On the Showers of Rays Which Produce Bursts of Cosmic-Ray Ionization

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An expression for the probability that a shower of a given number of rays will discharge a number of Geiger-Müller counters is derived and experimentally verified by observing the bursts of ionization in an ionization chamber and the simultaneously occurring discharges of three counters placed under the chamber. An estimate is made of the relative importance of the contributions of showers of various numbers of rays to the counting rate of counters placed out of line. The view that there is no sharp distinction between small showers as observed in cloud chambers and the very large showers which produce the bursts of ionization, or Stösse, in ionization chambers is emphasized. The results have a significant bearing upon the interpretation of counter experiments on showers.

IN the investigation of the showers of ionizing rays which are produced by cosmic radiation, many observers[1] have employed the simultaneous discharges of several Geiger-Müller counters placed out of line. Indeed, the first clear evidence of the existence of showers was obtained by means of them.[2] Now, only if showers consisted of an infinite number of rays, would the counters discharge simultaneously every time a shower occurred. The fraction of the number of showers which are recorded depends not only upon the geometrical configuration of the counters, but also upon the density of rays in a shower. In general, a small number of counters have been used in these investigations, and it has been natural to suppose that the counters are usually set off by a group of a small number of rays. An ionization chamber, on the other hand, will record all sizes of showers, but observations are limited to those sizes which are not obscured by the statistical fluctuations in the cosmic-ray ionization, i.e., to groups of rays containing numbers by no means small. It has been customary to imply that there is a sharp line of demarcation in frequency of occurrence and perhaps in other matters, between those sprays of small numbers of rays which have been usually referred to as showers, and the large groups commonly referred to as atomic bursts, or Stösse. It is proposed here to give evidence to support the view that, when the proper statistical considerations involving the probability of a counter discharge as a function of the number of counters, ray density, etc., are taken into account, the proper interpretation of the experiments so far performed lead to the conclusion that there is no marked discontinuity between small and large showers, but that all are represented in a more or less continuous gradation in size and are such as to suggest a family of phenomena intimately related as regards their origin and significance.

By making suitable assumptions, it is possible to arrive at an estimate of the probability that a shower of a given size will discharge a group of counters. Let us suppose that we have a source of showers and r counters placed under it. Let the number of rays in a shower be N, and suppose that these N rays are randomly distributed over a solid angle which includes all the counters. Let the *a priori* probability of one of the rays passing through the ith counter be p_i. The values of these *a priori* probabilities depend, of course, upon the conditions of the experiment: the positions of the counters, the solid angles subtended by the counters, etc. If $n_1, n_2, \cdots, n_r$ are the number of rays of the shower which pass through counters one, two, $\cdots$, r, respectively, then the probability of such a configuration of rays is the familiar multinomial distribution

$$\frac{N!p_1^{n_1}p_2^{n_2}\cdots p_r^{n_r}p_0^{n_0}}{n_1!n_2!\cdots n_r!n_0!}. \qquad (1)$$

n_0 and p_0 are defined by the relations $n_0+n_1+n_2+\cdots+n_r=N$, and $p_0+p_1+p_2+\cdots+p_r=1$, and refer to those rays which miss all r counters. If the counters are arranged so as to record a count when one or more rays pass through each

[1] B. Rossi, Zeits. f. Physik **82**, 151 (1933); E. Fünfer. Zeits. f. Physik **83**, 92 (1933); J. C. Street and T. H, Johnson, Phys. Rev. **42**, 144 (1932).
[2] B. Rossi, Physik. Zeits. **33**, 304 (1932).

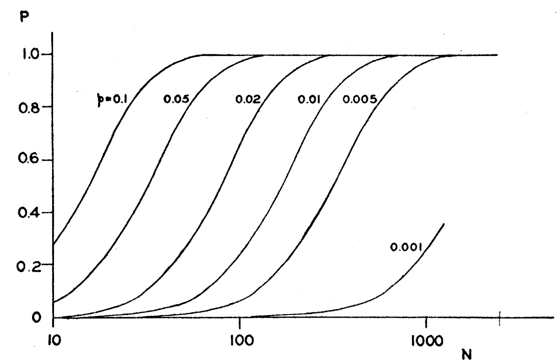

FIG. 1. The probability of the simultaneous discharge of three counters for various sizes of showers and *a priori* probabilities.

