The Excitation Function of Lithium Under Proton Bombardment

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(Received October 18, 1935)

The yield of alpha-particles in the bombardment of lithium with protons is calculated for different depths and widths of the "potential well." It is found that agreement with experiment can be obtained either by using the part of the incident wave having an angular momentum $L=0$ or the part having $L=1$. For the first condition with a radius of 0.35×10^{-12} cm one needs a "potential well" about 35 MEV deep. For $L=1$ and the same radius a "well" about 21.5 MEV is needed. The latter depth fits in nicely with approximate estimates of this depth from nuclear binding energies. The calculations are made more carefully than is customary in the usual type of potential barrier penetration consideration. It is found that this is necessary and that even the order of magnitude of the collision cross section requires the more accurate type of calculation. The influence of the depth and width of the "potential well" is found to be pronounced and it is found possible to vary the shape of the excitation curve as well as the absolute value of the cross section by changing the depth and width of the "well." For energies sufficient to allow the proton to slip over the potential barrier classically, the collision cross section may increase or decrease with the energy depending on the "well." Thus the saturation of the yield with the voltage is not a good measure of the height of the barrier and of the nuclear radius. The position of stationary and resonance levels is found to be important for the shape of the excitation curve. The effect of the decay of the incident wave inside the nucleus is estimated and is found to be small in the present case. The asymptotic form of the dependence of the collision cross section on velocity at low velocities is const.$\times v^{-2}$ $\exp\{-2\pi ZZ'c/"137"v\}$ within the limitations of the present theory. Estimates of the theoretically expected variation of the yield with velocity are made for Li^7+H^2 and compared with the experiments of Oliphant, Kinsey and Rutherford. A correspondence between different nuclear reactions is established by means of which one can use calculations for one reaction to obtain yields for another reaction with a corresponding "potential well." The "potential well" necessary for the quantitative representation of the alpha-particle reaction is compared with the mass of Be^8. It is found possible to fit both requirements by attributing the alpha-particle reaction to $L=0$ and the formation of Be^8 from Li^7 to the addition of a proton into a p level.

THE disintegration of lithium into two alpha-particles under proton bombardment was observed by Cockcroft and Walton[1] and the data were later extended to higher energies by Henderson.[2] Recently careful measurements of yields in thick as well as thin targets were made by Herb, Parkinson and Kerst.[3] According to these recent observations the yield of protons in thin targets increases steadily up to 400 kv. Measurements made up to 1000 kv by Hafstad and Tuve[4] show that here also the yield increases with the voltage. In this respect the data of Herb, Parkinson and Kerst as well as that of Hafstad and Tuve contradicts the observations of Henderson. In terms of the usual theoretical interpretation the results indicate that the potential barrier is not reached below 1000 kv.

It is tempting to connect these results with Goldhaber's discussion[5] of the relative improbability of this reaction because according to him the reaction can be reasonably attributed to incident protons having an orbital angular momentum $L=1$ (in units $\hbar$) and because the barrier for $L=1$ is appreciably higher than that for $L=0$. It is also of interest to know to what extent collision processes due to incident particles having $L=1$ are less probable than collision processes due to incident particles having $L=0$ and colliding "head on" with the bombarded nucleus. It is supposed by Goldhaber that the $L=0$ part of the incident wave is much more likely to produce a disintegration than the $L=1$ part and it would be very nice and simple if one could always use this point of view.

The probability of nuclear reactions is usually discussed[6] in terms of the following factors: (a) The probability that the bombarding particle

[1] J. D. Cockcroft and E. T. S. Walton, Proc. Roy. Soc. **A137**, 229 (1932).
[2] M. C. Henderson, Phys. Rev. **43**, 98 (1933).
[3] R. G. Herb, D. B. Parkinson and D. W. Kerst, Phys. Rev. **48**, 118 (1935).
[4] L. R. Hafstad and M. A. Tuve, Phys. Rev. **48**, 306 (1935). We are indebted to the authors for informing us of their results before publication.
[5] M. Goldhaber, Proc. Camb. Phil. Soc. **30**, 561 (1934).
[6] J. D. Cockcroft, International Conference on Physics, London (1934).

should perform a nuclear collision (this probability is usually computed by assigning to the process a collision cross section of the order of the square of the wave-length of the incident particles); (b) the probability that, once a collision within the area of the collision cross section takes place, the particle penetrates through the potential barrier; (c) the chance that once the particle penetrates through the barrier, a disintegration takes place. In terms of this description of the process one would expect the $L=0$ part of the incident wave to be more effective than the part represented by $L=1$ and one would further expect the process to reach saturation once the incident particles have an energy higher than the top of the barrier.

The incident particles must be supposed in many cases to be subjected to an attractive field of force when they are inside the bombarded nucleus because, for example, in the bombardment of Li by protons the final product of two alpha-particles is produced as a result of the binding of the incident proton into a H^3 combination in the Li^7 nucleus. The influence of the attractive forces can be treated schematically by means of a "well" in the potential. The influence of such a "well" is not taken care of by the usual description of the collision process. It is desirable to have a discussion in which this influence is taken into account. In order to do so we discuss below the same problem from a different point of view which is more closely related to a straightforward solution.

It is supposed in the calculations presented here that the incident particles can be treated as a wave incident on the bombarded nucleus. As usual the density of the particles can be represented by the square of the absolute value of the wave function ψ. The total chance of finding the particle inside the bombarded nucleus is $\int|\psi|^2 d\tau$ and can be calculated for any assumed form of the potential energy. The number of disintegrations per second is assumed to be $P\int|\psi|^2 d\tau$ where P is a constant characteristic of the colliding particles and of the end products of the reaction. The constant P may be described as the chance per second that a disintegration should take place when the incident particle is in the bombarded nucleus. Its order of magnitude may be roughly estimated as the relative velocity of the disintegration products divided by the nuclear radius and multiplied by the transparency of the potential barrier for the disintegration products. This point of view is closely related to the discussion in Chapter IV, §2 of Gamow's book.[7] One can justify it by a more general discussion in which the nucleus and the colliding particle are considered as a many-body problem.[8]

The essential limitations on the method from the point of view of the general wave equation for the composite parts of the nucleus are:

(a) It is sufficient to consider the wave function as represented by a sum of the functions representing the initial and final state.

(b) The wave functions representing the final state do not change appreciably with the velocity of the incident particles. This condition is satisfied if the energy liberated in the reaction is large compared to the incident energy and may be expected to be satisfied for $Li^7+H^1\rightarrow He^4+He^4$. This condition is not satisfied even approximately if the disintegration products separate with an energy which corresponds to a resonance level for their mutual potential energy.[8] In such a case we may speak of *resonance to the disintegration products* and it is possible that the resonance observed for the γ-rays from Li under proton bombardment is of this type.[9] It would then be attributable to the formation of one normal and one excited α-particle. The maximum energy of the rays is according to Crane, Delsasso, Fowler and Lauritsen[9a] nearly equal to the energy available in the formation of the 8 cm α-particles. We may provisionally suppose with these authors that in the emission of γ-rays one of the particles is excited to a level about 16 MEV high and the kinetic energy of the two particles is therefore small. The smallness of the kinetic energy is favorable for sharp resonance. Estimates show that a kinetic energy of about 500 kv would correspond approximately to a half-value breadth of 80 kv and this half-value breadth is sensitive to the kinetic energy. It is also possible that an excited Be^8 nucleus is formed. It is thus not necessary to consider the resonance to γ-rays in connection with the yield of 8 cm α-particles because the process involved may be essentially different.

(c) The weighting of the function which represents the incident particle is not important. The point is that this function enters the expression for the collision cross section through the matrix element of the interaction energy. It is thus weighted through the nucleus by quantities which involve the interaction energy and the wave function of the final state. It is laborious to make calculations with

[7] G. Gamow, *Nuclear Structure and Radioactivity.*

[8] Appendix I. (Justification of method used from point of view of many body problem.)

