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The Corpuscular Theory of the Primary Cosmic Radiation

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The approximately exponential law of cosmic-ray absorption is, at first sight most readily accounted for by the hypothesis that the primaries are composed of photons. However, latitude and asymmetry effects demand that at least 31 percent of the primary radiation be attributed to charged particles. This is enough to spoil the exponential law given so readily by photons unless the charged particle hypothess be developed so as to give an exponential law. The naïve hypothesis that the exponential law is to be provided by a suitable range of penetrations of the incoming rays, resulting from a corresponding range of energy distribution is found untenable, mainly because such a hypothesis would lead to a condition in which the quality of the measured radiation was independent of altitude. The paper reviews a hypothesis already published by the writer to the effect that the primary rays produced secondaries in number per centimeter of path proportional to the primary energy. These secondaries perpetuate the primary path. They represent the rays actually measured, and the theory gives an exact exponential law. The theory gives, in fact, a constant "apparent" coefficient of absorption of the measured effect which is independent of the primary energy. This result requires modification to harmonize with the data on the latitude and directional effects. It is shown that harmonization is provided by a modification of the theory which permits an increase of coefficient of apparent absorption with decrease of primary energy. It is found that such a modification can be readily made in more ways than one. It is then shown that the apparently contradictory phenomenon involved in the "hardening" of the average radiation with passage through matter can readily be accounted for by adding to this hypothesis already made, the known fact of a distribution of energies in the incoming primary corpuscular radiation. A modification of the foregoing theory permits the "apparent" primaries referred to in it to be really photons produced by impact of true primary charged particles with the upper atmosphere. The writer reviews a former hypothesis made by him with regard to the dependence of shower production and atomic burst production in lead, a hypothesis designed to provide an explanation that these phenomena increase with altitude much more rapidly than does the measured cosmic-ray intensity. It results that the extension of the main theory concerned with absorption of the primary energy when combined with the hypothesis concerning burst and shower production results in harmonization of all the details of these phenomena as far as they are known. A list of eleven facts concerned with cosmic-ray absorption, latitude and directional effects, burst and shower production is made. The list comprises all of the outstanding facts concerned with the subject; and it is shown that the extended theory harmonizes all of them.

INTRODUCTION

THE purpose of this paper is to amplify the views stated by the writer in former communications as to the nature of the primary cosmic radiation, and to develop them to the point of correlating in one consistent story the experimental phenomena upon which they have bearing.

Probably the most powerful argument in favor of photons as primary rays arises from the relative ease with which such an assumption accounts for the approximately exponential law of absorption shown by the cosmic-ray intensity. However, the latitude and directional effects seem to demand that, even as a minimum, such a large[1] percentage of the primary radiation shall be of the charged corpuscular type that the simple explanation of the exponential law inherent in the assumption of photons becomes spoiled; and it becomes necessary to seek in the corpuscular hypothesis itself an explanation of a similar type of absorption law. With the situation at this stage, it becomes of interest to see how far the various facts concerning cosmic-ray phenomena may be correlated completely by the assumption that the entire primary cosmic radiation is of the charged corpuscular type.

The Conventional View as to the Properties of a Primary Corpuscular Radiation, and the Difficulties Associated with that View

