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Fluctuations in Cosmic-Ray Ionization as Given by Several Recording Meters Located at the Same Station

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A graphical comparison is given of the intensity variations of cosmic rays as registered on several different recording meters located in the same room during a period of ten weeks. As one might expect, the agreement is statistical rather than coincident. The barometer effect as calculated from the data of the individual meters ranges from 1 to 2 percent per cm Hg. A diurnal analysis carried out for a ten-day period during which barometric changes were small indicates a maximum of intensity at about 9:00 A.M. having a magnitude about 0.19 percent greater than the average. A pronounced increase in ionization has been observed during rainfall when the meters were operated with top shield removed. This has been ascribed to γ-radiation from active deposits brought down from the upper atmosphere with the raindrops. The average number of cosmic-ray ionizing particles traversing unit area of the ionization chamber in unit time, as estimated from statistical fluctuation data, is shown to compare favorably with counter observations. As regards ionization bursts: Analysis of distribution-in-time of the bursts shows that they occur in a purely random manner. Size frequency distributions covering a period of several weeks on eight different meters can be represented for the most part by a single exponential function. An analysis of nearly 700 bursts occurring over a 25 mm Hg range of pressure gives no evidence of a barometer effect on burst frequency, although the consistency of the data is not sufficient to rule out the possibility of an effect as large as 8 percent per cm Hg. The frequency is a function of shield thickness, being much greater for a reduced top shield than for the full 12 cm of lead.

PART I—INTENSITY VARIATIONS

THE construction of seven precision cosmic-ray recording meters for permanent installation in various parts of the world[1] provided a unique opportunity for comparing the results obtained from several meters running concurrently at the same location. While it is obviously desirable to make such intercomparisons as an aid in estimating the significance of future differences in data obtained at the various permanent stations, the results are also of general interest because of the information they yield in connection with certain cosmic-ray problems.

The instruments used in these tests have previously been described by Compton, Wollan and Bennett.[2] Briefly, they consist of a spherical ionization chamber of 19.3 liters capacity, filled with 50 atmospheres of highly purified argon and shielded with 2500 lbs. of lead shot equivalent in thickness to 12 cm of solid lead. The ionization is measured by means of a Lindemann electrometer and the optical arrangement is such that the image of the electrometer needle can either be observed visually or recorded on a moving strip of bromide paper. A compensating

[1] This is in accordance with a program for the routine collection of cosmic-ray data originally proposed by A. H. Compton and R. D. Bennett (Bulletin No. 9, Union Géodesique et Géophysique Internationale 1933, p. 311) and sponsored by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. The permanent stations include the magnetic observatories at Cheltenham, Md., and Huancayo, Peru, as well as locations in Mexico, Greenland, and New Zealand.

[2] A. H. Compton, E. O. Wollan and R. D. Bennett, Rev. Sci. Inst. 5, 415 (1934).

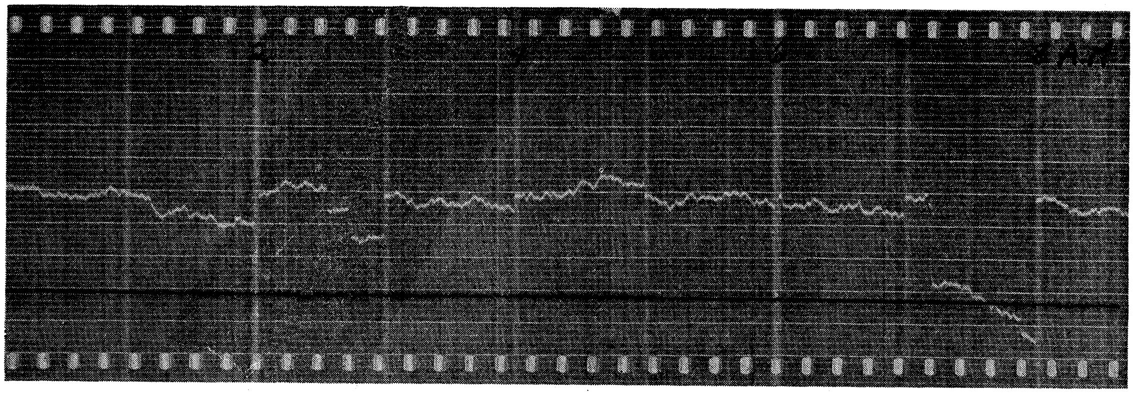

FIG. 1. Portion of record from meter No. 1 for September 21, 1934. Note the bursts in the third and eighth hours. The black line is the barometer trace.

arrangement is employed whereby most of the cosmic-ray ionization is balanced out by the β-ray ionization from a short adjustable uranium rod located in an auxiliary chamber inside the bomb, so that what is actually recorded is the differential between the ionization due to the cosmic rays and that due to the β-ray source. The latter remains constant after it has once been adjusted to the desired intensity and any changes in the intensity of the cosmic rays are indicated on the record. The collecting system is grounded automatically once each hour and the electrometer sensitivity that may be employed is limited only by the necessity of keeping the needle image on the scale during the whole period.

A typical record is shown in Fig. 1. The principal feature to be noted is that the needle drifts from its zero position during the 57.5 minutes that the collecting system is insulated, the amount of the drift being a function of the intensity of the cosmic rays during that period. The total ionization in any period is subject to statistical fluctuations due to the finite number of ionizing particles traversing the chamber, and such fluctuations are evident on the record. An occasional abrupt displacement of the needle image represents a burst of ionization presumably occasioned by the simultaneous passage of a large number of ionizing particles. In the discussion which follows, the bursts are treated separately from the average ionization and so far as possible they have been deducted in making calculations involving variations in ionization.

The hourly scalings of the records from six meters during March and part of April and May are given in Figs. 2 and 3. In each case the top trace indicates the variations in barometric pressure while the lower points show the net unbalance of the electrometer needle in each instrument at the end of the hourly period. The most prominent feature of these plots is the inverse relationship between cosmic-ray intensity and atmospheric pressure. It is also interesting to note that the barometer variations during March were considerably greater than for the later period. For several days around April 19 the total variation in atmospheric pressure was only 2 or 3 mm Hg which is exceptionally small for Chicago. Another point of interest in connection with Fig. 3 is the apparent decrease in cosmic-ray intensity with time which continued throughout the whole period of the test with the possible exception of the last few days (May 20–22) although most of the increase that occurs at this time can probably be attributed to the fall in the barometer.

Statistical fluctuations

Let us now consider the scatter of the points in Figs. 2 and 3 and inquire whether the observed variations in the hourly scalings are such as may reasonably be attributed to statistical fluctuations due to the limited number of ionizing events. The problem of statistical fluctuations of cosmic-ray ionization in a spherical chamber has been worked out by Evans and Neher[3] on the assumption that the ionizing agents are single

[3] R. D. Evans and H. V. Neher, Phys. Rev. 45, 144 (1934).

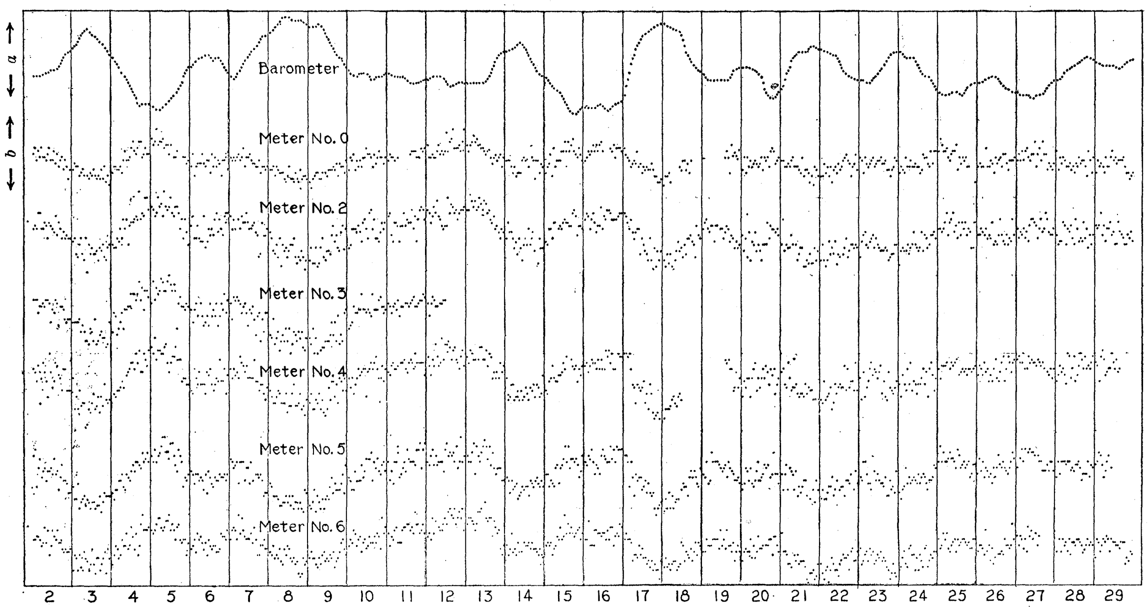

FIG. 2. Variations of cosmic-ray ionization, March 1935 (Chicago). a, 20 mm Hg; b, 5% I_{750}.