[9] L. R. Hafstad and M. A. Tuve, Phys. Rev. **47**, 507 (1935).

[9a] Crane, Delsasso, Fowler and Lauritsen, Phys. Rev. **46**, 531 (1934); **47**, 410 (1935).

the matrix element and attempts to do so show that the results are sensitive to assumptions about the interaction energy as well as assumptions about the state and constitution of the bombarded nucleus and the final products. In the present state of relative ignorance about the state of the particles inside nuclei it was felt desirable to have calculations in which the weighting of the incident wave is neglected. By doing so, one introduces in P an arbitrary adjustable constant and loses definiteness of theoretical prediction, but one gains freedom from special assumptions about nuclear binding forces.

(d) It is assumed that one may neglect the absorption of incident particles due to the fact that they cause disintegrations.[10] This restriction would make the theory inapplicable if the nuclear diameter and P were large. The incident wave would then decay approximately exponentially inside the nucleus and calculations neglecting the effect of such a decay would be invalid. It is complicated to try to fit experimental material by taking this effect into account in all of the calculations. It is simpler to first neglect the effect of absorption on the incident wave and to derive from comparison of theory and experiment a value of P. The calculations can then be repeated with this value of P and the experimental data fitted by successive approximations. This is the plan followed below and the corrections due to absorption are found to be negligible in the case considered.

In standard notation an incident plane wave

$$e^{ikZ}=(\pi/2\rho)^{\frac{1}{2}}\sum_0^\infty(2L+1)i^L P_L(\cos\theta)J_{L+\frac{1}{2}}(\rho) \quad (1)$$

is modified by the Coulombian field of the nucleus into

$$\sum_0^\infty(2L+1)i^L P_L(F_L/\rho)e^{i\sigma_L}. \quad (2)$$

Here $k=\mu v/\hbar$ where μ = reduced mass, v = relative velocity; $\rho=kr$ with r = distance between colliding particles; P_L = Legendre polynomial of order L; $J_{L+\frac{1}{2}}$ = Bessel function of order $L+\frac{1}{2}$. The function F_L is the regular solution of

$$[d^2/d\rho^2+1-2\eta/\rho-L(L+1)/\rho^2]F_L=0, \quad (3)$$

where $\eta=1/ka$, $a=\hbar^2/\mu ZZ'e^2$, Ze, $Z'e$ are the charges on the particles and e is the electronic charge. The function F_L is normalized so as to be asymptotic for large ρ to a sine wave with unit amplitude. We also use G_L which is asymptotic to a cosine wave with unit amplitude. The constant σ_L depends on k but not on ρ and drops out in the present application. The nuclear well inside the nuclear radius r_0 modifies the function F_L into $\bar{F}_L$ which also satisfies (3) for $r>r_0$ and satisfies

$$[d^2/d\rho^2+1+2U/\mu v^2-L(L+1)/\rho^2]\bar{F}_L=0, \quad (3')$$

where U is the depth of the "well" for $r<r_0$.

$$\bar{F}_L=[F_L/(1-F_LG_L\delta_L-iF_L^2\delta_L)]_{r=r_0}u, \qquad \delta_L=(F_L'/F_L-\bar{F}_L'/\bar{F}_L)_{r=r_0}, \quad (4)$$

where u is the regular solution of (3′) normalized so that $u(r_0)=1$. Replacing F_L by $\bar{F}_L$ in the expression (2), one can calculate the probability of disintegration on the assumptions already discussed. The effective collision cross section is then found to be

$$\sigma=(\Lambda^2/\pi v)P\sum_0^\infty(2L+1)\int_0^{r_0}\bar{F}_L^2dr \quad (5)$$

or

$$\sigma=\frac{4\pi Pr_0^3}{v}\sum_0^\infty\frac{(2L+1)(F_L^2/\rho^2)\overline{u_L^2}}{(1-F_LG_L\delta_L)^2+F_L^4\delta_L^2};$$

$$\overline{u_L^2}=\frac{1}{r_0}\int_0^{r_0}u_L^2dr. \quad (6)$$

In the last formula F_L, G_L are supposed to be taken at $r=r_0$. Formula (5) is reminiscent of the procedure used by Cockcroft. In it $\Lambda=\hbar/\mu v$ occurs in the combination Λ^2/π and one may treat this factor separately. The quantity $\bar{F}_L^2$ under the integral sign depends on the transparency of the barrier to the incident waves. It will be noted, however, that the integral in Eq. (5) brings in the nuclear radius r_0 so that in addition to the usual factors there is present a factor Pr_0/v which has the significance of the chance of a disintegration due to a sojourn of the proton during a time r_0/v in the nucleus. It is not present in the usual discussions and brings in an additional factor $1/v$ into the velocity dependence of σ. It should be also remembered

[10] The possible importance of this factor was emphasized by Fermi in a colloquium of the Ann Arbor 1935 summer school where some of the present results were discussed. We are very grateful to Professor Fermi for this and several other valuable discussions.

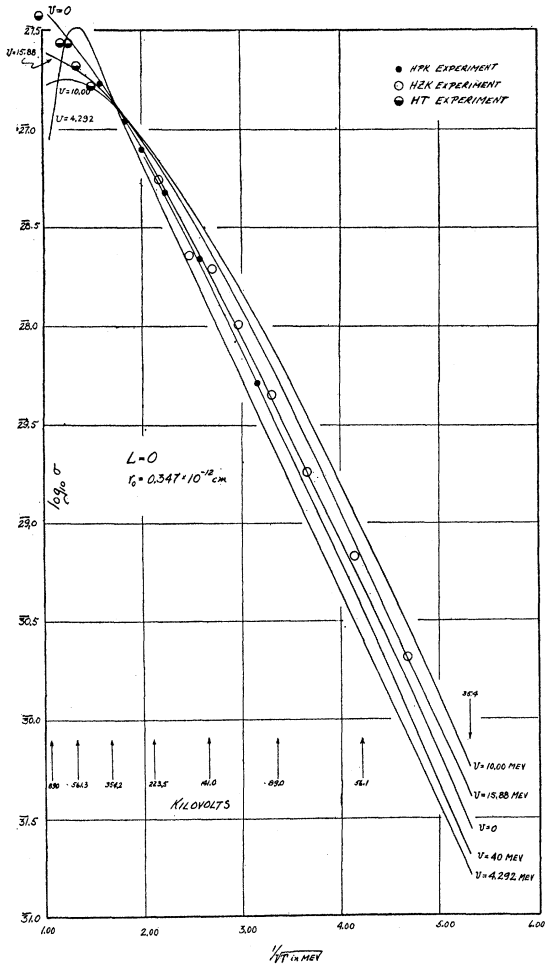

FIG. 1. $L=0$, $r_0=0.347\times10^{-12}$ cm, $U=0$, 4.292, 10.00, 15.88 MEV, $\rho/ak=0.316$.

that the integral in Eq. (5) is a function of the velocity quite apart from the penetrability of the barrier. Thus for $L=0$ and in the absence of a "nuclear well" one can approximate F_0 by $C_0\rho$, $C_0=[2\pi\eta/(-1+\exp 2\pi\eta)]^{\frac{1}{2}}$. The factor C_0 for large η [small energies] represents something like the penetrability of a one-dimensional barrier on account of the presence of $\exp(2\pi\eta)$. In addition C_0 contains $(2\pi\eta)^{\frac{1}{2}}$ which varies as $v^{-\frac{1}{2}}$. Finally ρ varies as v. The integral in Eq. (5) thus varies roughly as $v\exp(-2\pi\eta)$ and the cross section as $v^{-2}\exp(-2\pi\eta)$. This rough estimate thus leads to the type of velocity dependence used by Cockcroft. The reasons for the dependence are seen, however, to be different inasmuch as both the factors $(2\pi\eta)^{\frac{1}{2}}$ and $\rho/r=v$ are not usually considered. In this limiting case of small r_0 and "no well" one has approximately due to $L=0$

$$\sigma\cong(4Pr_0/3v)\pi r_0^2(2\pi\eta)/[-1+\exp 2\pi\eta]. \quad (7)$$

This formula shows that there is not very much point in speaking of Λ^2 rather than r_0^2 determining the order of magnitude of the cross section.[11] The factor πr_0^2 present in (7) shows, as one would expect, that σ vanishes with r_0 which is not the case in the method used by Cockcroft. On the other hand, the velocity dependence is the same in the two methods to within the limitations of Eq. (7).