The primitive picture of the behavior of a primary charged corpuscular radiation represents each of its entities, or rays, as producing ionization continually along its path at the rate of about 30 ions or more per centimeter of path (at sea level, atmospheric pressure), and losing, at each act, an amount of energy very small compared with the total energy possessed by the ray. Such a condition is the determining feature which gives to the ray a definite range, characteristic of its energy, and sensibly constant for all rays of the same type and energy. The increase of ionizing efficiency at the end of the range is a phenomenon associated with only the last meter or so of the range at normal pressure, and may be left out of consideration in the matters which concern us here. On this primitive view of the nature and properties of the primary corpuscular radiation, penetration of different numbers of the rays through different thicknesses of the atmosphere, for any assigned direction, is to be secured only by the existence of a range of energy in the particles which enter the atmosphere from above; and the existence of an exponential law for the measured intensity in any direction necessitates a corresponding distribution of numbers of rays among the energies calculated to give such a law. Apart from the artificiality[2] of postulating such an arbitrary situation, and in contrast to what a superficial picture of the phenomena would suggest, the result, if secured, would be accompanied, as the writer has shown[3] by a condition in which the *quality* of the radiation, as symbolized by the energy distribution among the rays would be the same at all altitudes. Such a condition would result in an exact proportionality between the intensity of the radiation at different altitudes (as measured by number of rays per second per square centimeter per unit solid angle), suitably averaged for all angles, and the frequency of occurrence of any phenomenon initiated by the primary radiation. Thus, for example, on this hypothesis, we should expect that the number of particle showers, or atomic bursts, of any size, produced by the cosmic radiation under specified conditions in lead would increase with altitude in exact proportion to the directionally averaged intensity of the radiation, whereas, as a matter of fact, the experiments of C. G. and D. D. Montgomery[4] of this laboratory, on showers of large size, show that shower production in lead increases with altitude more nearly in proportion to the square of the average cosmic-ray intensity as measured by ionization in a closed chamber. Similar conclusions result from the work of R. D. Bennett, G. S. Brown and H. A. Rahmel.[5] These experiments practically *demand* a theory which causes the quality of the radiation to vary with altitude; and they are supported by similar experimental results obtained, for small showers, by T. H. Johnson[6] and by B. Rossi and S. de Benedetti.[7]

[1] The latitude observations at high altitudes seem to demand that at least 25 percent of the cosmic-ray intensity, measured at high latitudes, shall be initiated by primaries of the charged corpuscular type. This lower limit becomes increased to 31 percent by the azimuthal, equatorial asymmetry measurements of T. H. Johnson, which are able to lay hold of a contribution from the equatorial intensity and designate it also as corpuscular. Even this 31 percent is still but a lower limit.

[2] A certain amount of the artificiality is removed on realizing that the necessary law of distribution relating numbers of rays to energy is an exponential one; and an exponential law is not an unreasonable one.

[3] W. F. G. Swann, Phys. Rev. **47**, 575 (1935).

[4] C. G. and D. D. Montgomery, Phys. Rev. **47**, 429 (1935).

[5] R. D. Bennett, G. S. Brown and H. A. Rahmel, Phys. Rev. **47**, 437 (1935).

[6] T. H. Johnson, Phys. Rev. **45**, 569 (1934).

[7] B. Rossi and S. de Benedetti, Ricerca Scient. **5**, 1 (1934).

Modification of Corpuscular Theory—Elementary Form

It has been shown by the writer,[8] that a variation of quality with altitude may be secured by a condition in which the primary rays come right through the atmosphere, losing energy on the way, and producing long range secondaries all along their paths.[9] If it is supposed that these secondaries perpetuate the direction of motion of the primaries,[10] and that they are the entities actually measured in our Geiger counters, *an exponential law for the measured intensity in any assigned direction* is secured[8] provided that we assume that the number of such secondaries produced per centimeter of path of the primary is proportional to the energy of the primary. It results, further, that if production of showers or atomic bursts in lead, or other elements of high atomic weight, depends upon a higher power of the ray energy than the first,[11] shower and burst production will increase with altitude more rapidly than does the measured radiation, so that such experiments as those of the Montgomerys, receive a rational interpretation. In line, moreover, with this idea, it is to be expected that the primary rays, which, at the earth's surface, travel in a direction of say 45° to the zenith, will be less efficient in the production of atomic bursts than will vertical rays, on account of the smaller average energy of the former resulting from the greater distance they have traveled through the atmosphere. An experiment confirming this view has[12] recently been performed by D. Cowie and the writer. This experiment explains, moreover, the oft-quoted observation to the effect that shower particles travel, for the most part, in a vertical direction, or in directions symmetrical to the vertical, a conclusion difficult to understand if an appreciable number of the primaries with direction inclined to the vertical are effective in producing atomic bursts.

[8] W. F. G. Swann, Phys. Rev. **46**, 828 (1934).

[9] We wish to leave as wide a latitude as possible in the nature of the primary charged corpuscles. Moreover, there is no need for us to commit ourselves as to whether the production of the secondaries takes place in one act, or through intermediaries, such as short range photons.

[10] A very exact perpetuation of direction results from the principles of conservation of energy and momentum, provided that the energies of all the particles concerned are such that the mass of each is large compared with its rest mass, and provided that potential energy changes are negligible. (See W. F. G. Swann, J. Frank. Inst. **220**, 373 (1935).)

