had to be made in deriving the formulas for these separations. For instance, in the triplet formulas the average value of the quantity $(1/r_{12})^3$ enters, in the term for the spin-spin interaction. Strictly speaking, this average is infinite if separable wave functions are used; to obtain the correct formula one must use the correct, nonseparable functions, which are zero when the two electrons are coincident.[7] The procedure chosen here is the one suggested by Bethe:[8] When electron number two is outside number one, to use, instead of $(1/r_{12})^3$, a constant times $(1/r_2)^3$, the constant being determined by the relative screening of the other electron as indicated by the value of $2\mu c$. In the expression for the spin-orbit interaction, the value of $2\mu c$ is also used, as a measure of the field on the $2p$ electron.

The expressions for the doublet separations which have been used are:

$$(1s^2,\ 2p)\quad {}^2P_{1/2}-{}^2P_{3/2}=2.911(\mu c)^3(Z-2)\ \text{cm}^{-1},\tag{6}$$
$$(1s^2,\ 2s^2,\ 2p)\ {}^2P_{1/2}-{}^2P_{3/2}=5.822(\mu c)^4\quad \text{cm}^{-1}.$$

[7] Breit, Phys. Rev. **36**, 383 (1930).

The fundamental expressions for the triplet separations,[7, 8] including spin-spin interaction, for the (sp) configuration, are

$$\begin{aligned}{}^3P_0-{}^3P_1&=ZC-9D,\\ {}^3P_1-{}^3P_2&=2ZC-(18/5)D,\end{aligned}\tag{7}$$

where

$$C=0.9704(\overline{1/r^3})\quad\text{and}\quad D=0.9704(\overline{1/r_{12}{}^3})\ \text{cm}^{-1}.$$

For the $(1s,\ 2p)$ state, we set

$$(\overline{1/r_{12}{}^3})=(\mu c)^3(Z-2\mu c),$$

and obtain

$$\begin{aligned}{}^3P_0-{}^3P_1&=1.941(\mu c)^3(9\mu c-4Z)\ \text{cm}^{-1},\\ {}^3P_1-{}^3P_2&=0.7763(\mu c)^3(9\mu c-2Z)\ \text{cm}^{-1}.\end{aligned}\tag{8}$$

For the $(1s^2,\ 2s,\ 2p)$ state we substitute $(Z-2)$ for the Z in Eqs. (8). The spin-spin correction for the $(1s^2,\ 2p^2)$ state is small and complicated, so we use only the spin-orbit term,

$${}^3P_0-{}^3P_1=1.941(\mu c)^4,\quad {}^3P_1-{}^3P_2=3.882(\mu c)^4,\tag{9}$$

which gives a fairly good check with experiment.

[8] Bethe, *Handbuch der Physik*, Vol. 24, 2nd edition (Springer), p. 380.

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The Initiation of Electrical Discharges by Field Emission

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The initiation of highly overvolted discharges produced in effectively ion-free gases by voltage impulses of short duration is shown to be independent of the gas between the electrodes, also of the pressure from one atmosphere down until the vacuum spark stage is reached. In asymmetrical fields values of the field intensity at the cathode required for initiation are the order of 5×10^5 volts/cm, while at the anode the field for initiation is some 20 percent higher. Oscillograms showing the time lags and rates of fall of potentials of such highly overvolted discharges between spheres, and between points and planes have been obtained as well as those of an ordinary discharge for comparison. When the breakdown occurs because of high fields at the cathode, the initiation is readily explained by field emission of electrons. In the case of high fields at the anode it can be said that the initiation is determined by the character of its surface. When the discharge occurs the rate of fall of potential is greater than ordinary discharges in gases. Oscillograms of the wave fronts produced by 60-cycle discharges between magnesium electrodes, and the rate of fall of potential across such discharges have been obtained.

THE time lag of the spark discharge may be defined as the interval between the attainment of normal breakdown potential and the beginning of the rapid fall of potential across the gap. It may in general be divided into two parts.[1, 2] The first part represents the time before

[1] Campbell, Phil. Mag. **38**, 214 (1919).