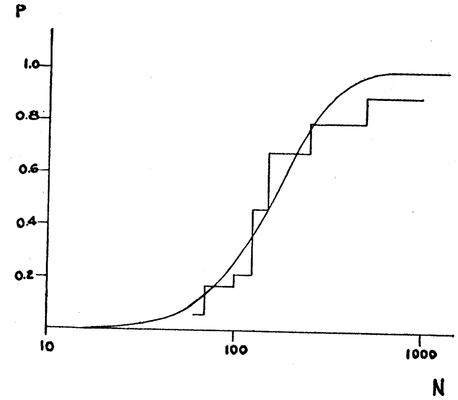

FIG. 2. Observed and calculated fractions of the number of bursts of ionization occurring simultaneously with a discharge of the counters.

counter simultaneously, the fraction of the number of showers of N rays which will produce a count is given by the summation of expression (1) over all values of $n_i \geqslant 1$, where $i \neq 0$. In the special case[3] when all the *a priori* probabilities, $p_1 \cdots p_r$, are equal, this sum may easily be found to be

$$\sum_{j=0}^{r} \frac{r!(-1)^j}{j!(r-j)!}(1-jp)^N. \qquad (2)$$

Fig. 1 shows how this probability varies with the number of rays in the shower for various values of p in the case of three counters.

It is possible to test this frequency formula experimentally by utilizing the fact that the sizes of bursts as measured in an ionization chamber are proportional to the numbers of rays in the corresponding showers. The fraction of the total number of bursts of ionization of any given size which occur simultaneously with the discharges of a group of counters suitably arranged is calculable through (2), and can also be measured experimentally, so that the principles underlying (2) can be tested. The apparatus used consisted of a spherical ionization chamber of magnesium immediately above which was placed enough lead shot to be equivalent to one centimeter of solid lead. The shot was contained in a square box whose side was approximately a diameter of the sphere. Under the sphere were placed three groups of three Geiger-Müller counters, the counters in each group being connected in parallel. The three groups were placed out of line with the center group below the other two, so that a ray passing through the sphere downwards could pass through only one group of counters. A simultaneous discharge of the three groups was made to flash a lamp, making a trace on the same photographic paper on which were recorded the bursts in the ionization chamber. The ionization chamber and its vacuum tube electrometer have been previously described.[4] The fraction of the number of bursts of ionization which were accompanied by a discharge of the counters was determined for bursts greater than 1.2×10^6 ions, in nitrogen at 14.5 atmospheres pressure. If we assume that the ionization was produced by rays of the ionizing power of high energy electrons (60 ions/cm at atmospheric pressure[5]), then we can determine the number of rays, N, in each burst which went through the sphere. The ratio of the mean solid angles subtended by the sphere and by one group of counters at the lead is the value of the *a priori* probability, p, associated with that group. This ratio was the same for all groups of counters and was equal to 0.01.

Fig. 2 shows how well the data obtained agree with expectation. In view of the fact that there are no arbitrary constants to be adjusted, and that some of the constants used in the reduction

[3] A more complete discussion of the derivation of these formulae is in preparation for publication in the Journal of The Franklin Institute.

[4] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **47**, 429 (1935).

[5] W. F. G. Swann, Phys. Rev. **44**, 961 (1933).

of the data have a large uncertainty (the specific ionization of high speed electrons, for example) the agreement between theory and observation is excellent. This agreement amounts, then, to a verification of the assumptions made. These may be restated in the following way:

(a) The ionization in a burst is almost wholly produced by rays having the ionizing power of high energy electrons.

(b) The number of rays per unit solid angle is proportional to the total number of rays in the burst which pass through the chamber. (The fitting of theory to experiment did not necessitate the assumption that p is a function of N.)

(c) The rays are distributed at random.

Since the solid angle subtended by a counter group is 0.028, the density of rays in a shower giving 100 rays in the vessel is $100\times 0.01/0.028$, or about 40 rays per unit solid angle.

Similar experiments have been performed, with other ionization chambers and other arrangements and numbers of counters, which, although the data are more fragmentary, agree with expectation as well as the experiment here described.[6] It is to be noted that the above method of measuring the experimental probability that a burst of ionization of any given size will be accompanied by a discharge of the counters gives an average value taken over all regions of origin of the showers and directions of the line of symmetry of the shower particles. However, since only rays which have passed through the ionization chamber are recorded either by the counters or by the electrometer, the results are largely independent of the distribution in angle of the showers, and of their place of origin. In all the above considerations the fraction of the time during which the counters were insensitive was assumed to be negligible. If this assumption were eliminated, and the necessary small correction made, it would take the form of a constant factor, slightly greater than unity, multiplying all the experimental probabilities and would thus tend to improve the agreement with theory.

In the foregoing section we have given an answer to the question, "How many bursts of ionization of a given size are accompanied by a discharge of the three counter groups?" Let us now invert the question and ask, "What fraction of the total number of the simultaneous discharges of the counters is produced by showers of a given size?"

If showers containing N rays occur at the rate of $R(N)$, and if the probability of a shower being recorded by some arrangement of counters is $P(N)$, then the counting rate, C, will be given by

$$C=\sum_{N=1}^{\infty} R(N)P(N).$$

Since we know $R(N)$ from the ionization data, it should be possible to compute the total counting rate of the counters used in the experiment described above, and compare it with the observed rate. It is necessary to know the distribution function, R, for values of N smaller than it is possible to observe in this chamber, and these values must be obtained by extrapolation.