and multiple electron rays which shoot clear through the chamber, the ionization due to a shower of n particles being assumed to be n times that due to a single particle. This leads to the following expression for the total ionization:

$$I = AN\Sigma n r_n / \Sigma r_n, \quad (1)$$

where A is the average ionization produced by a single particle and N the number of ionizing events, an event consisting of the passage of n related particles. The ratio of the frequency of showers to that of single-particle events is designated by r_n.

On the basis of the above picture a series of observations made upon I should show statistical fluctuations around the average value proportional to the inverse square root of N. The relation between N and the standard deviation σ as defined by the root mean square of the deviations from the mean is given by

$$\sigma = FI(N)^{-\frac{1}{2}}, \quad F^2 = (9/8)[\Sigma r_n \Sigma n^2 r_n / (\Sigma n r_n)^2]. \quad (2)$$

We have thus a means of estimating the rate of passage of ionizing particles through the ionization chamber from measurements of the standard deviation providing we can determine the value of F, which as indicated above, depends upon the distribution of showers and single particles. About the best that we can do here at present is to make an estimate based on the observed distribution of multiples and singles in drop track photographs. If we use the analysis of Anderson,[4] which included about eight hundred events, we get

$$\Sigma r_n = 1.15; \quad \Sigma n r_n = 1.42; \quad \Sigma n^2 r_n = 2.21,$$

from which $F = 1.32$.

This value of F is undoubtedly smaller than that which applies to a spherical ionization chamber completely surrounded by lead, because in the latter case the closer proximity of the shielding material should result in a relatively larger number of showers. It is uncertain as to just what value should be taken but we shall probably not be far off if we use $F = 1.5$, and this is the value adopted in the subsequent calculations.

The standard deviation is obtained from an analysis of the hourly records from the various cosmic-ray meters. If Y_m is the drift of the electrometer needle in scale divisions during the mth hourly period, then σ can be calculated from

$$\sigma^2 = \sum_1^n Y_m^2/n - (\Sigma Y_m/n)^2, \quad (3)$$

where n is the number of hours considered.

[4] C. D. Anderson, Phys. Rev. 44, 406 (1933).

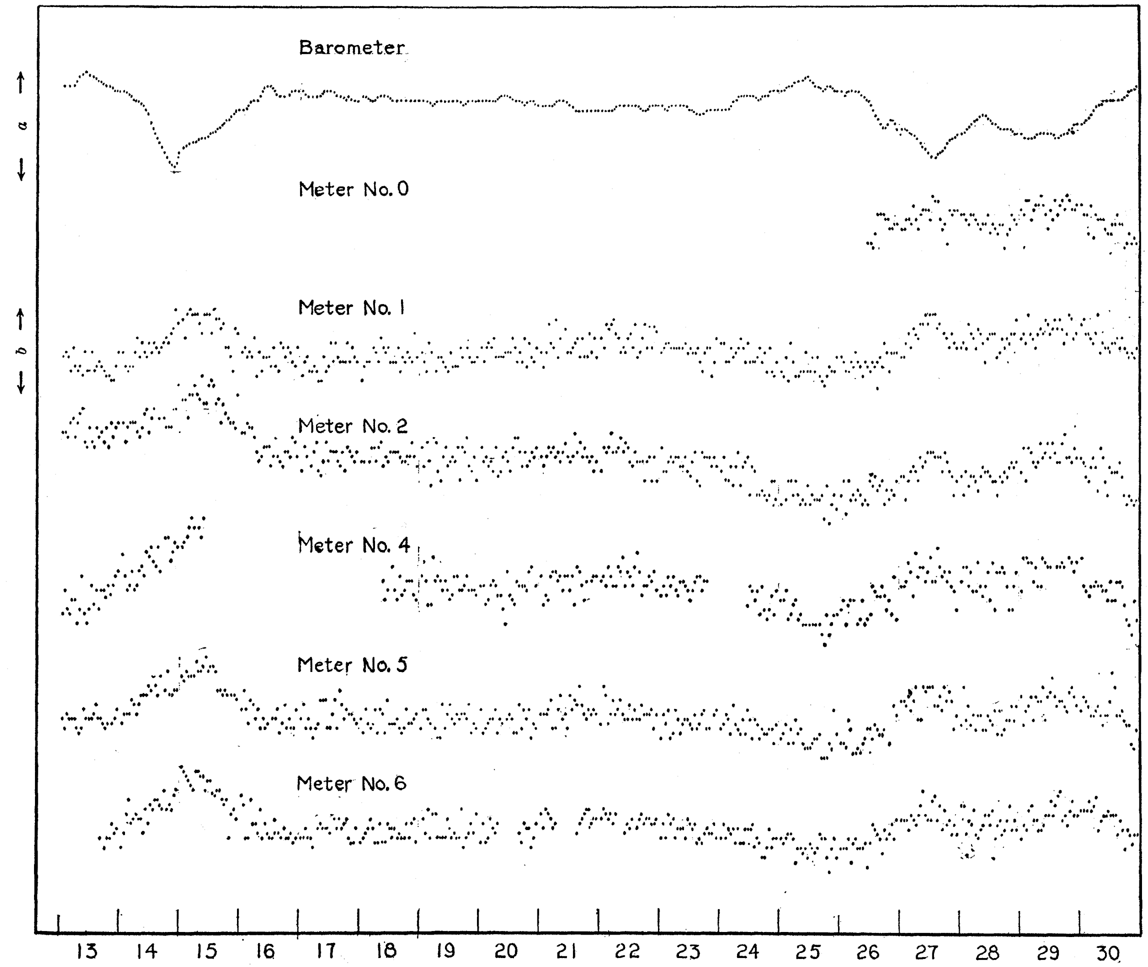

FIG. 3A. Variation of cosmic-ray ionization, April 1935 (Chicago). a, 20 mm Hg; b, 5% I_{750}.

Since Y varies considerably with the barometer such a calculation can best be made for a period during which the barometer is essentially constant. From Fig. 3 it may be seen that the period April 17–21 was free from barometer changes of any consequence. For this period of approximately 100 hours, the standard deviations as calculated from the records of four meters running simultaneously were as follows:

Meter No.	σ (ions per hour)
1	$33.4(10)^6$
2	34.4
5	35.1
6	30.4

Since the collecting systems were insulated for 57.5 minutes out of each hour the standard deviation for the period of one minute is obtained by dividing the "hourly" deviation by $(57.5)^{\frac{1}{2}}$. Using the average of the above values we obtain

$$\sigma = 4.4(10)^6 \text{ ions per minute.}$$

The mean value of the total ionization as determined by visual observations on the absolute cosmic-ray intensity comes out to be

$$I = 94.8(10)^6 \text{ ions per minute.}$$

Thus, from (2), taking $F = 1.5$ we get

$$N = ((1.5 \times 94.8)/4.4)^2 = 1045 \text{ per min.}$$

The cross-sectional area of the ionization chamber is 995 cm² so that the number of primary ionizing particles per cm² per minute is 1045/995 or 1.05, a value in close enough agree-