[2] J. J. Thomson, *Conduction of Electricity Through Gases*, Vol. 2 (1933), pp. 471, 474.

initiation of the spark mechanism, or the time required for the formation of a favorable distribution of ions. This is a statistical time which is influenced by many factors and may be the order of seconds just at the sparking potential.[3, 4] Ultraviolet light of sufficient intensity producing photoelectrons at the cathode removes this part of the lag by providing an initiating agent. The second part is sometimes called a "regular lag" and is the time required to establish the spark mechanism after electrons have been formed in the gap. When ultraviolet light initiates a discharge this lag is a very definite time for any particular sphere gap at the breakdown potential. Overvoltages much reduce not only the total time lag,[5, 6] but each of the parts described above as well, where by overvoltage is meant any applied potential across the spark gap greater than the static breakdown value. Measurements of the time lag will be given in this paper.

Street and Beams[7] have shown that a sphere gap may be overvolted some 16 times when it is inclosed in dry, dust-free, and effectively ion-free gas at atmospheric pressure. Such treatment of the gas essentially delays the initiation of the discharge for it was shown that irradiation of the cathode with ultraviolet light caused breakdown near normal values. The following experiments have been conducted to determine the characteristics of such a discharge and the nature of the initiation.

Experimental Procedure

In order to determine the part played by the cathode, experiments with discharges in asymmetrical fields were carried out.[8, 9] The circuit employed is shown in Fig. 1. The capacity C_1 of about $0.002\mu f$ is slowly charged through R_1 until G_1 breaks down. The circuit constants are of such values that the potential is applied to G_2 and G_3 in less than 10^{-7} sec. This potential is

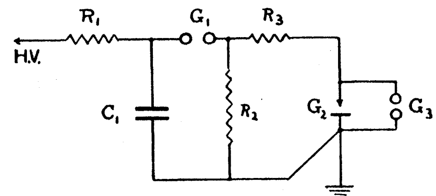

FIG. 1. Circuit for applying potential rapidly to a gap, G_2, to be overvolted and gap, G_3, for measuring potential.

removed by the discharge of G_2 or G_3. G_2 is the gap to be studied and G_3 is a sphere gap used to measure the potential applied to G_2. G_3 was irradiated by the ultraviolet light from an iron arc to reduce its time lag. G_2 was in an enclosure containing gas which had been dried by P_2O_5 and filtered through a bacteriological filter of diatomaceous earth. An auxiliary electrode near G_2 was used to sweep ions from the vicinity of the electrodes. For the point and plane gap this consisted of a ring near the point while, for spheres, the metal case of the container served. Using 2.5 cm brass spheres G_3 could be separated as much as 20 times the separation of G_2 without a discharge in G_2. This gives a breakdown field of 6×10^5 volts/cm. Substituting a point and plane gap for G_2 gave similar results. The discharge with the point positive always occurred at greater separations of G_3 than when negative.

In order to retain more uniform electrode surfaces and determine the effect of different gases, a gap was employed consisting of a wire and concentric cylinder. The cylinder was divided along its length, and a small potential applied between the portions to facilitate removal of ions. The wire was so arranged that it could be heated in vacuum before the gas was admitted. The results obtained by using a platinum wire of radius 0.0036 cm and steel cylinder of internal radius 0.43 cm are shown in Fig. 2. Here the maximum breakdown intensity at the wire surface as measured by G_3 is plotted as a function of the gas pressure for N_2 and He. The curves for air and H_2 fall very near to these. The corresponding field strengths at the surface of the cylinder were only of the order of 5×10^3 volts/cm. It will be observed that the breakdown potential is independent of the pressure from atmospheric down to about 40 cm of Hg. The oscillograph shows (Fig. 4) that this independ-

[3] Tilles, Phys. Rev. **46**, 1015 (1934).
[4] Zuber, Ann. d. Physik **76**, 231 (1925).
[5] Beams, J. Frank. Inst. **206**, 809 (1928).
[6] Rogowski and Tamm, Archiv f. Elektrotechnik **30**, 625 (1928).
[7] Street and Beams, Phys. Rev. **38**, 416 (1931).
[8] Flowers and Beams, Phys. Rev. **46**, 338 (1934).
[9] Flowers, Phys. Rev. **47**, 801 (1935).

