The Value of the Electronic Charge

Kellström's[1] recent determination of the viscosity of air seems to furnish the final piece of evidence required to establish the essential correctness of the so-called grating value of the electronic charge e. As is well known, there has been for a number of years a sharp discrepancy between the value of e given by oil-drop work and that deduced from the absolute wave-lengths of x-rays, as determined by means of a grating. In recalculating the oil-drop data, in 1929,[2] I adopted Millikan's assumed value of the viscosity of air ($\eta_{23}=1822.7\times 10^{-7}$), and also his assumed probable error (0.05 percent), since I did not feel competent to express an expert opinion in that field. This value had been obtained by the method of rotating cylinders, a method that has generally been considered more reliable than the method of capillary tubes. I thus obtained $e=(4.768\pm 0.005)\times 10^{-10}$ e.s.u., as the final result of Millikan's oil-drop work.

Shiba[3] was perhaps the first to challenge the correctness of Millikan's assumption regarding the viscosity of air. He presented a table of values of η which indicated clearly that the capillary tube method gives in general a definitely higher result than the rotating cylinder method. Shiba himself adopted 1831.2×10^{-7} as the best average value. Now, however, Kellström, using the method of rotating cylinders, obtains $(1834.8\pm 3.0)\times 10^{-7}$, an even higher value. The use of this last value with Millikan's data leads to $e=(4.816\pm 0.013)\times 10^{-10}$ e.s.u.

Meanwhile, as Bearden[4] has recently shown, all measurements on the absolute wave-lengths of x-rays are remarkably consistent. These results can be used to determine a value of e only if one assumes a geometrically perfect crystal (in practice, calcite), and certain observed constants (density, etc.) for this crystal. With such an assumption, and with the latest values of the constants, one obtains, in agreement with Bearden,[4] $e=(4.8036\pm 0.0005)\times 10^{-10}$ e.s.u. Hence there is no longer any outstanding discrepancy between the values of e determined by these two distinctly different methods.

This fact, however, does not settle the problem of the values of the three interrelated constants e, e/m, and h. As a result of a number of recent investigations on e/m, it seems more than probable that its true value lies between 1.757 and 1.758 ($\times 10^7$ abs. e.m.u.). Let us adopt 1.7575 $\times 10^7$ as the best average. To get a value of the Planck constant h, one may now use Bohr's formula for the Rydberg constant, a formula that is still believed by theoretical physicists to be correct to a high degree of accuracy. With $e=4.8036$ and $e/m=1.7575$, one obtains $h=6.6286\times 10^{-27}$ erg·sec.

About a year ago[5] I presented a diagram showing all important experimental results involving e, e/m, and h. This diagram indicated only too clearly the impossibility of assigning *any* set of values to these three constants that would even reasonably satisfy all of the experimental results. This situation still remains essentially unchanged. The set of values that I gave then was $e=4.768$, $e/m=1.7574$, $h=6.547$. A few of the experimental results are equally well satisfied by either set of values, but in general the two sets predict very different experimental results. Thus if the new set ($e=4.8036$, etc.) is correct, the experimental results for the limit of the continuous x-ray spectrum, as a function of voltage, are all in error by amounts up to 20 times the assigned probable error. These various results, however, are quite consistent, and the method is usually considered especially reliable. Similarly the average value of h/e, from determinations of ionization and resonance potentials, is in error by some five times its apparent probable error. The determinations of h/e from the photoelectric effect and from the value of the radiation constant c_2, are equally in error.

It appears to me that the most desired experiment in this field, at the present time, is a really reliable determination of h/e. Since the two sets of values of e and h that have been discussed give values of h/e differing by 0.58 percent, it should be possible to distinguish between them.

It may be noted, in closing, that the new set of values of e, h, and e/m, leads to $1/\alpha=137.06$. If one retains the same value of e/m (1.7575), but shifts e from 4.8036 to 4.810, $1/\alpha$ becomes exactly 137, and h becomes 6.6433.

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Note on Majorana's Exchange Energy

The exchange energy introduced by Majorana is $\Sigma_{i>j}J(r_{ij})P_{ij}=V$, where i, j refer to protons and neutrons, respectively, and P_{ij} is the exchange operator which interchanges the space coordinates x_1^i, x_2^i, x_3^i of the proton i with the space coordinates ξ_1^j, ξ_2^j, ξ_3^j of the neutron j, and r_{ij} is the distance between i and j. The interaction energy operator V is Hermitian and it commutes with the three components of the total momentum operator

$$(\hbar/i)(\Sigma_i\partial/\partial x_s^i+\Sigma_j\partial/\partial\xi_s^j)=G_s.$$

It is satisfactory in these respects. There is, however, an undesirable feature of this interaction energy which is due to the difference of the mass μ of the neutron and the mass m of the proton. It shows itself in an improper behavior of the center of mass and a lack of invariance to Galilean transformations. The coordinates of the center of mass $X_s=(\Sigma m x_s^i+\Sigma\mu\xi_s^j)/(\Sigma_i m+\Sigma_j\mu)$ have the following equations of motion

$$\frac{dX_s}{dt}=\frac{G_s}{M}+\frac{i(m-\mu)}{\hbar M}\sum_{(i>j)}J(r_{ij})(\xi_s^j-x_s^i)P_{ij}.$$

Here M is the total mass. If $m=\mu$ this is the usual relation between the velocity of the center of mass and the momentum. Because of the extra term in $m-\mu$ there is an additional tremblatory component in the motion of X_s somewhat analogous to Schrödinger's "Zitterbewegung."

The lack of invariance to Galilean transformations can be seen by considering the deuteron. The solutions corresponding to an energy E and a momentum G are obtained by the substitution $\psi=\varphi(y)\exp\{(i/2\hbar)\Sigma(x_s+\xi_s)G_s\}$ with $y=\xi-x$ which gives