The nuclear radius and the wave-length of the protons inside a reasonable "potential well" are of the same order of magnitude and the approximation of Eq. (7) is very poor. In applications to experimental material it is found necessary to use Eq. (6). According to Goldhaber it is probable that nuclear reactions are due to at most a few values of L. The contributions due to different L are, therefore, treated separately and the discussion in Appendix I shows that the values of P which can be expected to correspond to different L from the point of view of the many-body problem are different. The contribution to σ due to an individual L is denoted below by σ_L.

The denominator of an individual term of the summation in Eq. (6) determines the proximity of resonance.[12] Thus for small energies $F^2\delta$ is small and there is a maximum in σ_L whenever $1-FG\delta=0$. The function $\bar{F}$ is in this case identical with G. For large energies $F^2\delta$ is of the same order of magnitude as $1-FG\delta$ and one has no sharp resonance under these conditions. Nevertheless the variation of σ_L with velocity may be strongly affected by having a blunt resonance at high energies. By arranging the "potential well" in this manner one can decrease the ratio of σ_L at low velocities to σ_L at high velocities without introducing a pronounced resonance peak. Again by properly arranging the depth and width of the potential well it is

[11] Cf. G. Breit, Phys. Rev. **34**, 817–818 (1929) for a very similar formula $\sigma=\pi r_0^2(2\pi\eta)/[-1+\exp(2\pi\eta)]$ which applies for small r_0 on supposition that the flux of particles through the nucleus determines the probability of disintegration. The difference of a factor v is due to the fact that at present the density is used instead of the flux.

[12] G. Breit and F. L. Yost, Phys. Rev. **48**, 203 (1935).

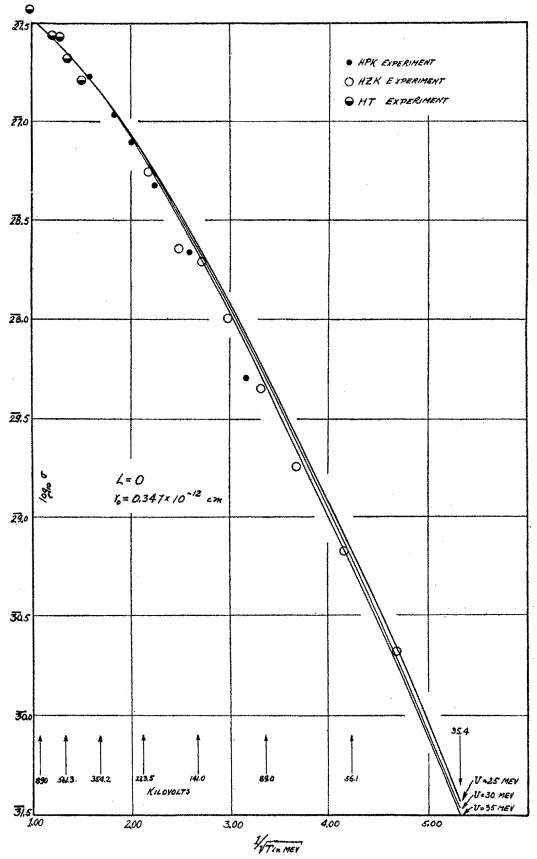

FIG. 2. $L=0$, $r_0=0.347\times10^{-12}$ cm, $U=25$, 30, 35 MEV, $\rho/ak=0.316$. Single line to the left of 354 kv is for $U=35$ MEV.

possible to produce the opposite effect of increasing the ratios of the low velocity values of σ_L to the values at high energies. This can be accomplished by arranging for a stationary level to lie slightly below the level of zero energy. The effect is then very similar to what one would have if there were resonance at very low energies. Both of these effects are obvious in the curves showing the theoretical dependence of σ_L on the energy reproduced above in Figs. 2 and 3. The theoretical value of σ_L for fixed P can be varied by *orders of magnitude* by changing the depth and width of the "well" without introducing pronounced resonance in the experimental region. It is thus at times rather unsatisfactory to speak of reactions as "probable" or "improbable" in the manner of Goldhaber unless one specifies a given width, depth and shape of the "potential well."

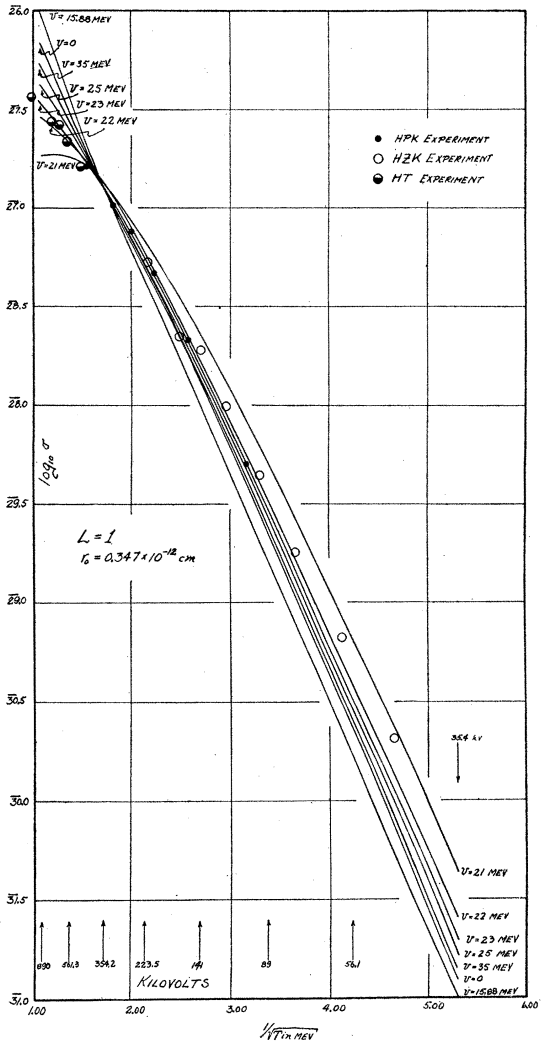

FIG. 3. $L=1$, $r_0=0.347\times10^{-12}$ cm, $U=0$, 15.88, 21, 22, 23, 25, 35 MEV.; $\rho/ak=0.316$.