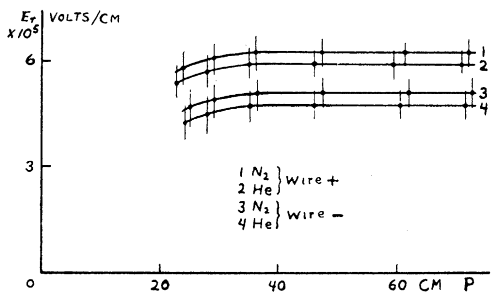

FIG. 2. Overvoltage breakdown intensity-pressure relations between a wire and a concentric cylinder by using circuit in Fig. 1.

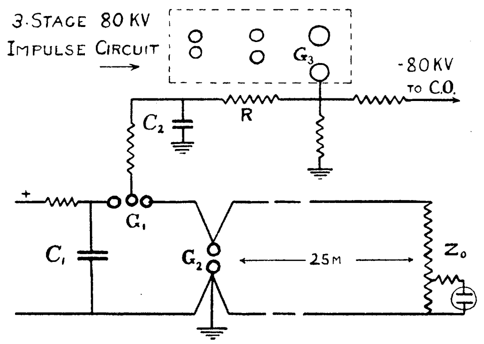

FIG. 3. Circuit for synchronization of the discharge to be studied at G_2 with the oscillograph.

ence continues to low pressure. The static sparking potential is proportional to the pressure and in He is of the order of 1/10 that in N_2 while that in H_2 is about half that in N_2. Thus the very small differences in the breakdown potential observed in Fig. 2 for the different gases indicate that the gas between the electrodes has very little effect on the breakdown potential.

With the wire negative the value of the field at the surface of the wire at breakdown is the order of 5×10^5 volts/cm which is in the region where field emission is observed from metals in high vacua,[10] and is of the order required to produce the vacuum spark.[11, 12] For the discharge in gases this field emission need not be large as a single electron probably suffices to initiate the discharge.[13]

The apparent breakdown at lower field intensities as the pressure is reduced, shown in Fig. 2, is evidently due to the method of measurement or to the construction of G_2 since the oscillograms do not show this decrease.

Oscillographic Measurements

Following these experiments a high voltage oscillograph of the Dufour type was set up. This one employed gas focusing of the electron beam and had deflection plates of about 1 cm calculated capacity. Writing speeds as high as 3×10^8 cm/sec. could be recorded, and all oscillograms shown are continuous on the films. The circuit for timing the discharge, and connecting to the deflection plates of the oscillograph is shown in Fig. 3. The capacity C_1 and the impulse circuit are slowly charged. When the last gap, G_3, of the impulse circuit discharges, high potential is applied to the oscillograph (C.O.), and to the third electrode of G_1 at a rate depending on the value RC_2. G_1 is thus tripped, and C_1 discharges into G_2, the gap to be studied, and the line connecting G_2 to the deflection plates of the oscillograph. The line is terminated by the surge impedance Zo (525 ohms) which also functions as a potential divider. A one-piece electrolytic resistor was found to serve best as surge impedance, potential divider, and critical damping resistance for the deflecting plates. The time constant for the deflection system used was not greater than 10^{-10} sec.

The first two oscillograms (Fig. 4) show the discharge of intensely irradiated zinc spheres of 2 cm diameter in air. The light from G_1 and a 10-ampere iron arc irradiated the cathode G_2 from distances of 10 cm. No. (1) shows the discharge when the applied potential wave just reaches static breakdown voltage. The steeply applied wave permits good measurement of the time lag of the spark. The results of measurements of this and similar oscillograms give about 8×10^{-8} sec.

No. (2) shows the results of reducing the separation of G_2 which tends to overvolt it. The time lag becomes inappreciable (about 10^{-8}) but still sufficiently great to allow an overvoltage of about 10 percent. The gap G_2 is set to discharge statically at 20 kv. The regularity of these oscillograms indicates uniform initiation by photoelectrons.

