

The Tension Coefficients of Resistance of the Hexagonal Crystals Zinc and Cadmium

MILDRED ALLEN, *Physics Department, Mount Holyoke College*

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The tension coefficients of resistance of the hexagonal crystals, zinc and cadmium, have been measured. These coefficients have been found to be independent of the secondary orientation. This is in agreement with a theory set forth by P. W. Bridgman, as corrected by J. W. Cookson; furthermore the experimental points lie within experimental error of the curves given by the theory representing them as functions of the primary orientation.

Cookson's correction gives the same formal dependence of the coefficients on the orientation as Bridgman's original theory, but demands additional constants fully to define the effect of deforming forces on the electrical resistance. Further experimental work will be necessary completely to determine this entire set of constants, and theory is sketched to show that torsion experiments would furnish the necessary data.

IN two previous papers^{1, 2} a study has been made of the tension coefficients of electrical resistance of the trigonal crystals, bismuth and antimony, as functions of the primary and secondary orientations of the crystals with respect to the cylindrical axis of the casting. The findings of these experimental papers have been in agreement with a theory of the change of resistance produced by stress developed by P. W. Bridgman³ on the basis of crystal symmetry. The experiments presented here were undertaken in order to extend this work to crystals of a different type of symmetry and so to verify the theory further; hence the hexagonal crystals, zinc and cadmium, have been studied. In the meantime, a correction of Bridgman's theory has been made by John W. Cookson⁴ and it must be examined to see how far it invalidates or changes the interpretation of the already published results for bismuth and antimony. Inasmuch as hexagonal crystals may be considered as a degenerate case of trigonal symmetry, the modification of the theory involved will first be shown as it applies to the trigonal case. The elastic constants enter into the generalized equations of Hooke's law in the same way in both the trigonal and hexagonal cases, but in the hexagonal case one of the six elastic constants of the trigonal case is zero. That means that to determine the change of resistance with stress fewer constants will be necessary for hexagonal than for trigonal crystals and that to pass from the trigonal to the hexagonal case it is only necessary to equate to zero one or more of the coefficients involved.

CORRECTION OF THE THEORY FOR TRIGONAL CRYSTALS

In his theory P. W. Bridgman *assumed* that the coefficients ρ_{rs} relating the change in resistance to the stresses should behave exactly like the elastic constants relating the strains to the stresses, that is, that they should be representable by a symmetrical matrix in which $\rho_{rs} = \rho_{sr}$. In January, 1935 Cookson published a note in the *Physical Review* showing on the basis of the known symmetries of the tensors, tension and electrical resistance, that Bridgman's theory was incorrect in that the matrix of the piezo-resistive coefficients was not necessarily symmetrical and that as a result more constants are necessary to define the change of resistance under tension than Bridgman indicated. In the case of trigonal crystals, two more constants are necessary making a total of eight instead of six.

In comparing the Bridgman and Cookson coefficients it is well to note that they are defined differently by the two men. The coefficients ρ_{11} , ρ_{12} , ρ_{13} and ρ_{33} are the same in the two cases; the Bridgman coefficients ρ_{14} and ρ_{44} are twice as great as those defined by Cookson. In accord with the earlier results reported, those used by Bridgman will continue to be used in this work.

According to the original Bridgman theory the tension coefficient β depends on the primary orientation θ and the secondary orientation φ in the case of a trigonal crystal in the following way, the trigonal symmetry showing up in the term in φ :

$$\beta = (1/\rho_{\theta}) \{ \rho_{11} \sin^4 \theta + \rho_{33} \cos^4 \theta + (2\rho_{13} + \rho_{44}) \sin^2 \theta \cos^2 \theta - 2\rho_{14} \sin^3 \theta \cos \theta \cos 3\varphi \}. \quad (1)$$

¹ Mildred Allen, *Phys. Rev.* **42**, 848 (1932).

² Mildred Allen, *Phys. Rev.* **43**, 569 (1933).

³ P. W. Bridgman, *Phys. Rev.* **42**, 858 (1932).