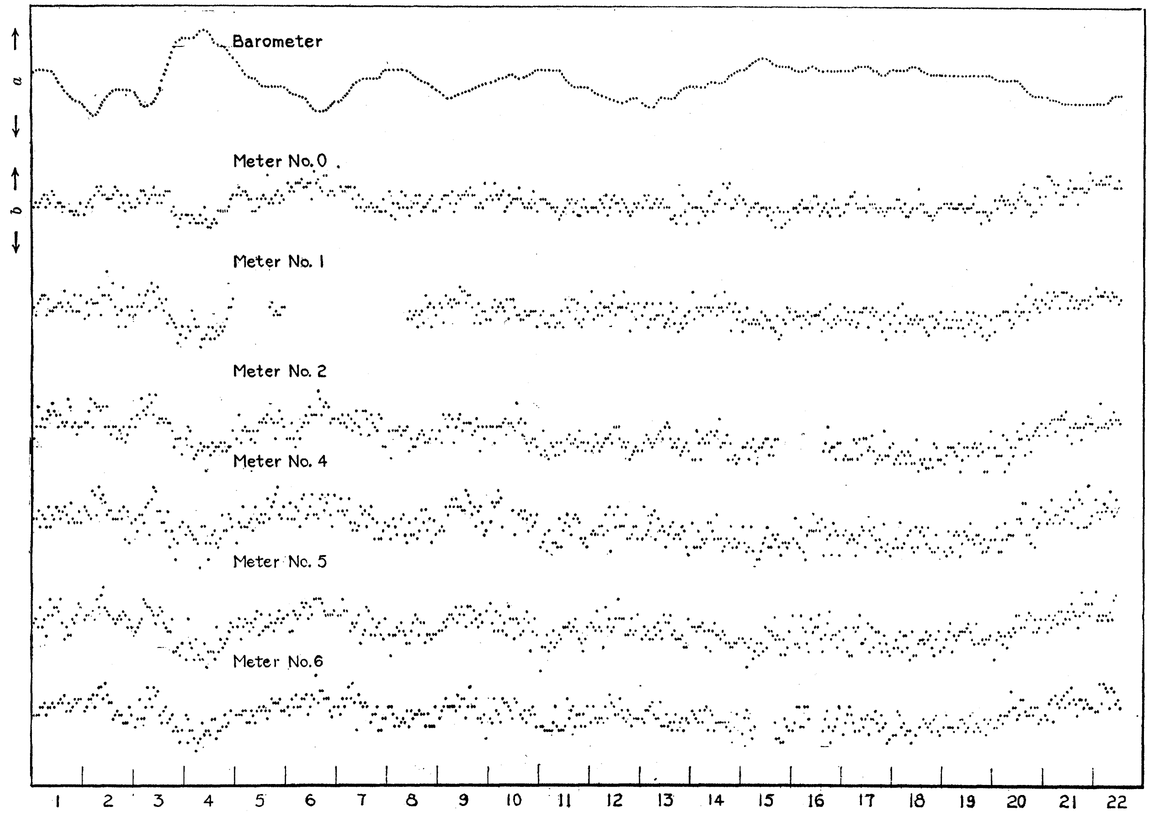

FIG. 3B. Variation of cosmic-ray ionization, May 1935 (Chicago). *a*, 20 mm Hg; *b*, 5% I_{750}.

ment with counter observations[5] to support the conclusion that the observed statistical fluctuation, while too great to be accounted for on the assumption that the ionization is due to randomly distributed single ionizing particles, may reasonably be attributed to a distribution of single and multiple ionizing rays not greatly different from that given by cloud chamber observations. This must mean that the distribution of tertiaries produced by lead secondaries is quite similar to that produced by air secondaries, which is what one might expect if the secondaries in each case are photons but not necessarily so if they are charged particles originating in the nuclei of the respective atoms.

Barometer effect

The variations of cosmic-ray intensity with atmospheric pressure at a given station can be inferred from curves of intensity *vs.* altitude; but for several reasons a straightforward correlation of ionization and barometer differentials does not lead to a constant result when carried out at different times. In the first place the barometer reading gives the weight of air directly above the instrument and therefore gives no accurate measure of the air path of the cosmic rays which come in at appreciable zenith angles. This difficulty is accentuated by the fact that the various high and low pressure areas which pass over the observer do so at different speeds and in different relative positions and have all sorts of pressure distribution. At Chicago during certain times of the year the "highs" and "lows" follow one another in fairly rapid succession as indicated to some extent in Fig. 2. It is not uncommon for the pressure to drop rapidly to a minimum and rise equally fast, and on some of these occasions the cosmic-ray records show that

[5] E.g., The measurements of Street and Woodward gave 0.80 per cm² per min. from unit solid angle at the vertical or 1.48 per cm² per min. from all directions. (Phys. Rev. **46**, 1029 (1934).)

the barometer has passed the low and is rising while the cosmic-ray intensity is continuing the increase which began with the falling barometer. An inspection of Fig. 2 will show that on March 3, 8 and 14 the midpoint of the ionization trough does not coincide with that of the barometer peak but is displaced slightly to the right. This, however, does not appear to be the case for March 17–18.

Another factor which makes the barometer correction difficult if too long a period of time is considered, is an occasional rather large change in cosmic-ray intensity independently of the barometer. Such a change is illustrated in Fig. 3 and has already been mentioned. These so-called "variations of the second kind" have not had a satisfactory explanation.

It is thus well known that the barometer effect as defined by the percentage change in cosmic-ray ionization per cm mercury (or sometimes per meter of water) change in atmospheric pressure may vary widely depending upon the period chosen for analysis. This has led to the practice by European investigators of calculating the barometer effect for short rather than long periods and making the barometer corrections accordingly.

If one plots hourly ionization as ordinates and barometer readings as abscissae a scatter diagram is obtained through which three straight lines may be drawn, two of which represent least-squares adjustment of one variable in terms of the other and *vice versa*, the third being a least-squares adjustment for normal deviations. If we suppose that the barometer readings are relatively exact and that most of the error is in the ionization, then presumably the best line for determining the barometer effect is the one given by averaging all of the ionization values in the various vertical columns of the scatter diagram. The slope of this line which is the so-called regression of y on x, where y represents the ionization and x the barometer, can be calculated from the data obtained from the records. To do this the correlation coefficient r is first calculated making use of the relation

$$n^2\sigma_x\cdot\sigma_y\cdot r = n\Sigma(xy) - \Sigma x\Sigma y, \quad (4)$$

where n is the number of hours considered and the σ's are the respective standard deviations of x and y as defined previously. It should be noted here that the y's and x's are scaled directly from the records and therefore represent departure from an arbitrarily selected base line rather than from the mean.

TABLE I. *Barometer coefficients for March.*

METER No.	r	P_r	a	P_a	$(a/I)\times 100$
0	−0.616	±0.016	0.89	±0.003	1.07
2	− .630	.015	1.33	.005	1.60
3	− .845	.012	1.76	.005	2.11
4	− .762	.011	1.43	.004	1.72
5	− .786	.010	1.57	.004	1.89
6	− .572	.017	1.12	.005	1.35

Once the correlation coefficient is obtained, the slope of the regression line is given by

$$a = r(\sigma_y/\sigma_x). \quad (5)$$

The probable error of the correlation coefficient[6] is

$$P_r = 0.6745(1-r^2)/n^{\frac{1}{2}} \quad (6)$$

while that of the regression coefficient is

$$P_a = 0.6745((1-r^2)/n)^{\frac{1}{2}}\cdot\sigma_y/\sigma_x. \quad (7)$$

In Fig. 4 are plotted the hourly ionization differentials against the barometer readings for meter No. 5 as taken from the March records. The full line drawn through these points is the regression line calculated as described above from the equation

$$y = \bar{y} + r(\sigma_y/\sigma_x)(x-\bar{x}),$$

where $\bar{y}$ and $\bar{x}$ represent mean values. The dashed lines give the probable error of y so calculated and show the limits within which the points fall half of the time. In other words it is equally probable that any given point fall inside or outside the region defined by these lines. Thus the probable error of y

$$P_y = 0.6745\sigma_y(1-r^2)^{\frac{1}{2}}$$

is useful in giving some idea of the scatter of the points along the regression line.

The correlation and regression coefficients as given by six different meters for the month of March are grouped in Table I, together with the calculated probable errors. With the exception of Meter No. 3 all of the instruments were

[6] *Handbook of Mathematical Statistics* (Houghton, Mifflin Co.).