[11] In general, we might suppose that shower production is a function of the primary energy E, of the form $A_1E+A_2E^2+\cdots$, etc., where the A's are functions of the atomic number of the shower producing material. If A_2 were proportional to the atomic number, we might have a condition where the second term was negligible compared with the first for air, while, for lead, the first was negligible compared with the second.

[12] W. F. G. Swann and D. Cowie, Phys. Rev. **47**, 811 (1935). A full account of this investigation is given in Phys. Rev. **48**, 649 (1935).

Bearing of the elementary theory upon the observed energies of cosmic-ray particles in relation to the energies suggested by the observed influence of the earth's magnetic field

The simple theory, involving proportionality between energy and energy loss per centimeter of path (at normal pressure), gives the *same measured coefficient of absorption regardless of the energy of the primaries*, as is implied, indeed, in the fact that it gives the exponential law. Moreover, the theory makes that *coefficient of absorption the same as the coefficient of absorption of the primary energy*.[8] The latter fact is of interest because it suggests, between the energies of the rays entering the atmosphere and the energies as measured at sea level, a relation calculable in terms of the relation between the cosmic-ray intensity at, or near, the top of the atmosphere and the corresponding intensity at sea level. The energies of the primary rays should, in fact, increase with altitude in the same proportion as the intensity as measured by counters increases. The recent stratosphere flight made by Major W. E. Kepner, Captain A. W. Stevens, and Captain O. A. Anderson, and also that made by Professor and Mrs. Jean Piccard, both with Geiger-counter apparatus designed by G. L. Locher and the writer, agree in showing that the vertical intensity, when extrapolated to the top of the atmosphere, is about 90 times the sea-level value.[13] The minimum electron energies

[13] It is preferable to use the actual measured relative intensities as obtained by counters rather than the values obtained from published coefficients of absorption, or even those given by ionization measurements directly, since there is evidence, particularly from the stratosphere flights, that the ionization measured intensity does not increase in exact proportion to the intensity as measured by the number of rays, even when the latter are averaged appropriately for all angles. The lack of proportionality in question is, of course, only to be expected in view of the change of quality of the primary radiation with altitude.

necessary to permit vertical entry in opposition to the earth's magnetic field in these latitudes, is about 4.5×10^9 volts. The corresponding sea-level value would consequently be about 5×10^7 volts. This is a reasonable value in the light of the requirements demanded of it. The energy necessary for vertical entry of the equator is about 3×10^{10} volts, and if we could assume the same law of increase of intensity with altitude at the equator as at these higher latitudes, the corresponding minimum sea-level energy for the primaries would be about 3×10^8 volts. Of course, these are only *minimum* values for the energies of the vertical rays, and higher energies at entrance will correspond to higher sea-level energies. Apart from any other considerations, however, the principle here exemplified is sufficient to explain why cloud chamber experiments, made, of course, at low altitudes, have failed to reveal corpuscular energies as great as those which would be suggested by consideration of the requirements of the earth's magnetic field. Moreover, consideration of these matters serves to emphasize the importance of cloud chamber energy measurements at high altitudes.

An element of difficulty exists from a consideration of the case of rays which are inclined appreciably to the vertical, and which at entry are of sufficiently low energy to show azimuthal asymmetry. These rays, traveling as they do through distances in the atmosphere considerably greater than the vertical rays, would be expected to become reduced in energy at sea level to values below those permissible for the performance of their functions. However, this difficulty is only transitory, and will tend to disappear in the more complete formulation of the theory to be outlined below, according to which the effective coefficient of absorption of the energy, while of the same order of magnitude as that of the measured intensity, may be in actuality, somewhat less. Only a small difference between the two coefficients is sufficient to impart complete harmony into the situation; and, in the meantime, the orders of magnitude above-cited are significant in illustrating the basic fundamentals involved.

Formulation of a More Complete Theory

The elementary theory, formulated in the foregoing, while containing the fundamentals underlying the more complete theory, is naturally to be regarded only as an approximation to that theory. Before considering the more complete formulation, it will be well to make a list of the outstanding facts which it must be the purpose of any successful theory to correlate. The list is as follows:

(a) The intensity for any zenith angle θ obeys, approximately, an exponential law. According to the analysis of Millikan, 90 percent of the radiation, as measured by its ionizing effect, is to be accounted for by a component with coefficient of absorption μ equal to 0.5 per meter of water.