⁴ John W. Cookson, *Phys. Rev.* **47**, 194 (1935).

The corrected theory gives the modified relation

$$\beta = (1/\rho_\theta) \{ \rho_{11} \sin^4 \theta + \rho_{33} \cos^4 \theta + (\rho_{13} + \rho_{31} + \rho_{44}) \sin^2 \theta \cos^2 \theta - (\rho_{14} + \rho_{41}) \sin^3 \theta \cos \theta \cos 3\varphi \}. \quad (2)$$

It is to be noted that β is the same formal function of the orientation angles as before, and that therefore the conclusions drawn regarding the fact that the tension coefficients show the characteristic symmetries of the crystal still hold. In my second paper,² however, an attempt was made to evaluate the piezo-resistive coefficients numerically and only part of the numerical results can be accepted under this new interpretation. Eqs. (2) on page 571 of that paper are empirical data drawn directly from the experimental points and the functional relation (1) of this paper; the functional relation (2) of the present paper can be equally well satisfied if there be substituted in these equations for $(2\rho_{13} + \rho_{44})$ the new relation $(\rho_{13} + \rho_{31} + \rho_{44})$ and for ρ_{14} the expression $(\rho_{14} + \rho_{41})/2$. Since these data for *bismuth* do not appear explicitly in either of the earlier papers, they are given here:

$$\begin{array}{l} \text{Uncorrected} \\ \rho_{11} = (-7.2 \pm 0.16) \times 10^{-9} \\ \rho_{33} = (-6.2 \pm 0.41) \times 10^{-9} \\ (\rho_{13} + \rho_{31} + \rho_{44}) = (-7.1 \pm 0.99) \times 10^{-9} \\ (\rho_{14} + \rho_{41})/2 = (13.5 \pm 0.27) \times 10^{-9} \\ \text{Corrected for strain} \\ \rho_{11} = (-7.7 \pm 0.15) \times 10^{-9} \\ \rho_{33} = (-6.8 \pm 0.40) \times 10^{-9} \\ (\rho_{13} + \rho_{31} + \rho_{44}) = (-9.3 \pm 0.95) \times 10^{-9} \\ (\rho_{14} + \rho_{41})/2 = (13.2 \pm 0.26) \times 10^{-9}. \end{array} \quad (3)$$

It is obvious that as before the experiments with linear tension can be expected to furnish only four relations between the coefficients.

In the previous work two additional relations between the constants were obtained from the values of the pressure coefficient of resistance for the two extreme orientations of the crystals, i.e., with the corrected notation,

$$\left(\frac{1}{p} \frac{\Delta R}{R} \right)_{\theta=90} = -(\rho_{11} + \rho_{12} + \rho_{13})/\rho_\theta, \quad (4)$$

$$\left(\frac{1}{p} \frac{\Delta R}{R} \right)_{\theta=0} = -(2\rho_{31} + \rho_{33})/\rho_\theta. \quad (5)$$

Even with these, there is now an *insufficient* number of relations to determine all the coefficients involved. Thus the values of the individual ρ 's given in the paper on antimony are invalidated (i.e., Eqs. (6) and (27)).

FURTHER THEORY

This insufficiency in the number of relations then requires the finding of additional data to give further relations between the constants. The most obvious procedure is to twist the crystal about its longitudinal axis. The resulting change in resistance must show the characteristic symmetry, if Neumann's general law is to hold. This has not been done experimentally, but perhaps it is worth while to sketch the underlying mathematics and so to indicate the experimental problem that must be solved before these piezo-resistive coefficients can be completely known.