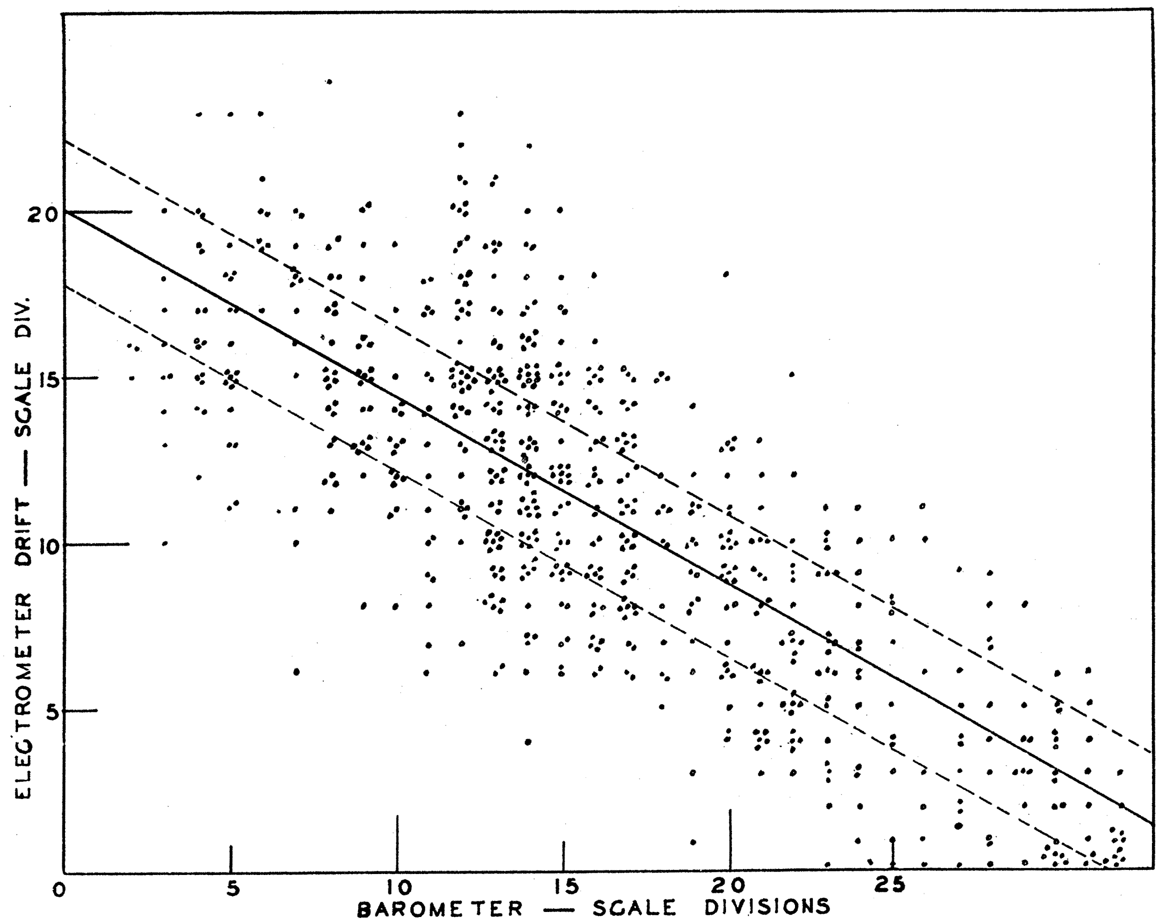

FIG. 4. Variation of cosmic-ray intensity with atmospheric pressure. Data from meter No. 5 for March 1935.

running for the entire month, as may be seen from Fig. 2. The correlation coefficient is given as a negative fraction while the regression coefficient is expressed in terms of ions per cc per sec. per cm Hg and also as percent of the total cosmic-ray ionization per cm Hg.

It will be seen that for the most part the regression coefficients for the various meters differ from one another by more than their probable errors, which is suggestive of the considerable differences noted by other observers when using the same meter at different times. Actually, however, a glance at Fig. 4 will show that the probable error of the calculated ionization is so great that the differences between the various meters indicated in the above table are somewhat exaggerated. A better indication of these differences is obtained by comparing the central regression zones (region between the dashed lines in Fig. 4) of the meters when adjusted to the same mean value. This has been done in Fig. 5 and it will be noted that there is a considerable area common to all zones. Thus in the pressure range 5 to 25 scale divisions (about 20 mm Hg) all of the meters can be said to be giving consistent results about 67 percent of the time within the limits of accuracy of the measurement. If Meter No. 0 be excepted the agreement is much better.

Diurnal variation

The existence of a diurnal variation of cosmic-ray intensity has never been conclusively demonstrated although numerous investigators have reported positive results. Among the best of such studies is that of V. F. Hess and R. Steinmaurer,[13] which covers a period of one year. The situation as of 1933 is summarized very

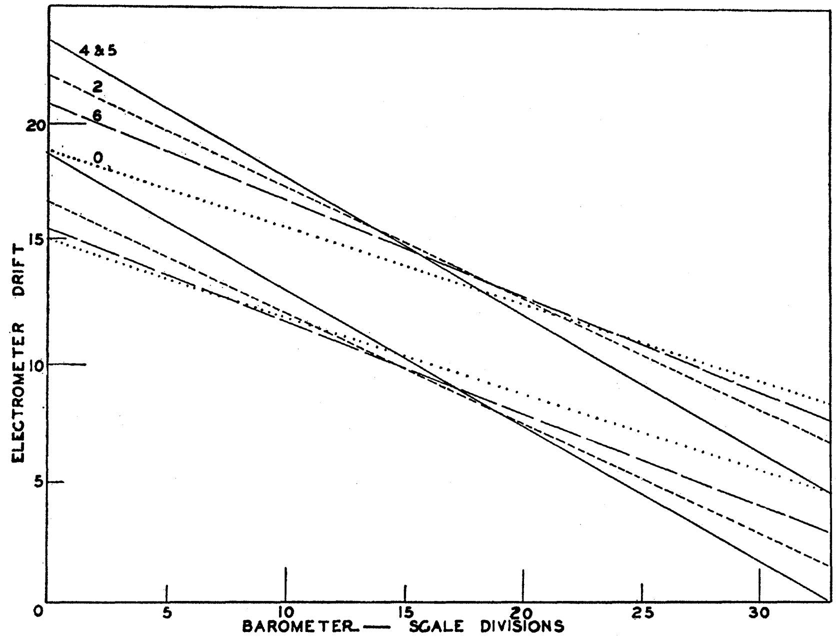

FIG. 5. Barometer variation as given by different cosmic-ray meters.

nicely by Broxon, Meredith and Strait[7] who themselves were unable to detect any significant diurnal variation in a series of experiments carried out on the campus of the University of Colorado only a short distance from the location where Bennett, Stearns and Compton[8] had previously found such a variation with a maximum about noon.

If a solar diurnal variation exists it may mean that a small component of the cosmic rays has its origin either in the sun as suggested by Hess and Pforte[9] or in the general neighborhood of the sun[8] although Gunn[10] has suggested that a diurnal variation might be due to fluctuations in the earth's magnetic field.

Variations according to sidereal time were looked for several years ago, and a sidereal periodicity was reported by Steinke[11] in 1929 as well as by other observers more recently. The search for a sidereal variation has received a new motive in a recent paper by Compton and Getting[12] in which are developed the consequences of the rotation of the galaxy on the intensity of cosmic rays originating in remote space outside the galaxy. A predicted maximum at about 20 hr. 40 min. sidereal time has received a tentative confirmation in some of the results of Hess and Steinmaurer[13] but in general the data needed for a satisfactory test are not yet available.

There are several difficulties in the way of establishing definitely the existence of a diurnal variation. Obviously if the ionization measurements are made during a period when there are any considerable barometric fluctuations, the results must be corrected to a common atmospheric pressure and such a process can easily introduce errors which may mask or distort the relatively small diurnal variation. There is also the possibility that any observed diurnal variation may be largely, if not solely, a temperature effect. Finally, if the measurements cover a long period of time, the situation may be complicated

[7] J. W. Broxon, G. T. Meredith and L. Strait, Phys. Rev. **43**, 687 (1933).
[8] R. D. Bennett, J. C. Stearns and A. H. Compton, Phys. Rev. **41**, 681 (1932).
[9] V. F. Hess and W. S. Pforte, Zeits. f. Physik **71**, 171 (1931).
[10] R. Gunn, Phys. Rev. **41**, 111 (1932).
[11] E. G. Steinke, Physik. Zeits. **30**, 767 (1929).
[12] A. H. Compton and I. A. Getting, Phys. Rev. **47**, 817 (1935).
[13] V. F. Hess and R. Steinmauer, Preuss. Akad. Phys.-Math. Kl. **15** (1933).