Calculations with Eq. (6) are somewhat laborious and it is often desirable to have a more convenient formula. For $L=0$ one can obtain such a formula by neglecting the "well" altogether, i.e., by supposing that the inverse square field applies everywhere. The nuclear radius has then only the significance of defining a sphere of action within which the incident particle is effective in producing disintegrations. In the notation previously used[12]

$$F_0=C_0\rho\Phi_0\,;\quad C_0^2=2\pi\eta/[e^{2\pi\eta}-1]. \qquad (8)$$

Introducing this into Eq. (5) one has to perform

an integration involving Φ_0^2. For low energies Φ_0 can be approximated[13] by

$$\Phi_0 \cong J_1(ix)/(ix/2) = \hat{\Phi}_0; \quad x = (8\rho\eta)^{\frac{1}{2}} = (8r/a)^{\frac{1}{2}}, \quad (9)$$

where J_1 is the usual Bessel function. This approximation is fair even if one approaches energies in the neighborhood of the barrier energy for given r. Integrating one finds

$$\sigma = (4\pi P r_0^3/3v) C_0^2 f, \quad (10)$$

where

$$f = |2I_1(x)/x|^2 - |2I_2(x)/x|^2, \quad (11)$$

where the I are the usual Bessel functions of an imaginary argument. For work with tables given in Watson's *Bessel Functions* it is convenient to use

$$f = (2I_1/x)^2 - (2/x)^2(I_0 - 2I_1/x)^2. \quad (11')$$

A few values are given in Table I.

TABLE I.

x	1.0	1.2	1.4	1.6	1.8	2.0
f	1.20	1.30	1.43	1.59	1.80	2.04

For use of the tables of Φ being prepared for publication[13] it is convenient to express f in terms of $\hat{\Phi}_0$, $\hat{\Phi}_1$. One has

$$f = \hat{\Phi}_0^2 - (r/2a)(\hat{\Phi}_0 - (r/3a)\hat{\Phi}_1)^2. \quad (11'')$$

Substitution of numbers gives Table II.

TABLE II.

$f/\hat{\Phi}_0^2$ =	0.892	0.869	0.844	0.814	0.778	0.742
$\log_{10}(r/a)$ =	$\bar{1}.4$	$\bar{1}.5$	$\bar{1}.6$	$\bar{1}.7$	$\bar{1}.8$	$\bar{1}.9$

(The third figure is not quite certain in these ratios.) Similar formulas can be worked out for $L=1$. In the approximation of Eq. (10) one obtains the same velocity dependence as by means of Eq. (7). The absolute value can be estimated more accurately by means of Eq. (10). Numerical trial shows that Eq. (10) gives a velocity dependence of σ and absolute values similar to those obtained from Eq. (6) for $U=0$. It is good only as a starting point.

For very low energies ($\eta \gg L$) the velocity dependence for all L becomes the same in the general case. The asymptotic form of σ_L is then

$$\sigma_L \sim \frac{\pi^2 P r_0^3 2^{2L+3}(r/a)^{2L}(F_L'/F_L - G_L'/G_L)^2 \overline{u_L^2}}{(2L)!(2L+1)!(\bar{F}_L'/\bar{F}_L - G_L'/G_L)^2} \frac{\eta}{v} e^{-2\pi\eta}. \quad (12)$$

The first fraction approaches a constant value because $\rho F'/F$, $\rho G'/G$, $\rho \bar{F}'/\bar{F}$ as well as $\overline{u_L^2}$ approach constant values.

The effect of absorption of incident protons within the nucleus can be estimated by using a wave equation with a complex potential energy

$$\Delta\psi + (2\mu/\hbar^2)(E - V + i\hbar P/2)\psi = 0. \quad (13)$$

Here E is the energy, V is the real part of the potential energy, and the extra term in P was added so as to represent absorption. It follows from this equation that

$$(\hbar/2\mu i)\,\mathrm{div}\,[\psi^*\nabla\psi - \psi\nabla\psi^*] = -P\psi^*\psi. \quad (14)$$

Particles may be said to disappear at the rate $P\psi^*\psi$ per unit volume and per unit time. For $L=0$ and a constant V Eq. (13) gives

$$\sigma = \frac{2\pi(P/v)[Sh/k_1' - s/k_0]}{|k'|^2(Ch+c)(G^2+F^2) + k^2(Ch-c)(G'^2+F'^2) - 2ks[k_0'(GG'+FF') + k_1'] - 2kSh[k_1'(GG'+FF') - k_0']}, \quad (15)$$

where

$$k' = k_0' + ik_1' = (2\mu/\hbar^2)^{\frac{1}{2}}[E - V + i\hbar P/2]^{\frac{1}{2}}, \quad (15')$$

$$Ch = \cosh 2k_1' r_0, \quad Sh = \sinh 2k_1' r_0, \quad c = \cos 2k_0' r_0, \quad s = \sin 2k_0' r_0. \quad (15'')$$

[13] F. L. Yost, John A. Wheeler and G. Breit in preparation for publication.

For very low energies the terms in G^2 and GG' predominate strongly and one has the asymptotic form

$$\sigma\sim\frac{2\pi[Sh/\zeta_1-s/\zeta_0](Pr_0^3/v)(F^2/\rho^2)(FG/\rho)^{-2}}{\zeta^2(Ch+c)+(Ch-c)(\rho G'/G)^2-2\zeta_0 s(\rho G'/G)-2\zeta_1 Sh(\rho G'/G)}, \tag{16}$$

$$\zeta_0=k_0'r_0,\quad \zeta_1=k_1'r_0,\quad \zeta=|\zeta_0+i\zeta_1|. \tag{16'}$$

Comparing this with Eq. (12) the asymptotic dependence on the velocity for very low energies is seen to be the same because FG/ρ, ζ_0, ζ_1, $\rho G'/G$ approach finite limits. The factor in Eq. (16) which contains the main velocity dependence is $F^2/v\rho^2$ which for $L=0$ behaves as the $(\eta/v)\exp(-2\pi\eta)$ of Eq. (12). The argument applies also for $V=0$. Eq. (12) behaves as usual while in Eq. (16) the ζ approach finite limits $\zeta_0=\zeta_1=r_0(\mu P/2\hbar)^{\frac{1}{2}}$ so that $Sh/\zeta_1-S/\zeta_0$ does not vanish as long as $P\neq 0$. Thus in the limit of $v\to 0$ the dependence $v^{-2}\exp(-2\pi\eta)$ may be expected to apply with and without absorption. The quantity Pr_0/v is therefore not the primary determining parameter for the applicability of Eq. (6) to the velocity dependence as one is tempted to suppose.

It is instructive to consider the special case of $ZZ'=0$ which corresponds to neutron capture. Instead of Eq. (15) we now have

$$\sigma=2\pi[Sh/k_1'-s/k_0'](P/v)[|k'|^2(Ch+c)+k^2(Ch-c)-2kk_1's+2kk_0'Sh]^{-1}. \tag{17}$$

For very low velocities and vanishing V, $k_0'^2=k_1'^2=\mu P/2\hbar$ and

$$\sigma\sim(2\pi\hbar/\mu v k_1')(Sh-s)/(Ch+c). \tag{17'}$$

If on the other hand, $P\to 0$, V is finite, and $v\to 0$

$$\sigma=2\pi k_0^{-2}(1-s/2k_0r_0)(Pr_0/v)/\cos^2 k_0r_0. \tag{17''}$$

Here also the velocity dependence is the same for very low velocities and the cross section varies as $1/v$ with or without absorption. According to Eq. (17') high values of P give small σ for small v which is just the opposite of the behavior without absorption given by Eq. (17''). This is due to the fact that a large absorption shifts the phase of the sine curve for $\bar{F}$ towards zero at the nuclear boundary and thus decreases the absolute value of $\bar{F}$ at r_0. These special cases indicate that the influence of absorption of the incident wave is more likely to show itself in the dependence of σ on P than on v. In connection with the discussion of experimental material numerical calculations made by means of Eq. (15) will be presented and it will be seen that in the special case considered the effect of absorption is not important.