(b) Insofar as there is departure from the exponential law, it is in the direction of increased hardening (diminution of average coefficient of absorption) with distance traveled by the primary rays.

(c) The latitude diminution of cosmic-ray intensity in equatorial regions increases with altitude as shown by the experiments of A. H. Compton and his associates.

(d) The east-west effect defined as the ratio of the intensity, I_a, of the asymmetrical part of the radiation to the sum, I_0, of the intensities from east and west increases with altitude for a constant latitude, as shown by T. H. Johnson. It is of course understood that I_a and I_0 refer to a definite zenith angle.

(e) While the east-west asymmetry increases with zenith angle for small zenith angles, it finally attains a maximum and then diminishes with further increase of zenith angle as shown by the experiments of T. H. Johnson, Alvarez and Compton, and others.

(f) The east-west effect *increases* with decrease of latitude at a given altitude, as shown by T. H. Johnson.

Then we have a number of phenomena concerned with shower production or atomic bursts. They are:

(g) Calling $S(L)$ the number of showers or bursts of any character produced per unit volume in lead or other element of high atomic weight, and I the average intensity of the cosmic radiation, the experiments of the Montgomerys, and of Johnson and of Rossi, show that $S(L)/I$ increases with altitude.

(h) Calling $S(L)_\theta$ the number of showers produced per unit volume of lead by the cosmic rays received at the zenith angle θ, and I_θ the measured intensity of the rays at this angle, $S(L)_\theta/I_\theta$ diminishes with increase of θ. This results from the experiment performed by Mr. Cowie and myself, cited earlier.

(i) Writing R for the ratio of $S(L)/I$ at latitude $\varphi=\varphi_1$ to the corresponding value at $\varphi=0$ it appears that R is less than unity for all values of φ_1 at which observations have been made.

(j) If R_h is the value of R at h, and R_0 is the value at sea level, R_h/R_0 is greater than unity.

(k) The showers produced by rays from east and west show less asymmetry than do the east and west intensities themselves.

The phenomena under *i*, *j*, *k* result from measurements made by T. H. Johnson.

The existence of a law approximately exponential as required by (a) has been provided for by the elementary theory already cited. In fact, the theory provides for this law as an exact consequence. The fact (b) is not provided for in the elementary theory.

The elementary theory while accounting for an alteration of quality with altitude, provides for a coefficient of absorption which is the same for all energies and altitudes. Now it is fundamental to the theory of the action of the earth's magnetic field upon charged corpuscles that the effect of the field shall be greater the smaller the energy. Hence, phenomenon (c) tells us that the apparent coefficient of absorption for the more deviable, soft, low energy rays must be greater than for the less deviable, high energy, hard rays.[14] Hence, our theory must provide for an increase of coefficient of absorption with decrease of energy. If such is provided for, an explanation is provided immediately for (d), since the asymmetrical rays have, by the fundamentals of the theory of asymmetry, less energy than the others. By the same provision, (e) is also explained, since increase of zenith angle is equivalent to increase of primary path.[15] The phenomenon (f) is an immediate consequence of the known theoretical conclusion that the range of energies involved in the asymmetrical rays increases with decrease of latitude. While, therefore, it is in harmony with the theory proposed, it does not invoke any assistance from that theory, but stands on its own merits.

Thus, with a reservation as regards (b), which reservation will presently be removed, all of the phenomena (a) to (f) inclusive are consistent with an extension of the elementary theory in the direction of providing for an apparent coefficient of absorption which increases with decrease of energy of the primary rays.

Turning, moreover, to the phenomena of shower production and the like, phenomena (g) and (h) receive a natural explanation on the lines formulated by the writer,[8] to the effect that shower production in lead depends upon a power of the primary energy higher than the first,[11] or at any rate higher than that which determines the shower production in air, which in turn determines the apparent intensity as measured.

If, for purposes of mathematical illustration, we suppose that I is proportional to E^n, where E is the primary energy, and n is positive; and if, following the ideas already formulated with regard to shower or burst production in lead, we assume $S(L)$ proportional to E^{n+k}, we have that $S(L)/I$ is proportional to E^{n+k}/E^n, i.e., to E^k; and the quantity defined as R, under (i) is proportional to $(E_1/E_0)^k$, where, as under (i) subscript zero refers to the equator and subscript unity to the latitude $\varphi=\varphi_1$. Thus, since on the average E_1/E_0 is less than unity, R is less than unity, which is in harmony with the fact (i).