[10] Millikan and Eyring, Phys. Rev. **27**, 51 (1926).
[11] Hull and Burger, Phys. Rev. **31**, 1121 (1928).
[12] Snoddy, Phys. Rev. **37**, 1678 (1931).
[13] Tiedeman, Physics **1**, 354 (1931).

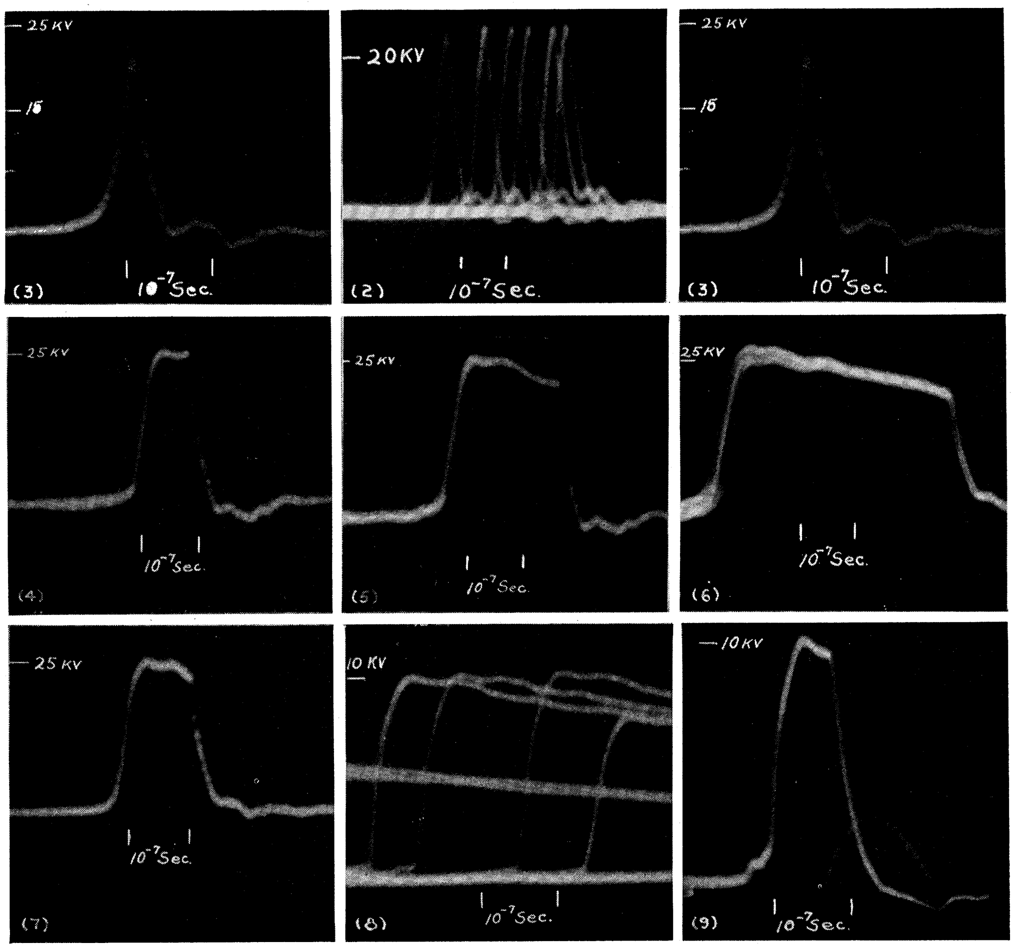

FIG. 4. Oscillograms. (1) Discharge of 2-cm Zn spheres irradiated strongly. Sweep speed 1 mm on film equals 4.5×10^{-9} sec. $d=8$ mm. (2) Same as Fig. 1 at slower sweep speed and gap set to discharge at 20 kv. $d=6$ mm. (3) Highly overvolted 2.5-cm brass spheres. Separation about 0.4 mm. (4) Same as 3. Separation slightly greater. (5) Same as 3. Separation slightly greater. (6) Discharge of negative point to plane, overvolted 100 percent. (7) Highly overvolted negative points. (8) Sixty-cycle wave fronts. (9) Breakdown of sixty-cycle magnesium gap.