Discussion of Experimental Material

The data considered here are those of Herb, Parkinson and Kerst[3] from 100 to 400 kv, of Hafstad and Tuve[4] in the region from 400 to 1000 kv and of Heydenburg, Zahn and King[14] from 200 kv to 46 kv. The data of Herb, Parkinson and Kerst give yields of α-particles in thick as well as thin targets and can be used to obtain absolute values of cross sections. The uncertain element enters in the range of protons in a solid Li target and in the dependence of the range on velocity. In deriving the absolute values of σ the 3/2 power dependence of range on energy was used. The data of Hafstad and Tuve and of Heydenburg, Zahn and King are used below to give values of σ by comparison with the absolute values of Herb, Parkinson and Kerst. Thus only ratios of σ's at different voltages are supposed to be given correctly by these data.[4, 14]

Assuming the 3/2 power dependence of range

[14] N. P. Heydenburg, C. T. Zahn and L. D. P. King, unpublished.

on kinetic energy, the number of disintegrations per proton is

$$Y=Nx_0\bar{\sigma},\quad \bar{\sigma}=(3/2)T_0^{-\frac{3}{2}}\int_0^T\sigma(T)T^{\frac{1}{2}}dT,\quad (18)$$

where $\sigma(T)$ = collision cross section for kinetic energy T; x_0 = range of protons at kinetic energy T; N = number of Li^7 nuclei per unit volume of target. Nx_0 can be obtained by comparison of a standard material (oxygen) with the stopping power of Li. Using the stopping powers of N_2, O_2, H_2O, CO_2, H_2 relative to oxygen, as given by Rutherford, Chadwick and Ellis, one obtains 0.948 for the stopping power of air relative to oxygen. From the Cavendish laboratory graphs the range of protons in air at 2 MEV = 7.2 cm. Using these numbers and 0.519 for the stopping power of Li relative to oxygen, one finds $Nx_0=6.03\times10^{19}$ at 400 kv. At this voltage the number of disintegrations caused by a single proton on a thick target[3] is $Y=46.4\times10^{-9}$. Hence $\bar{\sigma}=7.69\times10^{-28}$ cm^2 at 400 kv. From the thin target data of Herb, Parkinson and Kerst, the value of $\bar{\sigma}/\sigma$ was computed for 400 kv and was found to be 0.461 with a probable accuracy of 1 percent. Hence at 400 kv, $\sigma=1.668\times10^{-27}$ cm^2. No correction was made here for the 1 : 12 ratio of Li^6 to Li^7. From this σ the thin target results of Herb, Parkinson and Kerst give values of σ also for voltages down to 100 kv. The data of Heydenburg, Zahn and King were obtained for thick targets and were reduced by them to thin target data by differentiation. The use of these data thus presupposes the approximate validity of the 3/2 power range energy relation not only in obtaining the absolute value of σ at one voltage but also in obtaining the relative values at different voltages. One should not attach therefore as much weight to this set of data as to the thin target experiments.

The experiments of Hafstad and Tuve give preliminary values for the yield in a very thin film of lithium salts which probably had a negligible stopping power. We have summarized the data in Table III.

The number of significant figures is, of course, exaggerated in Table III both with respect to the absolute value of 400 kv and the relative values at different energies. Extra decimals were kept in the calculations only in order to avoid accumulation of errors. In the comparison of theory and experiment given in Figs. 1, 2, 3, 4, 5 the data of HPK are represented by closed circles, those of HZK by open circles and those of Hafstad and Tuve by circles, whose lower halves are black. The calculation of the theoretical values of σ was made with values of $\log_{10}ka=\bar{1}.6$, $\bar{1}.7$, $\cdots$ 0.3 and in some cases 0.4, 0.5. Computation for these values does not require interpolation of tables of Coulomb wave functions.[13] Throughout the kinetic energy of the bombarding particle was computed with

$$T=24.83mZ^2Z'^2(ka)^2\text{ kv}.\quad (19)$$

Here m is the mass of the bombarding particle in terms of the mass of the hydrogen atom and the kinetic energy is expressed in kilovolts and Z, Z' are the atomic numbers of the colliding particles. The nuclear radius is given by

$$r_0=2.88\times10^{-12}(\rho/ka)/(ZZ'/\mu)\text{ cm},\quad (20)$$

where μ is the reduced mass for the collision, also expressed in terms of the mass of the hydrogen atom. When the kinetic energy of relative motion is

$$ZZ'e^2/r_0=49.66Z^2Z'^2(\mu/m_H)(ka/\rho)\text{ kv},\quad (21)$$

the incident particle has enough energy to slip over the barrier. The kinetic energy which the incident particle must have in order to clear the

TABLE III. *Kinetic energy (T) of protons and collision cross section of (σ).*

T in MEV	(T in MEV)$^{-\frac{1}{2}}$	$10^{27}\sigma$	OBSERVER
0.100	3.162	0.0503	HPK
.150	2.582	.2061	
.200	2.236	.4692	
.250	2.000	.7693	
.300	1.826	1.070	
.350	1.690	1.367	
.400	1.581	1.668	
.450	1.488	1.64	HT
.550	1.347	2.21	
.625	1.264	2.70	
.700	1.194	2.78	
1.050	.976	3.72	
.2125	2.170	.546	HZK
.1875	2.309	.4395	
.1625(?)	2.481	.2258	
.1375	2.697	.1908	
.1125	2.981	.1008	
.0915	3.306	.04438	
.0748	3.656	.01787	
.0584	4.138	.006682	
.0460	4.662	.001908	

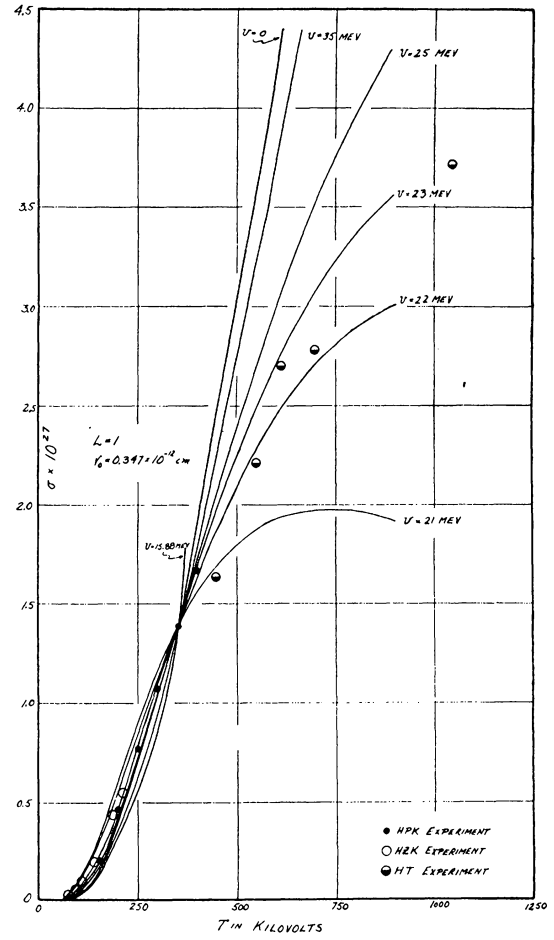

FIG. 4. $L=1$, $r_0=0.347\times10^{-12}$ cm; $\rho/ak=0.316$.