A moment of force N' is to be applied to twist the crystal about its axis of figure, i.e., about the Z' axis. Then N' will determine the values of the shearing forces according to the formulae given by Voigt,⁵

$$X'_{z'} = 2N'y'/Qa^2, \quad (6)$$

$$Y'_{z'} = -2N'x'/Qa^2, \quad (7)$$

where Q is the area of the circular cross section of the cylindrical casting and a the radius of this cross section, and x' and y' are the distances along the coordinate axes from the shearing force to the axis of the cylinder. It is to be noted that these shearing forces increase proportionately to the distance from the axis of the cylinder, so that the change in resistance will not be constant throughout the crystal, but those filaments parallel to the axis and near to the axis will change resistance relatively little compared with those far from the axis. The total change in resistance will then be the resultant of the changes of resistance for these individual filaments of small cross section. If $f_1(\theta, \varphi)$ represents the shearing coefficient of resistance $(1/X'_{z'}) \times (\Delta R/R)_f$ for one of the individual filaments of length l and of cross section $dx'dy'$ twisted by a force $X'_{z'}$ and $f_2(\theta, \varphi)$ the shearing coefficient for the same filament acted upon by the force $Y'_{z'}$,

⁵ W. Voigt, *Lehrbuch der Kristallphysik* (1928 edition), p. 636.

the total relative change in resistance for that filament will be

$$(\Delta R/R)_f = f_1(\theta, \varphi) X'_{z'} + f_2(\theta, \varphi) Y'_{z'}. \quad (8)$$

The problem then is to add up all the changes in resistance for these various filaments which are, of course, in parallel with each other. In the unstrained state the usual law for the addition of resistances in parallel gives for the value of R , the resistance of the entire crystal, the relation

$$\frac{1}{R} = \sum_f \frac{1}{R_f} \quad (9)$$

and in the strained state there is the similar relation

$$\frac{1}{R + \Delta R} = \sum_f \frac{1}{R_f + \Delta R_f}. \quad (10)$$

The latter may be expanded by the binomial theorem, since $\Delta R/R$ is a small quantity both for the individual filaments and for the entire crystal, to give the relation

$$\frac{1}{R} \left(1 - \frac{\Delta R}{R} \right) = \sum_f \frac{1}{R_f} \left(1 - \frac{\Delta R_f}{R_f} \right), \quad (11)$$

which becomes, in the light of (9) and of the fundamental definition of specific resistance as applied to the filament of length l and of cross section $dx'dy'$,

$$\frac{\Delta R}{R} = \int \frac{R}{\rho \delta l} (f_1 X'_{z'} + f_2 Y'_{z'}) dx' dy'. \quad (12)$$

The substitution in this of the values of the shearing forces given by Eqs. (6) and (7) gives for the *total* relative change in resistance the relation

$$\begin{aligned} \frac{\Delta R}{R} = & f_1(\theta, \varphi) \frac{2N'}{Q^2 a^2} \int_0^a \int_0^{(a^2-x'^2)^{\frac{1}{2}}} y' dy' dx' \\ & - f_2(\theta, \varphi) \frac{2N'}{Q^2 a^2} \int_0^a \int_0^{(a^2-y'^2)^{\frac{1}{2}}} x' dx' dy' \end{aligned} \quad (13)$$

and, when the indicated integration has been carried out, the torsion coefficient of resistance becomes

$$\frac{1}{F} \frac{\Delta R}{R} = \frac{2}{3\pi} f_1(\theta, \varphi) - \frac{2}{3\pi} f_2(\theta, \varphi), \quad (14)$$

$$\text{where} \quad F = N'/aQ \quad (15)$$

is the force per unit area applied tangentially to the periphery of the crystal to produce the twisting moment N' .

The problem then resolves itself into finding the functions $f_1(\theta, \varphi)$ and $f_2(\theta, \varphi)$ giving the relative changes in resistance of the individual rectangular filaments when subjected respectively to the shearing forces $X'_{z'}$ and $Y'_{z'}$. The primed axes are the axes of figure and the unprimed axes the crystallographic axes, with Z the trigonal axis and X the digonal axis, as before. The known laws of the transformation of tensors give the components of the shearing force $X'_{z'}$ in the unprimed set of axes:

$$\begin{aligned} X_x &= 2X'_{z'} \sin \theta \sin \varphi \cos \varphi, \\ Y_y &= -2X'_{z'} \sin \theta \sin \varphi \cos \varphi, \\ Z_z &= 0, \\ Y_z &= -X'_{z'} \cos \theta \sin \varphi, \\ Z_x &= X'_{z'} \cos \theta \cos \varphi, \\ X_y &= X'_{z'} \sin \theta (\cos^2 \varphi - \sin^2 \varphi). \end{aligned} \quad (16)$$