Again, if subscripts s and h refer, respectively, to sea level and to the altitude h, we have

$$R_h/R_s=\left[\frac{(E_1/E_0)_h}{(E_1/E_0)_s}\right]^k=\left[\frac{(E_1)_h}{(E_1)_s}\cdot\frac{(E_0)_s}{(E_0)_h}\right]^k.$$

Now if we introduce for the E's coefficients of absorption μ_1 and μ_0 we have

$$R_h/R_s=e^{kh(\mu_1-\mu_0)}.$$

If further, as already necessitated by the facts (c), (d) and (e) we permit an extension of the elementary theory in the direction of making the coefficient of absorption of primary energy increase with decreasing energy, then μ_1 is

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[14] In the case of rays which do not obey a true exponential law, it is convenient to introduce the concept of an apparent coefficient of absorption applicable to any point, and defined, for that point, by $-\mu_a=(1/A)dA/dx=d/dx(\log A)$, where A is the quantity measured and dx is an element of path.

[15] An interesting correlation, largely independent of the detailed form of the theory, here suggests itself. If asymmetry could be plotted against zenith angle in the absence of absorption, a continually rising curve would be obtained. This curve may be called the true asymmetry curve. In practice, this curve is disturbed by atmospheric absorption, so that, with increasing zenith angle, it shows first an increase and then a decrease in asymmetry. Now, for two different altitudes at depths h_1 and h_2 below the top of the homogeneous atmosphere, the path length is the same for corresponding zenith angles θ_1 and θ_2 related by $h_1/\cos\theta_1=h_2/\cos\theta_2$. If, therefore, in plotting asymmetry against zenith angle, we should choose for each zenith angle θ an appropriate altitude h for the measurement given by $h=\text{constant}\times\cos\theta$, and if we should plot the asymmetries so obtained against the corresponding values of θ, the result would be a graph of asymmetry *versus* zenith angle, freed from effects of atmospheric absorption at least to the extent that all rays measured would have traversed the same thickness of air.

greater than μ_2 and R_h/R_s is greater than unity, which is in harmony with (j). In all of the foregoing, it should be recognized that the magnitude of k depends, in all probability, upon the size of the shower or burst.

Turning now to fact (k), if E_a refers to the average energy of the rays which determine asymmetry and if E refers to the average energy of all the rays coming in the directions concerned, the normal asymmetry is determined by, say, $(E_a/E)^n$ where, as before, n is a positive number which, as a matter of fact is nearly equal to unity. In line with our fundamental hypothesis as to the dependence of burst and shower production upon primary energy, we conclude that the asymmetry in the shower production in lead is determined by $(E_a/E)^{n+k}$. Hence, regardless of the value of n since E_a is less than E we have, for the showers or burst, an asymmetry which is *less than the normal asymmetry for the intensity*, the ratio of the two being in fact $(E_a/E)^k$. This is in harmony with fact (k).

Summarizing what has been written above, it is clear that with the reservation already referred to in respect of (b), all of the phenomena from (a) to (f) inclusive may be correlated on the basis of an extension of the elementary theory in such a direction as to make the coefficient of absorption of the primary energy and the closely related apparent coefficient of absorption of the measured radiation increase with decrease of energy of the primaries. Moreover, all of the phenomena from (g) to (k) inclusive may be correlated on the basis of the said extension combined with the fundamental hypothesis already formulated as to the relation between primary energy and shower or burst production. It now remains to be shown how the coefficient of absorption concerned may be made to depend on the primary energy.

Realization of dependence of the coefficients of absorption upon primary energy

For purposes of continuity, we will first outline, briefly, the essentials of the elementary theory.

If n_θ represents the number of secondary rays produced per centimeter of path of the primary, inclined at zenith angle θ, and if E is the energy of the primary, the fundamentals of the theory give[16]

$$n_\theta = \alpha E \tag{1}$$

where α is a constant.