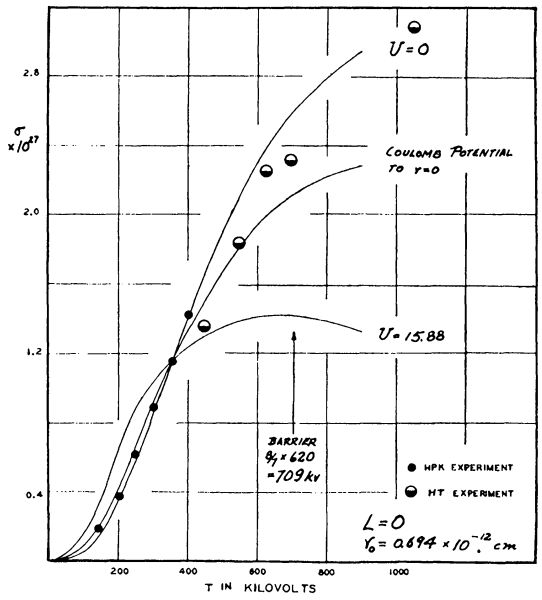

FIG. 5. $L=0$, $r_0=0.694\times10^{-12}$ cm, $U=0$, 15.88 MEV and Coulomb potential to $r=0$; $\rho/ak=0.631$.

barrier is $(m/\mu)ZZ'e^2/r_0$, i.e.,

$$T_{\text{barrier}}=49.66Z^2Z'^2(m/m_{\text{H}})(ka/\rho)\text{ kv.} \quad (21')$$

Calculations for constant U (see Eq. (3')) involve the use of the phase of the wave function for $r\leqq r_0$ at $r=r_0$. This is

$$z=r_0[(2\mu/\hbar^2)(U+\mu T/m)]^{\frac{1}{2}}. \quad (22)$$

Thus, e.g., for $L=0$, $\overline{F}=$const. sin z for $r<r_0$. The formula

$$z=(2\rho/ka)^{\frac{1}{2}}[U/(ZZ'e^2/r_0)+\tfrac{1}{2}ka\rho]^{\frac{1}{2}} \quad (23)$$

combined with Eq. (22) is convenient for the calculation of z.

A set of numerical calculations for protons colliding with Li or any other nucleus can be interpreted by means of Eqs. (19) . . . (23) together with Eq. (6) so as to give values of σ for protons and deuterons colliding with any other nucleus provided the value of U is properly chosen. Thus keeping ρ and ka fixed the terms in the summation in (6) are left unchanged provided z is also kept fixed. In order that this be the case we must have the same values of $U/(ZZ'e^2/r)$ in the two reactions according to Eq. (23). By Eq. (21) this is accomplished by keeping

$$U_1\mu_1^{-1}Z_1^{-2}Z_1'^{-2}=U_2\mu_2^{-1}Z_2^{-2}Z_2'^{-2}, \quad (24)$$

where the indices 1,2 refer to the two reactions. Collision processes in which this correspondence is maintained may be called *corresponding* processes. The only part of the formula for σ which is different for *corresponding* processes is the factor Pr_0^3/v. Values of σ in corresponding processes which belong to the same ka may be called *corresponding* values. The ratios of *corresponding* collision cross sections at different voltages are thus the same for *corresponding* processes. The values of U which must be used for deuterons colliding with the same nucleus as protons in order to give corresponding processes are nearly twice those for protons. This is convenient because the binding of a deuteron to a nucleus may be expected to be roughly twice the binding of a proton.

In Fig. 1 the values of $\log_{10}\sigma$ are plotted as ordinates and those of $(T$ in MEV$)^{-\frac{1}{2}}$ as abscissae for $L=0$, $\rho/ka=0.3162$, $r_0=0.347\times10^{-12}$ cm and $U=0$, 4.292, 10.00, 15.88 and 40.00 MEV. In this as well as the succeeding figures P was adjusted so as to make the theoretical and experimental curves intersect at 354 kv. It will be noted that $U=0$ agrees fairly well with the experimental data. The agreement is not perfect and $U=0$ is presumably an unreasonable value because it allows for no binding of the proton to the Li^7 nucleus. When U is increased the first effect is to spoil agreement with experiment. The values of U at low voltages become anomalously small as is seen from the curve for $U=4.292$ MEV. The maximum in σ shows that this effect of increasing U can be attributed to resonance above the barrier. As U increases further the resonance level moves through the experimental region towards lower voltages and finally becomes a stationary level. This condition is shown by $U=10.00$ MEV. Here the expected yield is too high for low voltages on account of the fact that $1-FG\delta$ decreases with the voltage. In order to obtain agreement with experiment the stationary level must be moved down into the "well." The effect of doing so is shown by $U=15.88$ and $U=40$ MEV. Relatively large changes in U are seen to be necessary to produce an appreciable effect. Fig. 2 gives similar curves for $U=25$, 30, 35 MEV. Above 400 kv the theoretical values appear to be definitely too high. However, it should be remembered that the experimental values are not very certain in this region. We may thus consider the agreement as being moderately satisfactory. $U=35$ MEV appears to be the best fit.

Fig. 3 shows similarly the condition for $L=1$. $U=0$, gives a too steep increase of σ with velocity. The immediate effect of increasing U is to increase the disagreement between theory and experiment as is shown by $U=15.88$ MEV. This is due to the fact that a resonance level is moving down into the experimental region as U increases. The level is at $T=0$ when $U=20.5$ MEV. By adjusting U in the neighborhood of this value one can obtain a large range of variation in the dependence of σ on T. Thus for $U=21$ MEV the deviation from experiment is in an opposite direction from that at $U=0$. The experimental points are seen to be bracketed by $U=21$ and $U=22$ MEV. Fig. 4 shows a direct plot of σ against T for the same conditions as Fig. 3.

Calculations for radii $r_0=0.694\times10^{-12}$ cm and 0.501×10^{-12} cm were made for σ for several values of U. The results are very similar to those given by the curves and can be foreseen by remembering that the important thing is the position of stationary and resonance levels.

It will be noted from the above graphs that it is not proper to describe the problem only as one of penetration through the barrier. The depth and width of the "well" are seen to have a very pronounced effect and one may have a decrease of σ with T at the barrier as well as an increase. The description of the disintegration phenomenon by means of penetration through a barrier is particularly poor for high energies and it begins to be poor before the energy exceeds the barrier height. For a fixed P the order of magnitude of the expected collision cross section varies with the "well." Thus the values of $(4Pr_0/3v)$ at 354 kv are as in Table IV for $L=0$, $r_0=0.347\times10^{-12}$ cm and in Table V similarly for $L=1$. Comparing these values with the general slope of the curves for $\log_{10}\sigma$ against $T^{-\frac{1}{2}}$ it is seen that there is considerable parallelism between them. Thus for example for $L=1$ the $4Pr_0/3v$ is nearly the same for $U=35$ and for $U=0$ and the slopes of the curves are also nearly the same. Both effects are due to the distance of the experimental region from the stationary level. It is also obvious that one cannot, in general, consider a reaction due to $L=1$ as much less prob-

TABLE IV. $L=0$.

U	$4Pr_0/3v$	U	$4Pr_0/3v$
0	0.034	20	0.135
4.29	.0028	23	.160
10	.0153	25	.164
15.88	.0820	27	.158
18	.111	30	.134

TABLE V. $L=1$.

U	$4Pr_0/3v$	U	$4Pr_0/3v$
0	1.415	22	0.0225
4.29	.887	23	.0453
10	.359	25	.114
15.88	.0560	35	.818
21	.00755		

able than for $L=0$. The variation of P for fixed L due to changing U can be just as important as the variation of P with fixed U due to changing L.

For the radius $r_0=0.347\times10^{-12}$ cm ($\rho/ka=0.3162$) the potential barrier is reached when $T=1413$ kv. It has already been pointed out that the shape and slope of the yield curves at the barrier varies with U. In order to show that this condition is not a characteristic of the radius used we give in Fig. 5 the theoretical yield curves for $\rho/ka=0.631$, $r_0=0.694\times10^{-12}$ cm, $L=0$. The barrier is reached in this case when $T=709$ kv which falls within the range of experiments of Tuve and Hafstad. For $U=0$ the yield goes on increasing at the barrier. For $U=15.88$ MEV it decreases after the barrier voltage is exceeded.