These values must be substituted in the six fundamental equations for the change in resistance due to the application of stresses to a trigonal crystal

$$\begin{aligned} \delta r_{11} &= X_x \rho_{11} + Y_y \rho_{12} + Z_z \rho_{13} + Y_z \rho_{14} + Z_x 0 + X_y 0, \\ \delta r_{22} &= X_x \rho_{12} + Y_y \rho_{11} + Z_z \rho_{13} - Y_z \rho_{14} + Z_x 0 + X_y 0, \\ \delta r_{33} &= X_x \rho_{31} + Y_y \rho_{31} + Z_z \rho_{33} + Y_z 0 + Z_x 0 + X_y 0, \\ \delta r_{23} &= X_x \frac{\rho_{41}}{2} - Y_y \frac{\rho_{41}}{2} + Z_z 0 + Y_z \frac{\rho_{44}}{2} + Z_x 0 + X_y 0, \end{aligned} \quad (17)$$

$$\delta r_{31} = X_x 0 + Y_y 0 + Z_z 0 + Y_z 0 + Z_x \frac{\rho_{44}}{2} + X_y \rho_{41},$$

$$\delta r_{12} = X_x 0 + Y_y 0 + Z_z 0 + Y_z 0 + Z_x \rho_{14} + X_y (\rho_{11} - \rho_{12}).$$

The total change in resistance is then to be found from the relation

$$\begin{aligned} \Delta R = & \delta r_{11} \sin^2 \theta \sin^2 \varphi + \delta r_{22} \sin^2 \theta \cos^2 \varphi \\ & + \delta r_{33} \cos^2 \theta + 2\delta r_{23} \sin \theta \cos \theta \cos \varphi \\ & + 2\delta r_{31} \sin \theta \cos \theta \sin \varphi \\ & + 2\delta r_{12} \sin^2 \theta \sin \varphi \cos \varphi, \end{aligned} \quad (18)$$

which gives for the final shearing stress coefficient the value

$$f_1(\theta, \varphi) = \frac{1}{X'_{z'}} \left(\frac{\Delta R}{R} \right)_f = \frac{1}{\rho_\theta} \{ \rho_{14} + 2\rho_{41} \} \sin^2 \theta \cos \theta \sin 3\varphi. \quad (19)$$

This is seen to have the necessary trigonal symmetry in φ . If the same procedure is followed through for an applied shearing force $Y'_{z'}$, the shearing stress coefficient comes out to be

$$f_2(\theta, \varphi) = \frac{1}{Y'_{z'}} \left(\frac{\Delta R}{R} \right)_f = \frac{1}{\rho_\theta} [(2\rho_{11} - 2\rho_{13} - \rho_{44}) \times \sin^3 \theta \cos \theta + (2\rho_{31} + \rho_{44}) \sin \theta \cos^3 \theta + \{ \rho_{14}(\sin^2 \theta - \cos^2 \theta) - 2\rho_{41} \cos^2 \theta \} \sin^2 \theta \cos 3\varphi]. \quad (20)$$

Thus the final shearing coefficient of resistance due to the application of a twisting moment N' becomes (according to Eq. (10))

$$\frac{1}{N'/aQ} \frac{\Delta R}{R} = \frac{2}{3\pi\rho_\theta} [(\rho_{14} + 2\rho_{41}) \sin^2 \theta \cos \theta \sin 3\varphi + (2\rho_{13} + \rho_{44} - 2\rho_{11}) \sin^3 \theta \cos \theta - (2\rho_{31} + \rho_{44}) \sin \theta \cos^3 \theta + \{ 2\rho_{41} \cos^2 \theta - \rho_{14}(\sin^2 \theta - \cos^2 \theta) \} \sin^2 \theta \cos 3\varphi]. \quad (21)$$

This gives *five* additional relations between the piezo-resistive constants which are more than are necessary. Thus for a complete solution of the problem of the relation of the change of resistance to stress, experiments involving the twisting of the crystals may be performed.