In oscillograms (3), (4), and (5) are shown the discharge of the highly overvolted 2.5-cm spheres in dry, relatively ion-free air at atmospheric pressure. The separation of the spheres is approximately 0.4 mm. For separations less than some critical value the discharge always appears as in No. (3). For slightly greater separations a statistical time lag appears as in (4) and (5). Time lags as long as 0.5 microsecond have been observed. In the case of No. (3) the field reaches a value sufficient for considerable field emission so that the time between extraction of electrons is relatively short. This would necessitate a field current of about one electron in 10^{-9} sec. or roughly 2×10^{10} amp. For lesser values of the field the statistical characteristics are observable as in Nos. (4) and (5). If the hypothesis of field emission is correct, the field current here is about one electron in 10^{-7} sec. on the average or about 2×10^{-12} amp. This is in agreement with currents observed in high vacua and their rapid variation with field strength. A discharge such as shown in No. (3) has a minimum time lag and the difficulty of measuring the peak voltage reached by a parallel sphere gap is obvious.

Oscillograms (6) and (7) show the discharge from negative points in air at atmospheric pressure. The positive discharge appears the same. No. (6) is in room air and overvolted about 100 percent to reduce time lags. The type of time lag shown is extremely erratic, probably because of difficulty of uniform initiation with a point. Even with this overvoltage the rate of fall of potential is less than that for spheres at normal breakdown. No. (7) is a point discharge in dry, ion-free air with about 15 times static breakdown potential applied. This shows the same characteristics as overvolted spheres. In all cases the discharge at high field strengths produces a very rapid fall of potential in the initial stage. For the overvolted spheres the fall to half-value is at least twice as fast as for irradiated spheres. The overvolted points seem to show an earlier diminution in the rate of fall than overvolted spheres.

The relation between pressure and the overvolted discharge potential as measured by the oscillograph is shown in Fig. 5. The breakdown potential is constant from atmospheric pressure down to 8 mm of Hg. Between 8 and 7 mm a rise of about 30 percent occurs. At this pressure it is believed that the electron mean free path becomes of the order of magnitude of the electrode separation, and the electrons removed from the cathode by the field reach the anode without appreciable ionization in the gas. The potential rises higher in the time required to set up the vacuum spark mechanism with attendant electrode temperatures. However, the potential drops at still lower pressures and then rises again. This has been observed repeatedly and may be due to variation of surface conditions with the pressure.

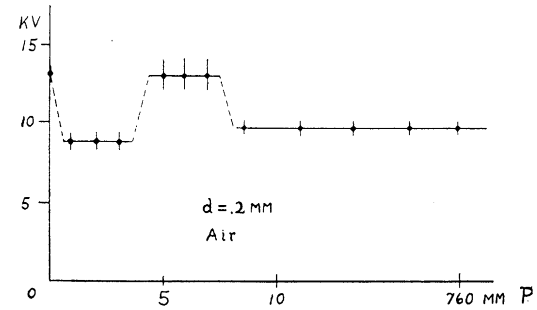

FIG. 5. Overvoltage breakdown potential-pressure relations between 2.5-cm brass spheres as determined by the oscillograph.

Discussion of Results

The initiation of the highly overvolted discharge is readily explained by the removal of electrons from the cathode by the electrical field when the discharge occurs between spheres or from negative points and wires. The field strengths and field currents have been shown to be of the same order of magnitude as observed in high vacua. The estimation of the field currents depends upon the ability of a single electron to initiate the discharge and the assumption that the fall in potential begins within a negligibly uniform time (time lag) following this event. At the high field strengths and high pressures the fall in potential should begin almost immediately with field emission since even slight overvoltages reduce the time lag of the ordinary spark (oscillogram (2)).