The effect of absorption of the incident waves discussed in connection with Eqs. (13), (14), (15) was estimated for $L=0$, $U=15.88$ MEV, $r_0=0.347\times10^{-12}$ cm. According to Table IV the experimental value of the cross section of 354 kv demands $4Pr_0/3v=0.0820$. The complex potential energy used in Eq. (15) is connected more directly with Pr_0/v_i where v_i is the velocity which the proton would have classically inside the "well." For the $Li^7+H^1\rightarrow He^4+He^4$ reaction we have thus $4Pr_0/3v_i=0.01134$. This reaction, in the classification of Goldhaber, is an "improbable" one. Compared with $Li^6+H^1\rightarrow He^3+He^4$ it is perhaps thirty times less probable. In order to have a more typical case we multiply the above value of $4Pr_0/3v_i$ by 40 which corresponds to using $Pr_0/v_i=0.3435$. We use this value for very small v and calculate the corresponding values for larger v. The formulas which determine the necessary quantities in Eq. (15) are:

$$\zeta_0=k_0'r_0;\quad \zeta_1=k_1'r_0;$$

$$\zeta=|k'|r_0=(\zeta_0^2+\zeta_1^2)^{\frac{1}{2}};\quad v_i=\hbar\kappa/\mu \tag{25}$$

$$\kappa^2=2\mu(E-V)/\hbar^2;\quad z=\kappa r_0 \tag{25'}$$

$$\zeta_0^2=(z^2/2)\{1+[1+(Pr_0/zv_i)^2]^{\frac{1}{2}}\};$$

$$\zeta_1^2=(z^2/2)\{-1+[1+(Pr_0/zv_i)^2]^{\frac{1}{2}}\}. \tag{26}$$

In Eq. (15) the difference of ζ_0 from z and the difference of ζ_1 from 0 determine the effect of absorption. A numerical calculation gives the following values:

TABLE VI.

T	z	ζ_0	ζ_1	σ/σ_{ab}
35.4 kv	2.852	2.857	0.1713	1.01
89.0	2.857	2.862	.1710	1.04
223.5	2.867	2.872	.1704	1.04
354.2	2.877	2.882	.1698	1.04
890	2.919	2.924	.1674	1.07
2235	3.020	3.024	.1618	1.09

The last column of the above table gives the ratio of σ, the theoretical cross section neglecting the decay of the incident wave, to σ_{ab} the theoretical cross section taking this decay into account. The first values may be somewhat inaccurate on account of inaccuracies in the Coulomb functions. As the kinetic energy increases σ_{ab} increases slightly in comparison with σ. However, the effect is not serious and is usually too small to be of interest. It is thus likely that calculations neglecting the decay of the incident wave inside the nucleus are usually sufficiently good. It is not intended to say, however, that this will be always so. Small z and large Pr_0/v_i will doubtless make the effect of absorption important and an estimate of its importance seems to be advisable.

It is of interest to compare the deuteron reactions on Li with the corresponding proton reactions. According to Oliphant, Kinsey and Rutherford[15] the increase with energy in the efficiency of the deuterons is more marked than that for protons. Thus according to their graph the deuteron yield is lower than the proton yield below about 120 kv and is considerably greater than the proton yield at 170 kv. This crossing over of the yield curves can be understood as the effect of the difference in velocity of protons and deuterons at the same voltage. The deuterons having the lower velocity, the exponent $-\pi ZZ'c/$ "137"v in the C_L is larger in absolute value for deuterons than for protons and varies therefore more rapidly. For the lower range of voltages used by Oliphant, Kinsey and Rutherford the exponential factor in C_L is the most important factor of σ and a crossing over of the yield curves is thus to be expected.

[15] Oliphant, Kinsey and Rutherford, Proc. Roy. Soc. **A141**, 722 (1933).

According to the consideration of corresponding collision processes discussed in connection with Eq. (24) we would expect the graphs of Fig. 1 to apply to the deuteron reactions provided the abscissa $(T_{\text{proton}})^{-\frac{1}{2}}$ is multiplied for each point by $2^{-\frac{1}{2}}=0.707$. The deuteron graphs are thus steeper than those for protons. The graphs of Fig. 1 ($\rho/ka=0.3162$) when applied to Li^7+H^2 correspond to a nuclear radius of 0.195×10^{-12} cm. This is appreciably smaller than the 0.347×10^{-12} cm used for Li^7+H^1. It is fairer to use a larger radius for the H^2 collision such as $r_0=0.39\times10^{-12}$ cm which corresponds to $\rho/ka=0.631$. The yield curves corresponding to this radius for Li^7+H^1 are shown in Fig. 5. According to Eq. (24) $U_{\text{proton}}=15.88$ is equivalent to $U_{\text{deuteron}}=28.2$ MEV. For such a well the factor by which the yield under deuteron bombardment should increase from 150 to 175 kv should be about 1.82 while actually it is about 2.5 for the 10.3 cm absorber and 1.8 for the 2.7 cm absorber. It is presumably the latter value that corresponds to the Li^7+H^1 reaction and there is at least a rough agreement between experiment and theory. However, it is felt that a more careful study of the deuteron reactions would be proper.

The value $U\leqq20$ MEV for Li^7+H^1, $L=1$ fits in reasonably with estimates from the nuclear binding energies. Using Wigner forces and statistical considerations one would estimate 18 MEV to be the value of U and roughly the same for Majorana-Heisenberg forces. This agreement may be partly accidental on account of the crudeness of the present theory as well as the theory of nuclear binding energies.

The two possibilities $L=0$, $U\leqq35$ MEV and $L=1$, $U\leqq22$ MEV can be compared with the probable mass of Be^8 which is approximately[16] 8.0071. With this mass, and Bethe's masses for Li^7 and H^1, the energy liberated in the formation of Be^8 when a proton is captured by Li^7 is 16.7 MEV. If $U=22$ MEV this capture cannot occur into the stationary level with $L=1$ because this level is too high by $16.7-1.5=15.2$ MEV. For this U the level with $L=0$ would be approximately in the right position to give the 16.7 MEV binding energy. However, this level of Li^7 is filled by two protons and the exclusion principle makes the addition of a third proton impossible. One can increase U so as to bring the second level for $L=1$ into about the same position as the first one is for $U=22$. This would give approximately the right dependence of the α-particle yield on velocity considering $L=1$ as responsible for the disintegration. However, the required depth would be about 80 MEV which is unreasonably large. For $U=35$ MEV the α-particle yield dependence on velocity can be fitted by using $L=0$ for the proton and simultaneously the mass of Be^8 can be accounted for by supposing that Be^8 is formed through the capture of a proton into the first level with $L=1$. In Li^7 there is only one proton in this level and the addition of a second proton to form Be^8 is possible and probable. The mass of Be^8 thus indicates that $U=35$ MEV and $L=0$ for the proton should be used for the α-particle yield. According to Tables IV, V for this U the $L=0$ process is more probable than the process for $L=1$ if P is the same. The angular momentum of the alpha-particles with respect to each other after disintegration must be $l=2$ if $L=0$ while it may be 0 or 2 if $L=1$. One generally supposes that $l=0$ is more probable than $l=2$. However, the wave-length of the relative motion of the alpha-particles is in this case of the same order as the nuclear radius and their kinetic energy is large. The two values $l=0$ and $l=2$ may be equally probable and there appears to be no general objection to regarding $L=0$, $l=2$ as the main process. It should be, nevertheless, remembered that in this process the total orbital angular momentum of the particles is changed from 1 to 2. This change occurs simultaneously with a change of total particle spin from 1 to 0. The interaction between orbital and spin angular momenta must therefore be supposed sufficiently strong to make this interchange possible. Such an interaction is not necessary for $L=1$, $l=0$, 2.

Briefly the above discussion indicates that one may suppose that there is only a weak interaction between orbital and spin angular momenta at the sacrifice of even an approximate agreement with the mass of Be^8 or else one may account for that mass but then one must suppose a strong spin orbit interaction.