The change of resistance due to the change of dimensions will be more complicated to compute than in the case of simple longitudinal tension, chiefly because the coefficients involved transform in a much more complicated way.

EXPERIMENTAL PROCEDURE

The study of the hexagonal crystals, zinc and cadmium, was undertaken for two reasons. The first reason, as has been indicated, is that hexagonal crystals may be considered as a degenerate case of trigonal crystals in which the theory already developed may be applied if one puts $\rho_{14} = \rho_{41} = 0$. On examination of the above theory, it will be seen from Eq. (2) that the tension coefficient for hexagonal crystals must be

independent of the secondary orientation φ . The second reason is that all real metals crystallize either in the hexagonal or the cubic system, and the results for these two metals will consequently be representative of those for a considerable number of other metals. Bismuth and antimony, on the other hand, are only quasi-metallic, having a higher resistance than real metals and with arsenic the only other element having the same type of symmetry.

The experimental procedure with zinc and cadmium was the same as that used with antimony, the change in resistance being measured by the deflection method.² These hexagonal crystals required greater care, partly because the effect to be measured is so small and partly because the crystals bend so very easily. The order of magnitude of the tension coefficient proved to be the same as for antimony, but since the specific resistance of zinc and cadmium is only about one-sixth that of antimony, the change in the galvanometer reading produced by the application of the tension was correspondingly smaller, and in fact varied between the limits 0.2 and 1.2 cm. The uncertainty in the measurement of the cross section of the crystal cylinder surely introduced as great an error as arose from the smallness of these galvanometer readings, since the diameters were 1/16 inch and the cross section was never accurately circular.

The zinc used was spectroscopically pure and was furnished by the New Jersey Zinc Company. The cadmium crystals were made from the pure Kahlbaum metal. Both were cast by the writer in cylinders 1/16" in diameter and 1½" or 2" long. The melting points of both these metals were well below that of Pyrex glass, so that they could be crystallized in glass molds in an atmosphere of CO₂ to prevent oxidation as they were lowered through the furnace. The metals would have stuck to the molds if these had not been washed out previously with a solution of lacquer in chloroform. The elastic limit of both is low: 500 grams tension, approximately 250 g/mm², for many orientations could be safely applied to the zinc crystals and 100 grams to the cadmium. The allowable force was determined by plotting the change in resistance against the force applied and finding how far the linear relation between the two held. It would be of interest to study

what happens beyond the elastic limit, but this has not been done as yet.

Zinc and cadmium are both normal as regards the sign of the pressure coefficient, it being negative in both cases. In this they differed from bismuth and antimony for which the pressure coefficient of resistance is positive. These abnormal elements showed abnormality also in the sign of the tension coefficient of resistance which was negative for some orientations and positive for others. Cadmium and zinc, however, are entirely normal in that the tension coefficient is found to be positive for all orientations.

ZINC RESULTS

Fifteen zinc crystals were measured; two others were measured but discarded, the values of the coefficients found being nearly twice that to be expected in comparison with the other values: such high values result easily from slightly cracked crystals.

According to the theory sketched above the tension coefficient β of electrical resistance will be related to the primary orientation θ (θ being the angle between the hexagonal axis of the crystal and the cylindrical axis of the casting) by the equation

$$\beta = (1/\rho\theta) \{ \rho_{11} \sin^4 \theta + \rho_{33} \cos^4 \theta + (\rho_{13} + \rho_{31} + \rho_{44}) \sin^2 \theta \cos^2 \theta \}. \quad (22)$$

