2}d\phi d\lambda. \tag{8}$$

We now think of the electric vector in the emitted beam as making an effective angle ψ with the direction of the electron's motion just prior to the collision in which the radiation is produced. The angle θ between the electric vector

[9] Kulenkampff, Ann. d. Physik **87**, 597 (1928).
[10] Bothe, *Handbuch der Physik*, Vol. 24 (Geiger and Scheel).

and the original direction of the cathode rays is then (Fig. 4) found from the relation:

$$\cos\theta=\cos\phi\cos\psi+\sin\phi\sin\psi\cos\zeta, \quad (9)$$

where ζ is the dihedral angle between the planes containing ϕ and ψ. The component of the emitted radiation with electric vector parallel to the cathode-ray beam is then proportional to

$$X=A\int^{\infty}\int_0^{\Lambda}\int_0^{2\pi}\frac{1}{\lambda}\cos^2\theta\sin\phi e^{-\phi^2/2\lambda^2}d\phi d\lambda d\zeta. \quad (10)$$

If we observe the radiation from the direction OZ (Fig. 4), the other component involved is the one with electric vector parallel to OY. This is proportional to

$$Y=\frac{A}{2}\int_0^{\infty}\int_0^{\Lambda}\int_0^{2\pi}\frac{1}{\lambda}\sin^2\theta\sin\phi e^{-\phi^2/2\lambda^2}d\phi d\lambda d\zeta, \quad (11)$$

Λ is of course the Bothe angle for the whole film, and A is a constant. Substituting in (10) and (11) the value of $\cos\theta$ from (9) gives

$$X=A\int_0^{\infty}\int_0^{\Lambda}\int_0^{2\pi}\frac{1}{\lambda}(\cos^2\phi\cos^2\psi$$
$$+\sin^2\phi\sin^2\psi\cos^2\zeta+2\cos\phi\cos\psi\sin\phi$$
$$\sin\psi\cos\zeta)\sin\phi e^{-\phi^2/2\lambda^2}d\phi d\lambda d\zeta, \quad (12)$$

and

$$Y=\frac{A}{2}\int_0^{\infty}\int_0^{\Lambda}\int_0^{2\pi}\frac{1}{\lambda}\sin\phi\cdot e^{-\phi^2/2\lambda^2}d\phi d\lambda d\zeta-\tfrac{1}{2}X. \quad (13)$$

Integrating with respect to ζ,

$$X=\pi A\int_0^{\infty}\int_0^{\Lambda}\frac{1}{\lambda}[2\cos^2\psi\sin\phi\cdot e^{-\phi^2/2\lambda^2}$$
$$+(\sin^2\psi-2\cos^2\psi)\sin^3\phi\cdot e^{-\phi^2/2\lambda^2}]d\phi d\lambda, \quad (14)$$

$$Y=\frac{A}{2}\int_0^{\infty}\int_0^{\Lambda}\frac{2\pi}{\lambda}\sin\phi\cdot e^{-\phi^2/2\lambda^2}d\phi d\lambda-\tfrac{1}{2}X. \quad (15)$$

The integration with respect to ϕ may be accomplished by using the approximation $\sin\phi\sim\phi-\phi^3/6$. While it may seem a logical inconsistency to make this approximation and still carry the integration from zero to infinity, it can be shown that the error introduced thereby, with the values of λ involved in this problem, is quite negligible.

Thus we get

$$X=\pi A\int_0^{\Lambda}[2\lambda\cos^2\psi+2\lambda^3(\sin^2\psi-(7/3)\cos^2\psi)]d\lambda \quad (16)$$

and

$$Y=\pi A\int_0^{\Lambda}(\lambda-\tfrac{1}{3}\lambda^3)d\lambda-\tfrac{1}{2}X. \quad (17)$$

These become, on integrating with respect to λ,

$$X=\pi A[\Lambda^2\cos^2\psi+\tfrac{1}{2}\Lambda^4(\sin^2\psi-(7/3)\cos^2\psi)], \quad (18)$$

and

$$Y=\pi A[\tfrac{1}{2}\Lambda^2\sin^2\psi-\tfrac{1}{4}\Lambda^4(\tfrac{1}{3}+\sin^2\psi-(7/3)\cos^2\psi)]. \quad (19)$$

The observed depolarization is then

$$D_0=\frac{Y}{X}=\frac{\tfrac{1}{2}\tan^2\psi-\tfrac{1}{4}\Lambda^2((4/3)\tan^2\psi-2)}{1+\tfrac{1}{2}\lambda^2(\tan^2\psi-7/3)} \quad (20)$$

$$=\frac{D_s-\tfrac{1}{4}\Lambda^2((8/3)D_s-2)}{1+\tfrac{1}{2}\Lambda^2(2D_s-7/3)}, \quad (21)$$

where $D_s=\tfrac{1}{2}\tan^2\psi$ is the depolarization for undiffused electrons. If we neglect the fourth and higher powers of Λ in Eq. (21) we get the concise expression:

$$D_0=D_s+\tfrac{1}{2}\Lambda^2(1+D_s-2D_s^2). \quad (22)$$

In Fig. 2, the modified curve was plotted by computing values of D_s from Sommerfeld's theory[5] and computing values of D_0 therefrom by means of Eq. (22). It will be seen that the correction thus introduced is not nearly sufficient to bridge the gap between theory and experiment. The discrepancy remaining is probably due, at least in part, to the fact that Sommerfeld's curve is for the case where $(V_K/V)^{\frac{1}{2}}$ approaches zero, and possibly also to the fact that Sommerfeld's theory neglects relativity.

Acknowledgments

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