[16] Oliphant, Kempton and Rutherford, Proc. Roy. Soc. **A150**, 241 (1935).

Appendix I

The collision of a proton with a Li^7 nucleus and the subsequent formation of two alpha-particles can be treated schematically by thinking of three particles A, B, C. Particles A and B are thought of as being bound to each other initially, particle C collides with the combination, knocks out B and attaches itself to A. The combination $A+B$ represents the Li^7 nucleus and C represents the proton. A may be thought of as an H^3 combination within Li^7 and B as an alpha-particle inside the same nucleus. The combination $A+C$ is then a newly formed alpha-particle recoiling from B. We introduce coordinates $x_A{}^i$, $x_B{}^i$, $x_C{}^i$, $i=1, 2, 3$ for the three particles. We introduce also

$$x_B-x_A=\xi, x_C-x_A=\eta, x=x_B-(M_Ax_A+M_Cx_C)/(M_A+M_C),$$
$$y=x_C-(M_Ax_A+M_Bx_B)/(M_A+M_B). \quad \text{(I 1)}$$

The energy of the system will be supposed to be the sum of the kinetic energy and of the potential energies $V_{AC}(\eta)$, $V_{AB}(\xi)$, $V_{BC}(\eta-\xi)$. Corresponding to the initial state of the system one can separate the wave equation in the variables ξ, η and the coordinates of the center of mass provided one neglects V_{AC}, V_{BC}. Similarly if one neglects V_{AB}, V_{BC} the wave equation can be separated in the variables η, x and the coordinates of the center of mass. Approximately the wave function can be represented for the initial state as $\Phi_0(\xi)g_0(y)$. Here Φ_0 represents the initial state of relative motion of A and B while $g_0(y)$ describes the state of relative motion of B with respect to the center of gravity of A and B. Similarly the final state can be represented by $\Psi_m(\eta)f_m(y)$. Here the index m refers to different possible final states. The wave function can be expanded into a linear combination of $\Phi_n(\xi)g_n(y)$ and $\Psi_m(\eta)f_m(y)$. It will be supposed that the coefficient of $\Phi_0(\xi)g_0(y)$ is much larger than the coefficients of the other functions and also that in determining the coefficients of the other functions one may neglect all coefficients except that of $\Phi_0(\xi)g_0(y)$. On these assumptions one finds

$$[T_y+E_0{}^\xi-E+\int\Phi_0{}^*(\xi)[V_{AC}(y+q\xi)+V_{BC}(y+(q-1)\xi)]\Phi_0(\xi)d\xi]g_0(y)=0, \quad \text{(I 2)}$$

$$[T_x+E_m{}^\eta-E+\int\Psi_m{}^*(\eta)[V_{AB}(x+(1-p)\eta)+V_{BC}(-x+p\eta)]\Psi_m(\eta)d\eta]f_m(x)$$
$$+\int\Psi_m{}^*(\eta)[V_{AC}(\eta)+V_{BC}(p\eta-x)-\{\int\Phi_0{}^*(\xi)[V_{AC}(y+q\xi)+V_{BC}(y+(q-1)\xi)]\Phi_0(\xi)d\xi\}_{y=-qx+s\eta}]$$
$$\Phi_0(x+(1-p)\eta)g_0(-qx+s\eta)d\eta=0, \quad \text{(I 3)}$$

$$p=M_A/(M_A+M_C), \quad q=M_B/(M_A+M_B), \quad s=1-q+pq \quad \text{(I 4)}$$

and where T_y, T_x are the kinetic energy operators for the relative motions represented by y, x; E=total energy; $E_0{}^\xi$, $E_m{}^\eta$ are the energies of internal motion represented by ξ, η. Eq. (I 2) determines the function $g_0(y)$. The potential energy is a properly taken average of $V_{AC}+V_{BC}$ over the initial state of the combination $A+B$. Eq. (I 4) is an inhomogeneous equation which allows one to determine f_m. By the method of Chapter VI of Mott and Massey's book on *The Theory of Atomic Collisions* one can obtain f_m by analyzing it and the integral involving g_0 in (I 4) into a series of spherical harmonics and then determining the coefficient of each spherical harmonic (a radial function) in the expansion of f_m by an integration over the radius $(x_1{}^2+x_2{}^2+x_3{}^2)^{\frac{1}{2}}$. The asymptotic form of this coefficient for large radii is proportional to the result of multiplying the $\int d\eta$ in (I 4) by the regular normalized solution of the radial homogeneous equation for f_m [obtained by omitting $\int d\eta$ in (I 4)] and then integrating over x. Thus g_0 enters into the expression for the asymptotic form of f_m under $\int d\eta$ and is weighted in accordance with operations just described. The expected dependence of σ on g_0 is therefore more complicated than a simple proportionality to the integral of $|g_0|^2$ through the interior of the nucleus. Qualitatively the difference of the approximation used from the more exact result given by (I 4) cannot be significant as long as the shape of $|g_0|^2$ through the nucleus does not vary much with bombarding voltage. If g_0 becomes large on account of resonance as determined by Eq. (I 2) f_m will also be large. Similarly, if the regular normalized solution of the homogeneous equation for a radial part of f_m shows resonance at the energy $E-E_m{}^\eta$, the solution of (I 4) for f_m will be large and we have then *resonance to the disintegration products* which may be the explanation of the resonance for γ-rays from Li^7+H^1.

Numerical estimates show that one may treat Eq. (I 4) approximately by passing to the limit $M_A\to\infty$, $p=1$, $q=0$, $s=1$. In this case V_{AC} does not enter into the $\int d\eta$ in (I 4). For a spherically symmetric Φ_0 and $V_{BC}=e^2/r_{BC}$ one has

$$\sigma_L=\frac{4\pi}{2L+1}\frac{v_B}{v_Ck_B{}^2(E-E_m{}^C)^2}\left|\int\bar{F}_L{}^B(\rho_B)R_L(\rho_B)\rho_Bd\rho_B\right|^2,$$
$$R_L(\rho_B)=e^2\Phi_0(B)\int\Psi_m{}^*(C)S_L(r_B, r_C)(\bar{F}_L{}^C(\rho_C)/\rho_C)d\tau_C, \quad \text{(I 5)}$$
$$S_0(r_B, r_C)=\frac{1}{r_2}-\frac{1}{r_C}\int_{B'<C}|\Phi_0(B')|^2d\tau_{B'}-\int_{B'>C}\{|\Phi_0(B')|^2/r_{B'}\}d\tau_{B'};\ S_L=r_1{}^L/r_2{}^{L+1},\ r_2>r_1;\ (r_1, r_2)=(r_B, r_C) \text{ or } (r_C, r_B).$$

The indices B, C here indicate that one uses the coordinates of particles B, C and $\rho_B=k_Br_B$, $\rho_C=k_Cr_C$ where k_B, k_C are the values of k for B and C; the functions $\bar{F}_L{}^B$, $\bar{F}_L{}^C$ are analogous to $\bar{F}$ of Eq. (3′). The example of Eq. (I 5) shows that $\bar{F}_L{}^C$ enters into σ_L in essentially different ways for different L because S_L, $\bar{F}_L{}^B$ vary with L. We thus expect P to vary with L.

From the above formula (I 5) one finds that if Φ and Ψ are each of the form $e^{-r/2R}$ and if $\bar{F}^C$ is small in the nucleus then σ is proportional to R^4 for $L=0$ and to R^6 for $L=1$. Besides, the value of the matrix element depends on the overlapping of wave functions. Thus the cross section is quite sensitive to the nuclear radius and relatively small changes in assumptions about the state of the nucleus may account for large changes in the values of P.