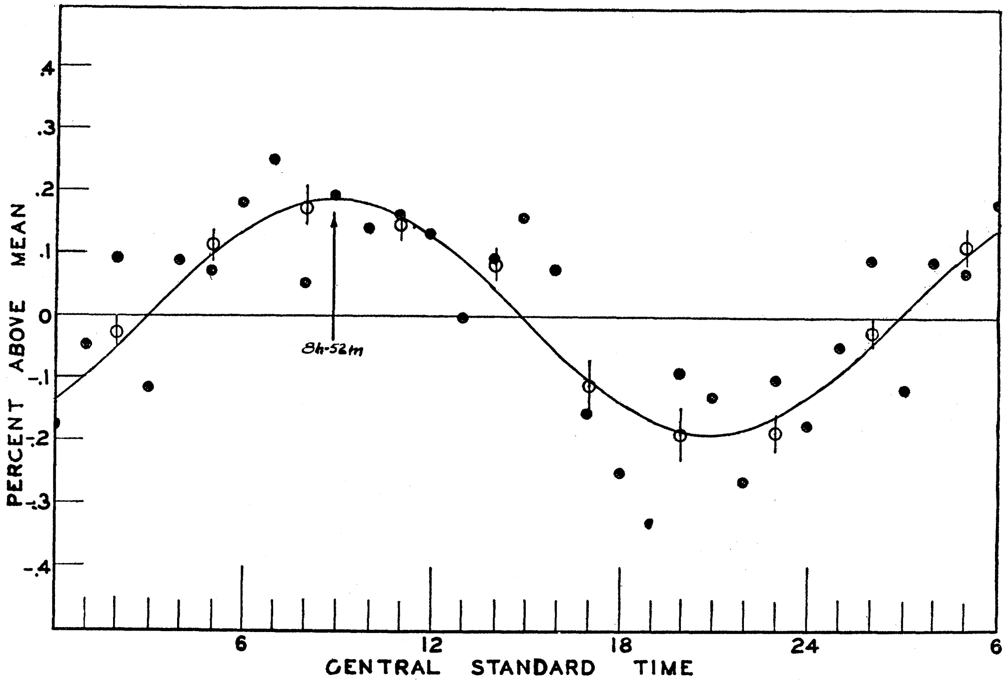

FIG. 6. Diurnal variation of cosmic-ray intensity.

by one or several "variations of the second kind."

In view of the above there should be certain definite advantages in searching for a diurnal effect at a time when the barometer fluctuations are small and in reducing the probable error of the observations by using several instruments rather than by prolonging unduly the period of observation. Ordinarily one cannot choose his experimental conditions so easily; but it just happened that during the intercomparison tests herein described there was a ten-day period, already mentioned, during which the barometer changed very little. Thus it seemed that this period, April 16–26, should be particularly suitable for diurnal analysis.

After making such small barometric corrections as were necessary, the results from all meters were grouped hour by hour on the basis of central standard time and the means calculated. These mean values are plotted as the black dots in Fig. 6. The open circles are the three-hour means, each being the arithmetic average of from 140 to 150 hourly readings on five different meters. The probable errors of the three-hour means as calculated from the relation

$$P=0.6745\sigma/\sqrt{n}$$

are indicated by vertical lines through the open circles. It will be seen that a definite periodicity is indicated with a maximum at about 9:00 A.M.[14]

The first harmonic of the Fourier analytic curve is shown as the full line, the coefficients having been calculated from the hourly means by least-squares methods. The calculated amplitude and phase together with their probable errors are as follows:

Amplitude = 0.189 percent ±0.035,
Phase = −2 hr. 52 min. ±42 min.

Thus the amplitude is from five to six times the probable error.

According to these results the maximum intensity occurs in the neighborhood of 9:00 A.M. and cannot therefore be accounted for solely as a solar diurnal variation. It is, however, of

[14] It is not possible to attribute this variation to a temperature effect in the apparatus since careful measurements have shown no detectable temperature coefficient when the temperature of the meter was varied over a range of more than 20°C. Then, too, the diurnal maximum comes several hours earlier than would be expected if it were a temperature effect.

interest to note that 9:00 A.M. Standard Time is about 23 hr. sidereal time on April 21 which is suggestively close to the time 20 hr. 40 min. predicted for the peak on the basis of Compton's theory of the effect due to motion of the galaxy. Perhaps one could consider the observed curve as a sort of combination of the two effects. If solar and sidereal variations are both present then it is to be expected that the diurnal variation will be a maximum about February 16 and a minimum about August 16.

Unshielded chamber; rain effect

Because there is some indication that the softer components of the cosmic rays show more pronounced barometer and diurnal effects than do the more penetrating rays, some European investigators have adopted the practice of operating their recording meters at least a portion of the time with the top shield removed. However, in view of the rather wide variations in γ-ray intensity due to radioactive deposits in the air, it would seem that the results obtained with no top-shielding would be difficult to interpret.

An interesting illustration of the pronounced increase in atmospheric radiation during a rain storm is shown in Fig. 7. The records (tracings) are from three different meters operating side by side, one with no top-shielding, another with 1 cm of lead above the bomb, in the form of a close-fitting hemispherical cap, and the third with several cm of lead shot as top shield. All meters were fully shielded from below.

Weather Bureau records show that during 12 hr. to 14 hr. there was heavy thunder with lightning but no rainfall. The cosmic-ray records show no change during this period. A heavy rainfall started at 16 hr. 28 min. and coincident with this a sharp increase in ionization is noted on the unshielded meter and somewhat later, on the meter shielded with 1 cm of lead. The precipitation was 0.39″ during a period of about 20 minutes, then dropped to 0.03″ and 0.01″ in the ensuing two hours. It may be noted that the ionization excess continued at about the same value for slightly more than two hours after which there was a rapid return to the level existing prior to the storm. No change is evident at any time on the third meter, which was shielded with lead drop shot to an average thickness of some 6 to 8 cm.

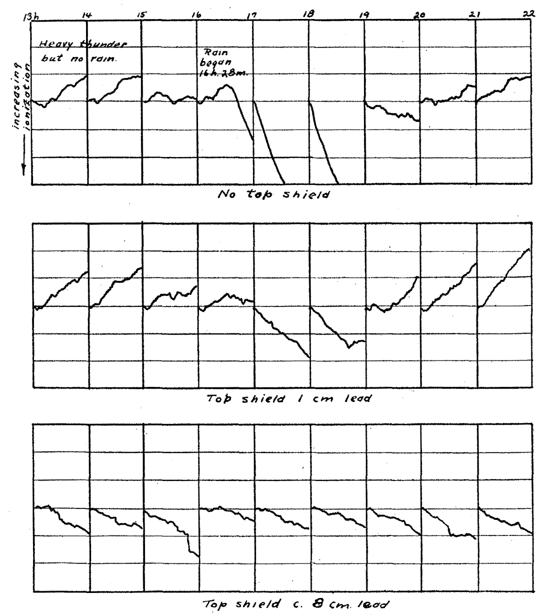

FIG. 7. Tracing of portion of record from three recording meters for August 17, 1935, showing effect of rainfall on ionization.

That freshly fallen rain is radioactive was observed several years ago by C. T. R. Wilson[15] who found a half-period of approximately 30 minutes for the active deposit. Since it would be impossible for α- or β-rays to penetrate the half inch of steel comprising the walls of our cosmic-ray ionization chamber, the whole of the observed increase of ionization was presumably due to γ-radiation. Ra B gives off some feeble γ-rays[16] but one would not expect these to be very effective so that the more penetrating rays from Ra C are indicated as the most probable cause of the increase in ionization. Confirming this view is the fact that the observed absorption in 1 cm of lead is just about what we would expect on the basis of the known absorption of γ-rays from radium. If we assume that most of the active deposit came down as Ra B which, being positively charged, would readily form condensation nuclei then the effective period during which γ-rays would be emitted would be about that observed on the records since the half-periods of

[15] C. T. R. Wilson, Proc. Camb. Phil. Soc. **11**, 428 (1902).
[16] Rutherford, *Radioactive Substances and Their Radiations* (1913), p. 488.