(The specific resistance for the orientation θ is indicated by the symbol ρ_θ , and ρ_{11} , ρ_{33} , ρ_{13} , ρ_{31} and ρ_{44} are five of the six constants relating the change in resistance to the applied stress.) Thus the data from the tension experiments furnish *three* relations between the *six* constants involved. Applying the method of least squares, the values of these three constants, using the observed experimental data *without* correcting for the change in resistance resulting from the change in shape of the crystal, are the following:

$$\begin{aligned} \rho_{11} &= (0.86 \pm 0.26) \times 10^{-11}, \\ \rho_{33} &= (14.09 \pm 0.45) \times 10^{-11}, \end{aligned} \quad (23)$$

$$(\rho_{13} + \rho_{31} + \rho_{44}) = (15.27 \pm 1.2) \times 10^{-11},$$

where the tension T is expressed in units of kg/cm². The values of these same constants *corrected* for the change in shape of the crystal are

$$\begin{aligned} \rho_{11} &= (0.00 \pm 0.29) \times 10^{-11}, \\ \rho_{33} &= (11.63 \pm 0.50) \times 10^{-11}, \quad (24) \\ (\rho_{13} + \rho_{31} + \rho_{44}) &= (15.19 \pm 1.35) \times 10^{-11}. \end{aligned}$$

The resulting empirical curves for β are shown in Fig. 1 together with the individual observed points. The difference between the points and the curve is in some instances as great as 2×10^{-6} , but in view of the small deflections which were measured this is well within experimental error, so that the agreement seems satisfactory. The average difference between the observed points and the curve is about 1.1×10^{-6} .

It is of interest to compare these results with those found by other observers for polycrystalline zinc. Tomlinson⁶ in 1883 found the value for the tension coefficient of zinc to be 2.75×10^{-6} and Rolnick⁷ quite recently, using a.c. technique, found the value 3.26×10^{-6} . Both these values lie within the range of those observed with single crystals of differing orientations. If the formula⁸ for the tension coefficient of a haphazard arrangement of crystals of all possible orientations be applied, i.e.,

$$\frac{\beta}{\rho} = \frac{1}{3} \left[\frac{2\beta_{\theta=90}}{\rho_{\theta=90}} + \frac{\beta_{\theta=0}}{\rho_{\theta=0}} \right], \quad (25)$$

the average tension coefficient comes out to be 7.8×10^{-6} . ρ is here the specific resistance of the haphazard arrangement computed according to the formula

$$\frac{1}{\rho} = \frac{1}{3} \left[\frac{2}{\rho_{\theta=90}} + \frac{1}{\rho_{\theta=0}} \right]. \quad (25a)$$

This is far greater than the observed value and leads to the conclusion that the orientation of the crystals is not absolutely at random, but that there is a tendency for the zinc crystals to form with their principal axes of symmetry perpendicular to the axis of the cylindrical casting. This is in agreement with the known fact that in making single zinc crystals they tend to crystallize with the principal axis of symmetry perpendicular to the length of the crystal.

Since zinc crystals are easily cleavable, the determination of their orientations could be readily made. The tension coefficients were found

⁶ Tomlinson, Phil. Trans. **174**, 1 (1883).

⁷ H. Rolnick, Phys. Rev. **36**, 506 (1930).

⁸ P. W. Bridgman, Proc. Am. Acad. **60**, 305 (1925).

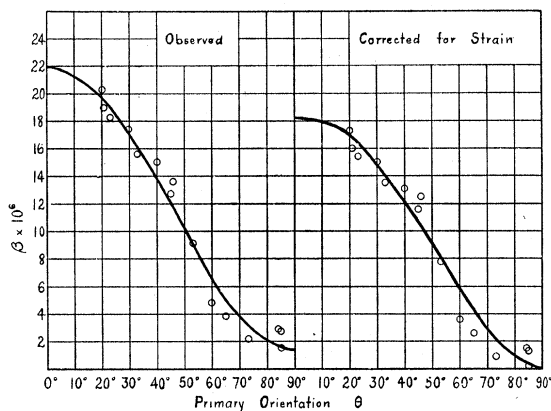


FIG. 1. Tension coefficient of zinc.

to be independent of the secondary orientations, as demanded by the theory. For instance, three crystals of approximately 85° primary orientation and of secondary orientations 3° , 10° and 14° had respectively the tension coefficients 2.71 , 1.58 and 2.91×10^{-6} which agree within the experimental error to be expected as indicated by the agreement of the experimental points with the empirical curve.