Fig. 3, curve A, shows the observed distribution curve, $R(N)$, plotted on a logarithmic scale. The number of single rays passing through the chamber in unit time (one hour) is 1.48×10^5.[7] From this, the number of showers of two, three, etc. rays are obtained by using the cloud chamber observations[8] of the relative frequency of these small showers, and are also plotted. All these data are satisfactorily represented by a relation of the form $R=A/N^s$, which is a straight line on our diagram. There is no evidence here that there is any fundamental distinction to be made between large and small showers. This method of extrapolation gives not only a satisfactory representation of the frequency of large and small showers, but also reasonable values for the ionization produced by showers (20 percent of the total ionization) and the size of the observed fluctuations in ionization (the standard deviation computed for single rays must be multiplied by 1.2 to correct for showers). A frequency distribution function of this same form also satisfactorily represents the experimental distributions in various other ionization cham-

[6] W. F. G. Swann and C. G. Montgomery, Phys. Rev. **44**, 52 (1933); W. F. G. Swann, J. Frank. Inst. **218**, 173 (1934). Report on the Work of the Bartol Research Foundation, 1933–1934.

[7] J. C. Street and R. H. Woodward, Phys. Rev. **46**, 1029 (1934).

[8] C. D. Anderson, Phys. Rev. **44**, 406 (1933).

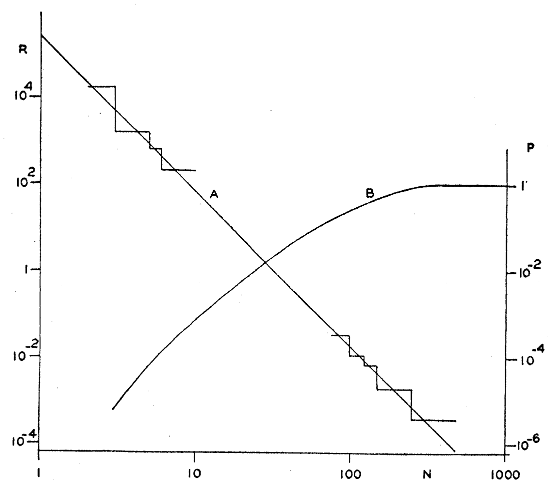

FIG. 3. Curve A, frequency distribution of showers, $R(N)$. Curve B, probability of a count, $P(N)$.

bers, including one experiment in which showers of from ten to one thousand rays were measured, and whose rates of occurrence varied over a factor of 10^5. Curve B, Fig. 3, is the function $P(N)$. The scale of ordinates is at the right. The function PR, obtained by multiplying the above functions, increases from zero, for two-ray showers, to a maximum for showers of eight or nine rays and then falls monotonically as N increases. The counting rate due to showers greater than 60 rays is the observed rate of occurrence of counts coincident with bursts of ionization, *viz.*, 0.9 per hour. The total counting rate computed from these data is 2.5 per hour. The observed counting rate was 5.5 per hour. In view of the large range of extrapolation, this agreement is quite satisfactory. Indeed, we should expect the observed counting rate to be greater than the rate computed in this manner, since no account has been taken of the showers which set off the counters without passing through the chamber. Further, we find that the contribution of showers below ten rays (0.4 per hour) is of the same order of magnitude as the contribution of showers greater than 100 rays, and that more than half of the counts are caused by showers greater than 30 rays. This situation is not a consequence of the particular arrangement of counters considered here, but holds for most cases where counters have been used to investigate showers. Thus the phenomena which counters record are, in general, quite complicated ones.

The importance of the foregoing considerations is attested by the fact that, in investigating showers, it has been customary to use two or three counters, and regard their simultaneous discharges as evidence of the occurrence of doubles, triples, etc., at the same time regarding any discharges produced by large showers or Stösse as of such rare occurrence as to be negligible, or at any rate to figure only as a correction. We must now realize that the rarity of occurrence of the large showers is largely compensated by the large chance which they have of operating the counters, in comparison with the much smaller chance of operation that three-ray showers have, for example. The net result is that the number of discharges of the counters produced by the large showers is comparable with the number produced by those which it is the object of the experiment to measure.

It has also been a common practice, when using counters to investigate showers, to correct for the showers observed when no shower producing material is present by subtracting the "zero" counting rate from all the others. This is, of course, a valid procedure only when the distribution functions, $R(N)$, combine linearly. It has been previously pointed out[9] that this is not true in general, and that one piece of matter can affect the showers from another piece not only by an absorption process, but also by producing additional ionizing rays. An inspection of the data on "transition" phenomena shows that this is quite a general property, and its effect must be carefully considered in drawing conclusions from counter data.

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[9] C. G. Montgomery, D. D. Montgomery and W. F. G. Swann, Phys. Rev. **47**, 512 (1935); C. G. Montgomery, Phys. Rev. **45**, 62 (1934).