Ra B and Ra C are 26.8 and 19.5 min., respectively.

These results emphasize the importance of using heavy shields in making cosmic-ray measurements if such atmospheric effects as the one illustrated here are to be avoided. With the 12 cm shield normally employed on the Compton-Bennett meters, no such effects are detectable.

Part II—Ionization Bursts

A burst of ionization is indicated on the cosmic-ray records by an abrupt displacement of the needle trace by an amount varying from one to twenty or twenty-five scale divisions (see Fig. 1). An event of this kind corresponds to the production of from ten to two hundred million ion pairs presumably by the simultaneous passage of several hundred ionizing particles through the ionization chamber. The actual number of particles involved is somewhat uncertain but can be estimated if we assume that the average rate of occurrence of ionizing events as obtained from statistical fluctuation data is somewhere near correct. This was found to be 1045 per minute for the whole chamber. Since an individual event might consist of the passage of a single particle or of several associated shower particles, the average number of ionizing particles traversing the chamber per minute would be

$$N = 1045 \Sigma n r_n / \Sigma r_n$$
$$= 1045 \times 1.23 = 1285$$

if we use the distribution of multiples and singles given by cloud chamber observations. Since the observed total ionization per minute is, after deducting the zero correction, $94.8(10)^6$ ions the average number of ions produced per particle is $7.36(10)^4$. Thus a burst of magnitude $5(10)^7$ ions corresponds to about 680 ionizing particles. This would be a burst of moderate proportions, a few scale divisions on the record, and many larger ones have been observed.

Table II. *Distribution of bursts according to time of occurrence.*

Hour	No. of Bursts	Hour	No. of Bursts
1	54	13	76
2	80	14	76
3	62	15	65
4	89	16	79
5	73	17	71
6	72	18	68
7	72	19	70
8	66	20	62
9	71	21	66
10	64	22	71
11	78	23	68
12	77	24	80

The data on bursts presented here were obtained by an analysis of the records from (a) three meters located under a slate and steel roof during parts of September, October and November, 1934; (b) six meters stationed under a glass roof in a campus greenhouse during March and parts of April and May, 1935.

In taking out the bursts the smallest one considered was $1.5(10)^7$ ions which according to the analysis of Bennett, Brown and Rahmel[17] is large enough to avoid confusion with the ordinary statistical fluctuations. However, it should be noted that a considerable amount of individual opinion is involved in picking out these smallest bursts, much more so than for the larger ones.

A casual examination of the records from any of the meters will show occasional days when as many as eight or ten bursts are in evidence while at other times not a single burst occurs in a complete day's record. This makes it of interest to examine the time-distribution of the interval between bursts. First in order to see whether the frequency of bursts was greater at certain times of day than at others all of the bursts from a total of 9206 hours of record were arranged according to the hour during which the burst occurred. The results are given in Table II.

Apparently there is no significant variation of burst frequency with the time of day, the results being more suggestive of a random distribution with respect to time. If the distribution is purely random the number of intervals between bursts should vary with the size of the interval in accordance with Poisson's formula

$$n = N(e^{-t_1/T} - e^{-t_2/T}),$$

where n is the expected number of intervals in the group defined by t_1 and t_2, N is the total number of intervals and T the average interval. A comparison of the calculated and observed intervals making use of Pearson's criterion should permit a decision as to whether or not the bursts

[17] R. D. Bennett, G. S. Brown and H. A. Rahmel, Phys. Rev. **47**, 437 (1935).

Table III. *Distribution of intervals between bursts.*

Length of Interval (hr.)	Number of Intervals Obs.	Calc.	Δ	$\Delta^2/N_{calc.}$
0–1	63	56.2	−6.8	0.82
1–2	47	45.1	−1.9	.08
2–3	39	35.7	−3.3	.30
3–4	28	28.1	0.1	.00
4–6	35	40.5	5.5	.75
6–10	32	42.1	10.1	2.42
10–15	25	19.1	−5.9	1.82
15–∞	9	9.1	0.1	00
				$\chi^2=5.89$

occur in a random fashion. The results of such a comparison using the data on bursts obtained from meter No. 0 for the period September 1 to November 17, 1934, are given in Table III.

Reference to a table of chi-square values[18] shows that the probability of departure from the theoretical distribution by an amount equal to or greater than that indicated by $\chi^2=5.89$ is between 0.65 and 0.70 for any random sample. This would indicate strongly that the assumption of random distribution of bursts is correct. A similar analysis carried out for the bursts occurring on five meters during March leads to the same conclusion, which is in agreement with previous results of the Montgomerys.[19]

Burst frequency *vs.* size

The chief importance of a study of burst frequency as a function of magnitude lies in the possibility of learning something about the nature of the burst mechanism. The reality of the burst as a cosmic-ray phenomenon has been questioned by Millikan, Anderson and Neher[20] and defended apparently successfully by Bennett[21] and Hoffman.[22]

If we accept the reality of the phenomenon then we may logically inquire as to its nature. Although several investigators including Hoffman[22] feel certain that a burst is not of the nature of a large "shower," there seem to be enough points of similarity between the two phenomena to justify the opinion that some of the smaller bursts at least must be showers. For example, cloud chamber observations show that the number of particles in a single shower may on occasion be quite large, enough so that the resultant ionization in a high pressure chamber would be comparable with that due to a small burst. Whether all bursts are of this nature is an open question, although it would seem that the frequency with which they occur is much too small to be consistent with observed shower frequencies. However, this objection becomes of less importance as a distinction between showers and bursts in view of the results of Bennett, Brown and Rahmel[17] showing that the relation between burst frequency and magnitude can be expressed as a series of exponentials, because one may reasonably suppose that for the bursts still smaller than those recorded as such the slope of the semi-log curve of frequency *vs.* size would become increasingly steep so that the extrapolation to fewer and fewer ionizing particles would give a more favorable comparison with cloud chamber data. When it is considered that counter-controlled expansion chambers probably give a ratio of multiples to singles which is too high, the picture is still more favorable. However,

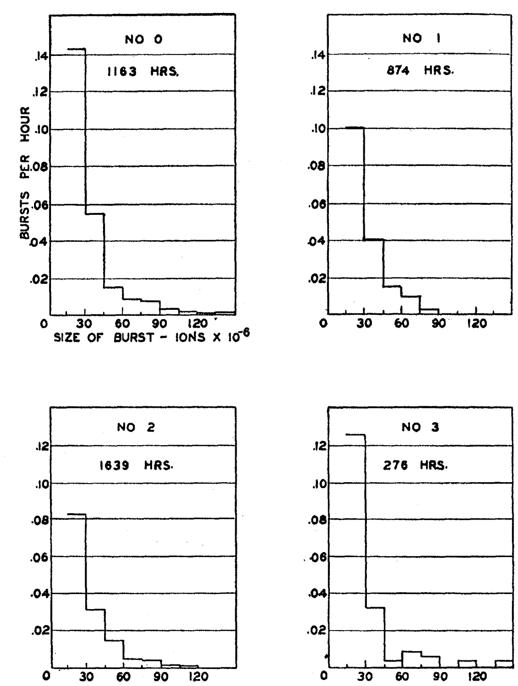

Fig. 8. Burst frequency as a function of magnitude.

[18] T. C. Fry, *Probability and Its Engineering Uses.*

[19] C. G. and D. D. Montgomery, Phys. Rev. **44**, 779 (1933).

[20] R. A. Millikan, C. D. Anderson and H. V. Neher, Phys. Rev. **45**, 141 (1934).

[21] R. D. Bennett, Phys. Rev. **45**, 491 (1934).

[22] G. Hoffman, Int. Conf. on Nuclear Physics (London, 1934), p. 226.

if it should be demonstrated that there is a maximum in the curve of frequency *vs.* size, the extrapolation argument would no longer hold and it would become more evident that the burst is an independent phenomenon. As a matter of fact such a maximum has been reported by Hoffman,[22] at $3.8(10)^6$ ions for lead and $2(10)^6$ ions for aluminum, but the curves of C. G. and D. D. Montgomery[23] covering approximately the same range show no maximum. The reason for the difference in results is not clear.