CADMIUM RESULTS

Sixteen cadmium crystals of various orientations were measured. The resulting data were again compared with the theory, as given in Eq. (2). Fig. 2 then shows the agreement of the observed experimental points with the curve determined according to this theory. In only one case is the discrepancy between an experimental point and the curve greater than 2×10^{-6} . This agreement is satisfactory in view of the fact that the deflections were again at least as small as in the case of zinc. The constants in Eq. (2) had the following values:

- (a) when no correction is applied for the change in shape of the crystal,

$$\begin{aligned} \rho_{11} &= (6.08 \pm 0.33) \times 10^{-11}, \\ \rho_{33} &= (16.3 \pm 0.53) \times 10^{-11}, \\ (\rho_{13} + \rho_{31} + \rho_{44}) &= (11.7 \pm 1.7) \times 10^{-11}, \end{aligned} \quad (26)$$

- (b) when the correction is applied for the change in shape,⁹

$$\begin{aligned} \rho_{11} &= (4.81 \pm 0.34) \times 10^{-11}, \\ \rho_{33} &= (11.6 \pm 0.55) \times 10^{-11}, \\ (\rho_{13} + \rho_{31} + \rho_{44}) &= (6.22 \pm 1.8) \times 10^{-11}. \end{aligned} \quad (27)$$

The tension coefficient for polycrystalline cadmium has not been determined and so the results

⁹ These corrections are computed from the strain coefficients for cadmium given by P. W. Bridgman, Proc. Am. Acad. 60, 305 (1925).

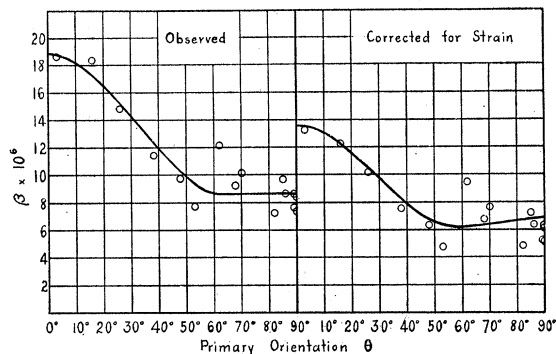


FIG. 2. Tension coefficient of cadmium.

of this experiment cannot be compared with it.

Inasmuch as cadmium crystals do not cleave, the determination of the crystal orientation was a matter of some difficulty. Various methods were tried. One was to observe under a microscope the angles made by the glide planes when the crystal was stretched.¹⁰ Another was to project the reflection pattern on a screen, supporting the crystal in a spherical flask of benzene which acted as a lens.¹¹ However, neither of these methods seemed to work easily and in the end there was adopted P. W. Bridgman's method¹² of observing the reflection pattern with the eye and recording it by marking on a wooden sphere (through whose center the crystal was rigidly attached) the points at which a mirror held parallel to the reflection planes of the crystal would touch the sphere. Cadmium crystals then showed hexagonal symmetry, their reflection pattern consisting of twelve dots in each hemisphere, six near the pole and six near the equator. Regular reflections from the glass interfered with the crystal reflections if the crystals were observed before the removal from their glass jacket. The crystal reflections could be noticeably increased by etching them in dilute HCl, where one part of water was added to two parts of acid. The resulting orientations are probably good to about 3° .

I am much indebted to Harvard University for permission to carry out these experiments in the Research Laboratory of Physics and to Professor P. W. Bridgman for advice in connection both with the experiments themselves and their interpretation.

¹⁰ Bruce Chalmers, Phil. Mag. 14, 612 (1932).

¹¹ E. Jakowlewa, Physik. Zeits. Sowjetunion 3, 429 (1933).

¹² P. W. Bridgman, Proc. Am. Acad. 60, 305 (1925).