The highly overvolted discharge has been shown to be independent of the pressure over wide ranges. This is to be expected to the extent that the pressure does not change the time lag or the field emission. At some low pressure the gas should cease to be predominant in the discharge mechanism. The characteristics then become those of the vacuum spark and the time after field emission until the potential begins to fall, i.e., the time lag depends upon the electrodes, the amount of energy supplied, and the extent of the field current.

The discharge potential has been shown to be relatively independent of the kind of gas present. It should depend upon the gas only to the extent that different gases change the surface condition and thus field emission. The possible differences observed are thus attributable to different work functions at the surface of the wire.

The conditioning of the electrodes by the passage of several discharges between new or polished electrodes before appreciable overvoltage is attained is a supporting characteristic.

In the case where the point or wire is positive the explanation is not as evident. There are some indications that positive ions may be removed from metal surfaces by electric fields,[14] and the occurrence of such should be sufficient

[14] Beams, Phys. Rev. **41**, 687 (1932).

to initiate the discharge, provided the positive ions can produce sufficient ionization at the existing field strengths. However, at the present such a process is somewhat speculative. It is reasonably certain that in neither the positive nor the negative discharge can the initiation be attributed to chance electrons or ions in the field. Any alternative process must of necessity be new or very complex. For the case of the negative discharge the evidence strongly supports the theory of field emission.

The 60-Cycle Discharge

In order to study a discharge under conditions very different from the overvolted one, oscillograms of the 60-cycle magnesium spark have been obtained. A knowledge of the rate of fall of potential across such a discharge and the wave front produced is important from an experimental standpoint. At high resolution speeds of the oscillograph the synchronization of such a discharge presents some difficulty. Fig. 6 shows a successful circuit used. The impulse circuit is slowly charged while C_1 is charged at sixty cycles by the transformer and rectifier. G_1 and G_2 discharge at the same rate and are allowed to run continuously. G_2 is a magnesium gap. The impulse circuit is started by the third electrode in G_4, the first stage of the impulse circuit. This occurs near the peak of the potential cycle applied to C_1. The discharge of G_3 which applies potential to the oscillograph trips the gap G_1 slightly ahead of its regular breakdown time in each cycle and thus times the wave.

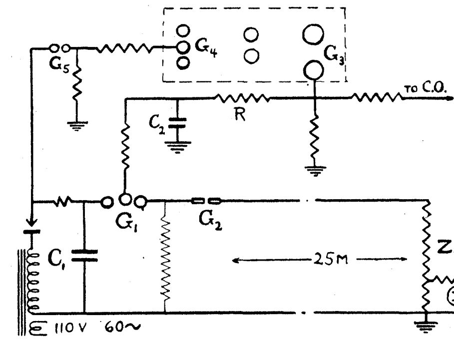

Fig. 6. Circuit for synchronization of the sixty-cycle discharge studied at G_2.

Oscillogram (8) shows the wave fronts produced by G_2 discharging into the line. More precisely this is the current-time relation of G_2. The current at any instant is given by V/Zo which gives the maximum of about 20 amperes. Such wave fronts are of interest in many recent experiments. No steep portions are in evidence and for the voltages used the gradients are less than that produced by static breakdown of spheres. No. (9) shows the breakdown of an individual 60-cycle discharge connected across the line as G_2 in Fig. 3. The discharge is damped only by the small resistance of the copper connecting wires and maximum current between breakdowns reaches the order of 500 amperes. Discharging continuously at 60 cycles this current produces considerable heating of the electrodes. This particular oscillogram shows a time lag which is usually absent because of slight over-voltages. A 3.5-megacycle wave for timing purposes is also shown.

Apparently the rapid rate of discharge has only slight effects on the sparking mechanism at these frequencies. Measurements of oscillograms (1) and (9) and many similar ones give a time constant of 3.0×10^{-8} sec. to the 60-cycle discharge and 2.7×10^{-8} sec. to the regular breakdown as shown in oscillogram (1).

This difference for the most part may be due to the shape of the electrodes which were magnesium rods 5 mm in diameter and slightly beveled at the ends.

The sixty-cycle discharge produces a wave front comparable to the ordinary spark discharge and is less steep than sometimes assumed.