The number of bursts observed in scaling the records from eight different cosmic-ray meters for a total of about 9000 meter hours during March–July 1935 is given for the separate instruments by the frequency polygons of Figs. 8 and 9. The meters were all located under a glass roof in a campus greenhouse and each was fully shielded with 12 cm of lead.

It will be noted that the bursts occurred considerably more frequently on some instruments than on others. Thus the highest total rate was observed on No. 0 at 0.238 per hour while No. 2 and No. 7 were about equally low at around 0.145 per hour. The total rate for all the other meters did not vary appreciably from 0.190 per hour.

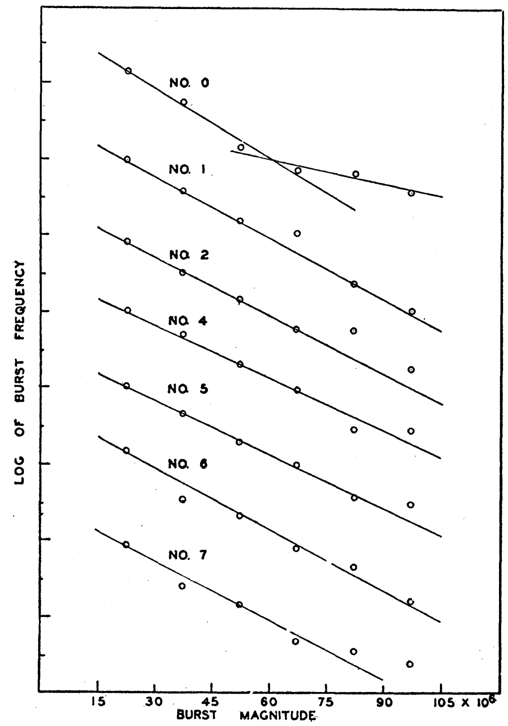

FIG. 10. Exponential relationship between burst frequency and size.

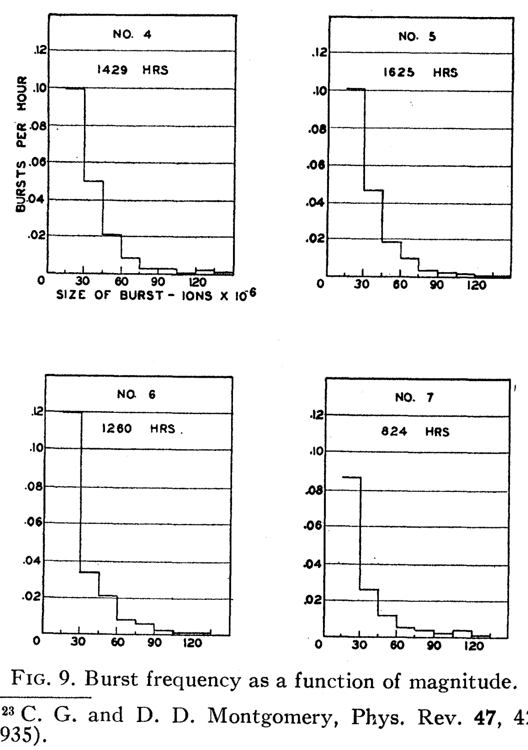

FIG. 9. Burst frequency as a function of magnitude.

If one plots the logarithm of frequency against burst magnitude the curves of Fig. 10 are obtained. Here for convenience the burst magnitude has been taken as the midpoint of the various size groups. In order to separate the curves the ordinates for each have been shifted a full unit with respect to the one above and below.

For the most part, the points for each meter fall on or near a single straight line although in the case of No. 0 a better fit is apparently obtained by using two intersecting straight lines. However, the position of the points in the two largest size groups is subject to considerable uncertainty because it is obviously impossible to say very much about the frequency of an event which occurs less than five times in more than 1000 hours unless the observations extend over a much longer period. Taking, then, the single exponential as giving the best representation of the data, the constant F_0 and a in the relation

$$F = F_0 e^{-aS}$$

[23] C. G. and D. D. Montgomery, Phys. Rev. **47**, 429 (1935).

TABLE IV. *Coefficients relating burst frequency to size.*

Meter No.	0	1	2	4	5	6	7
F_0	0.66	0.44	0.29	0.33	0.33	0.45	0.36
a	.067	.064	.057	.052	.051	.058	.065

are as shown in Table IV. F is the frequency in bursts per hour and S is the midpoint of a size group having a spread of $15(10)^6$ ions. S is expressed in millions of ions

It must be kept in mind that these coefficients are a function of the size group used in the analysis. If we had used a grouping smaller than $15(10)^6$ ions the frequency for each group would have been less. Therefore one cannot obtain the total burst frequency by integrating the exponential expression with respect to the burst magnitude. Neither is it permissible to interpret F_0 as the extrapolated frequency for very small bursts. With the grouping used we can extrapolate only to magnitude $7.5(10)^6$ ions at which point the calculated burst frequency (that is, the frequency of all bursts up to $15(10)^6$ ions) comes out to be 0.40 per hour or about 0.053 percent of the total ionization. Obviously the size group is much too large to attach any particular significance to this value but no smaller grouping was chosen because $15(10)^6$ ions represents one scale division on the record and this is therefore a convenient size of unit. The value 0.053 percent as the total contribution of all bursts up to about two hundred ionizing particles ($7.44(10)^4$ ions per particle) is of course much too small in view of cloud chamber observations on showers; but from the way this value was obtained it is not to be expected that it would have any meaning for very small bursts involving only a few ionizing particles.

The results of two independent investigations on much smaller bursts have been reported recently. C. G. and D. D. Montgomery[23] using a 50-liter ionization chamber filled with 14.5 atmospheres of nitrogen find about the same sort of size-frequency distribution for bursts ranging from $1.5(10)^6$ ions to $7.5(10)^6$ ions as described above for much larger bursts, while Hoffman reporting on some of Messerschmidt's work at the 1934 London International Conference on Physics shows that a maximum frequency is reached at $3.8(10)^6$ ions for lead and that below this size the bursts become decidedly less frequent. The charge sensitivity used was the same in both of these investigations, but no details are given on the ionization chamber used by Messerschmidt, so that it is not possible to state definitely that the range of burst sizes (as regards the number of ionizing particles) was the same in both cases although this seems likely.

If we take the results of the Montgomerys on burst frequency at Swarthmore, an interesting extension of our large burst data to the region below $15(10)^6$ ions is possible. The Dow metal ionization chamber used at Swarthmore had a volume $2\frac{1}{2}$ times as great as ours and was filled with 14.5 atmospheres of nitrogen which, according to Hopfield's curves[24] and some additional data of our own, has an ionization about 1/5 as great as that in 50 atmospheres of pure argon. Thus a burst of a given size in the Dow metal sphere corresponds to one twice as large in the argon-filled chamber, that is, the same number of ionizing particles would be involved in the two cases. At Swarthmore the observed number of bursts having a magnitude greater than $7.5(10)^6$ ions was 0.10 per hour as compared with 0.14–0.24 obtained on our instruments for all bursts larger than $15(10)^6$ ions. If one plots the Montgomery data, log frequency *vs.* magnitude, and extrapolates back to the size group $0-0.5\times10^6$ ions an estimated frequency of 13 per hour is obtained. For our ionization chamber this size group would include all bursts under 10^6 ions; that is, all those involving up to about 14 ionizing particles. Thus it appears that while the frequency of very small bursts is considerably less than the apparent frequency of showers estimated from cloud chamber and counter observations, it is nevertheless sufficiently high to make an appreciable contribution to the shower data and to this extent at least there seems to be little reason for making a distinction between the two.

In the region of larger bursts the situation may be different. During several months of operating from four to eight cosmic-ray meters here in Chicago, a few bursts were observed involving as many as $4(10)^8$ ion pairs, requiring for their production something like 5000 ionizing particles and a total energy of about $1.3(10)^{10}$ electron volts. Still larger bursts were observed by Bennett, Brown and Rahmel on Mt. Evans.

[24] J. J. Hopfield, Phys. Rev. **43**, 675 (1933).