If the primaries lose energy entirely by the creation of secondaries; and, if dx is an element of path of the primary, and β is a constant,

$$-dE/dx = \beta n_\theta = \beta\alpha E, \tag{2}$$

so that $E = E_0 e^{-\beta\alpha x}$; moreover, in view of (1)

$$n_\theta = (n_\theta)_0 e^{-\beta\alpha x}, \tag{3}$$

so that both E and n_θ obey an exponential law with the same coefficient $\beta\alpha$.

We will now proceed to extend the foregoing theory by modifying (1) to the form

$$n_\theta = \alpha E^{1-\lambda}, \tag{4}$$

where λ is a constant. Under these conditions, (2) assumes the form $-dE/dx = \beta\alpha E^{1-\lambda}$, and if we define the coefficient of absorption μ_e of the energy at any point as $\mu_e = -(1/E)(dE/dx)$, we find

$$\mu_e = \beta\alpha E^{-\lambda}. \tag{5}$$

We are concerned more particularly with the measured coefficient of absorption μ_n defined as $\mu_n = -(1/n_\theta)(dn_\theta/dx)$. In view of (4),

$$\mu_n = (1-\lambda)\mu_e, \tag{6}$$

so that on the basis of the hypothesis embodied in (4), Eqs. (5) and (6) show that both μ_n and μ_e increase with decrease of energy.

A less empirical method of securing dependence of absorption coefficient upon primary energy

A similar result may be secured in another manner which has the advantage of a more direct physical significance.

Suppose that, leaving (1) unchanged, we replace (2) by

$$-dE/dx = \beta n_\theta + \gamma. \tag{7}$$

The quantity γ represents a contribution to the loss of energy per centimeter of path which is constant along the path of the primary ray. It is symbolized, for example, by such loss as is

[16] The measured intensity involves the range of the secondaries as well as n_θ. In what follows, this range is assumed constant.

represented by ionization, although we do not wish to limit it to this explanation.

Combining (1) and (7), we obtain

$$\mu_e=\beta\alpha+\gamma/E \tag{8}$$

and since, by (1), $\mu_n=\mu_e$, we have $\mu_n=\beta\alpha+\gamma/E$, which again gives an expression which causes μ_n to increase with decrease of E. Here, μ_n approximates to a constant value for high energies, this constant value being $\beta\alpha$.

The relation between the coefficients μ_n and μ_e

It is of interest to observe that if (1) be modified to

$$n_\theta=\alpha E^s \tag{9}$$

we find $\mu_n=s\mu_e$, so that if s is greater than unity, μ_n is greater than μ_e. Such a provision has an advantage in causing the ratio of the energy of the rays entering the atmosphere to the energy of the rays at sea level to be less than the ratio of the corresponding measured cosmic-ray intensities. This consequence lessens the difficulties already referred to in the matter of the asymmetric rays, concerning the entering energies as computed from the earth's magnetic field, and the sea level energies as computed from the entering energies and the coefficient of absorption.

The incorporation of (9) into the theory as represented by (7), leads to

$$\mu_e=\beta\alpha E^{s-1}+\gamma/E \quad \text{and} \quad \mu_n=s\beta\alpha E^{s-1}+s\gamma/E.$$

This expression represents a μ_n which increases with decrease of E for low values of E, but which decreases with decrease of E for high values of E.

Realization of the refinement involved in the departure from an exponential law

We now turn to the elements involved in realizing the requirement (b) in the list of requirements cited above. At first sight, the requirement of decrease of average coefficient of absorption with distance traveled seems contrary to the principle that the effective coefficient of absorption of the individual rays increases as the rays lose energy,—a principle demanded by the asymmetry effect. The difficulty is only transitory, however, for now we introduce once more the idea of a wide range of primary energies entering the atmosphere, a hypothesis which is indeed required by the latitude and directional effects. Then, although each of these rays may *soften* as a result of its passage through the atmosphere, a suitably chosen energy distribution among the entering rays will insure that the measured radiation as a whole will harden with approach to sea level. The matter may be illustrated by considering the case of two distinct energies entering the atmosphere. Let the first, denoted by subscript unity be the higher energy. For purposes of illustration we shall write, for some assigned direction $n_1=n_{10}\exp(-\int\mu_1 dx)$. This rather artificial looking expression is adopted because it corresponds to a coefficient of absorption, defined as $-(1/n_1)(dn_1/dx)$, which is equal to μ_1 and is variable with x, if μ_1 varies with x.