In conclusion the author takes pleasure in acknowledging his indebtedness to Dr. J. W. Beams under whose direction this work was accomplished, and also to extend his thanks to Dr. L. B. Snoddy for his valuable advice and interest.

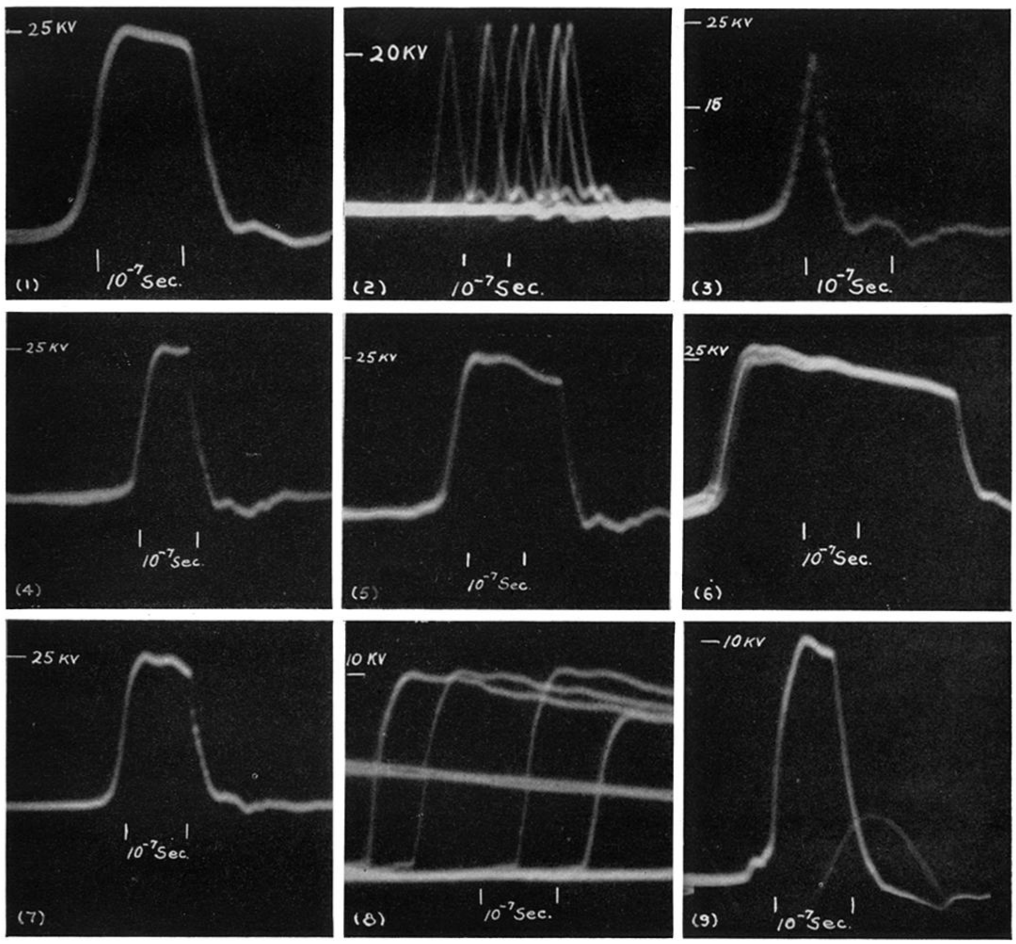

FIG. 4. Oscillograms. (1) Discharge of 2-cm Zn spheres irradiated strongly. Sweep speed 1 mm on film equals 4.5×10^{-9} sec. $d=8$ mm. (2) Same as Fig. 1 at slower sweep speed and gap set to discharge at 20 kv. $d=6$ mm. (3) Highly overvolted 2.5-cm brass spheres. Separation about 0.4 mm. (4) Same as 3. Separation slightly greater. (5) Same as 3. Separation slightly greater. (6) Discharge of negative point to plane, overvolted 100 percent. (7) Highly overvolted negative points. (8) Sixty-cycle wave fronts. (9) Breakdown of sixty-cycle magnesium gap.