Probably the simplest picture of the mechanism of such a burst is that of an incoming high energy primary transferring its energy in one event into a gigantic photon spray the components of which spend most or all of their energy in the production of positron-negative pairs which are the actual ionizing particles. The occasion for the event is assumed to be an interaction between the cosmic-ray particle and an atomic nucleus in the material surrounding the ionization chamber, and it may be that the nucleus itself makes some contribution to the burst, although it is of course out of the question to suppose that all of the ionizing particles come from the nucleus.

Barometer effect of bursts

It has been found by two different investigators[17, 23] that the frequency of bursts increases with altitude considerably more rapidly than does the total cosmic-ray intensity. Similar conclusions have been reached in regard to showers[25, 26, 27] although Gilbert[28] found the showers and vertical intensity to increase with altitude in about the same proportion. In general, then, one would expect a barometer effect for showers and bursts which would be larger than for the vertical or total intensity. This has been demonstrated experimentally for showers by Stevenson and Johnson[29] who found a barometer effect corresponding to an absorption coefficient of 0.41 per meter of water for out-of-line counter coincidences as compared with 0.28 for in-line coincidences. A less definite barometer effect for small bursts was observed by the Montgomerys[23] amounting to about 0.5 percent per mm of mercury, but an effect ten times this large has been reported by Steinke, Gastell and Nie.[30] No effect at all, however, was found for bursts larger than $7(10)^6$ ions.

An analysis of nearly seven hundred bursts larger than $15(10)^6$ ions during March on five different cosmic-ray meters at Chicago reveals no evidence for a barometer effect, although the consistency of the data is not sufficient to rule out the possibility of a rather large effect. The results are summarized in Table V. A calculation

TABLE V. *Total burst frequency vs. atmospheric pressure.*

Barometer (mm Hg)	Scale Div.	Total Bursts	Bursts per hr.	Barometer (mm Hg)	Scale Div.	Total Bursts	Bursts per hr.
735	18	14	0.21	749	34	17	0.21
	19	8	.12		35	32	.24
	20	18	.19		36	25	.24
	21	3	.08		37	31	.24
	22	18	.46		38	26	.26
	23	27	.25		39	18	.28
	24	19	.20		40	14	.26
741	25	18	.22		41	10	.29
	26	22	.25		42	11	.22
	27	44	.22		43	9	.18
	28	55	.22		44	9	.21
	29	72	.21		45	16	.19
	30	44	.21		46	15	.25
	31	28	.16	759	47	5	.13
	32	38	.20		48	3	.10
	33	20	.33				

of the correlation coefficient between the two variables leads to a value of 0.056 which is, for practical purposes, zero. The average burst frequency was 0.22 ± 0.0085 per hr. if one assumes zero correlation. However, it is more informative to note that the band within which the frequencies fell half of the time was about 0.045 unit wide or about 20 percent of the average frequency. Since the total range of pressure covered was 25 mm Hg, it may be said that the results are not inconsistent with a barometer effect as large as about 8 percent per cm Hg. It seems certain, however, that the effect can be no larger than this.

All of the meters were shielded with 12 cm of lead during these tests. In view of the known high absorption coefficient of shower producing radiation and short range of the shower particles it seems likely that most of the bursts registered on this apparatus are due to the higher energy particles and occur within a zone only a few centimeters wide in the immediate neighborhood of the bomb. In the Montgomery apparatus the thickness of lead above the chamber was small enough that one might expect a majority of the bursts to be due to tertiaries produced by air secondaries. They would therefore show a barometer effect in keeping with the known high

[25] T. H. Johnson, Phys. Rev. **45**, 569 (1934); **47**, 318 (1935).
[26] B. Rossi and S. de Benedetti, Ricerca Scient. (5) **1**, 594 (1934).
[27] P. Auger and L. Leprince-Ringuet, Comptes rendus **199**, 785 (1934).
[28] C. W. Gilbert, Proc. Roy. Soc. **A144**, 559 (1934).
[29] E. C. Stevenson and T. H. Johnson, Phys. Rev. **47**, 578 (1935).
[30] E. G. Steinke, H. Gastell and H. Nie, Naturwiss. **51**, 898 (1933).

absorption coefficient of the shower producing radiation. However, for much greater thicknesses of lead it seems necessary to adopt the view that the showers occasioned by the air secondaries do not reach the ionization chamber and that the burst-producing radiation originates in the lead, probably as a result of a close nuclear approach of a high energy primary. On this view any barometer effect would have to be due to the air absorption of the primary particles and should therefore be much smaller than for bursts involving air secondaries.

Burst frequency as a function of shield thickness

The shield of 17 cm of lead shot ordinarily used on the meters described above is apparently thick enough to absorb a considerable portion of the burst-producing radiation and resultant burst particles. Evidence for this is a very considerable, at least twofold, increase in burst frequency which ensues when a few hundred pounds of the shot is drained out.[31] The optimum shield thickness to give maximum burst frequency has not yet been determined but probably lies in the neighborhood of five cm of lead. This corresponds to a similar situation in the case of showers, where the frequency increases with additional top shielding up to about 2 cm of lead and then falls off for greater thicknesses. The size of the bursts seems not to be greatly affected by reducing the shield thickness.

[31] R. L. Doan, Phys. Rev. **48**, 470 (1935).

In conclusion the writer wishes to acknowledge his great indebtedness to Professor A. H. Compton for providing the opportunity of making this investigation and also for numerous suggestive discussions during its progress. Thanks are also due to the Carnegie Institution of Washington through whom the necessary funds were provided, and to Dr. J. A. Fleming, Director of the Institution, who emphasized the desirability of such a comparative study. Practically all of the statistical calculations presented here and many more that have not been discussed were carried out by Mrs. Ardis T. Monk with the help of J. O. Pyle, Jr., and James Geary. Needless to say, this assistance has been invaluable from the standpoint of expediting the investigation.

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Theory of the Effect of Temperature on the Reflection of X-Rays by Crystals

II. Anisotropic Crystals

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In anisotropic crystals the temperature factor for the reflection of x-rays is a function of the orientation of the reflection plane. The general case of anisotropic metals is here treated by an extension of a simple method recently discussed in this journal. The complete solution is found for metals with hexagonal symmetry. The temperature factor is generally written as e^{-M}. The constant M is explicitly calculated for Zn and Cd (hexagonal symmetry). It is found that for these two metals M is 1.80, 1.73, respectively, as large for the reflection plane normal to the principal axis as for reflection planes parallel to the principal axis.

§1. INTRODUCTION

IN view of the approximations made in the Debye theory of specific heats, its success for isotropic as well as for anisotropic crystals is surprising. An explanation may lie in the relative insensitiveness of the specific heat to the assumptions made about the lattice vibrations. In particular, the specific heat is a scalar quantity, and so does not directly reflect the anisotropic vibrations of the atoms in anisotropic metals. This anisotropy in vibrations may, however, be detected experimentally by a study of the temperature dependence of the atomic

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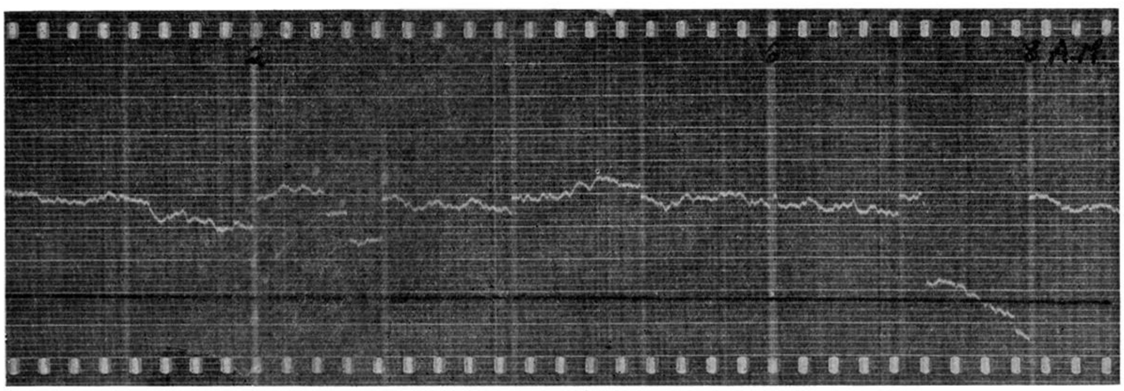

FIG. 1. Portion of record from meter No. 1 for September 21, 1934. Note the bursts in the third and eighth hours. The black line is the barometer trace.