In a similar manner, we write

$$n_2=n_{20}\exp(-\int\mu_2 dx).$$

The contribution of the two types of radiation is given by

$$n=n_1+n_2=n_{10}\exp(-\int\mu_1 dx)+n_{20}\exp(-\int\mu_2 dx).$$

The average coefficient of absorption of the combined radiation may be defined as $\mu=-(1/n)(dn/dx)$, so that

$$\mu=(\mu_1 n_1+\mu_2 n_2)/(n_1+n_2).$$

To illustrate the properties of this expression it will be sufficient to remark that if at $x=0$, i.e., at entry to the atmosphere, n_1 is small compared to n_2, μ approximates μ_2. On the other hand, at sufficiently large values of x, the quantity n_1 is large compared with the quantity n_2 on account of the required assumption that μ_2 shall be greater than μ_1. Hence, at large values of x, the coefficient μ approximates μ_1. Thus, under the condition cited we have an illustration of the fact that, while the individual quantities μ_1 and μ_2 both increase with x, the measured μ decreases from μ_2 to μ_1 with increase of x.

It is important to note that the invocation of a distribution of primary energies in the foregoing matter is on an entirely different plane from such an invocation as would be designed to give an exponential law. Here our exponential law, which represents the main story, is given as an approximation without any distribution of primary energies. In fact, to a first approximation, all primary energies give an exponential law with the same apparent coefficient of absorption.

In the foregoing discussion we have taken as the measured intensity, the intensity determined by the secondaries. We may regard each primary as accompanied by a number of secondaries equal to the product of the number emitted per centimeter of path and the range of the secondaries. It is this quantity which, in the foregoing theory, determines the contribution of a single primary to the measured intensity. The writer has given reasons for supposing that the primaries may not ionize at all until their energy has diminished to a sufficiently low value;[17] and, if such is the case, the theory as we have stated it requires no modification. If the primaries do ionize, then the contribution given, on the foregoing theory, to the number of measurable rays passing through a plane perpendicular to the path of a primary and attributable to that primary, should be increased by unity. This does not disturb the essentials of the theory, however.

It is of interest to note that if q is the number of secondaries produced per centimeter of path by a primary, and if each secondary has an energy of 10^8 electron volts, the loss of energy per centimeter of path would be $q\times 10^8$. On the other hand, the loss of energy per centimeter of path at a place where the primary energy is E, is, according to the elementary theory, equal to μE, where μ is the absorption coefficient. Estimating all lengths in terms of centimeters of air compressed to the density of water, we have $\mu=0.005$. Hence, $q\times 10^8=0.005E$. If E is of the order of 10^{10} electron volts, $q=0.5$. The range of 10^8-volt secondaries is of the order 20 cm in air compressed to the density of water. Hence the number of electrons accompanying each primary at the place where the primary energy is 10^{10} volts is of the order $20\times 0.5=10$. The question of whether the primary is, or is not, added becomes therefore of small importance; although the importance is increased for small primary energies.

Any difficulty concerned with the participation of the primary in direct ionization may be removed by a modification of the theory in which what we have above regarded as the primaries are really photons in their journey through the atmosphere. We must suppose that these photons receive their energies and their directional characteristics in their creation from the real charged particle primaries by impact of the latter with atoms of air in the higher regions of the atmosphere. If then we impute to the photons the same characteristics of shower production, energy loss, etc., as we have imputed to the primaries in the foregoing discussion, the whole theory already given follows again in all its essential details. As the photons lose energy, their frequencies of course change.

It is not my purpose to attempt to fix too definitely at this stage, the exact forms of the details of the various elements of the corpuscular theory here presented. It will suffice to say that, within the general spirit of the ideas outlined in the elementary form of the theory, it is possible to make modifications which will include all the experimental facts concerned with absorption in the atmosphere, and with the latitude and directional effects. By the further hypothesis already cited, to the effect that, in the case of the heavy atom elements, frequency of shower production and the like depend upon a higher power of the primary energy than the first, we are able to correlate with the theory the experiments already cited in relation to such phenomena, together with a number of other experiments relating shower production to altitude, and to the characteristics of the primary rays as determined by their direction, or by their behavior in the matter of asymmetry.

[17] W. F. G. Swann, Phys. Rev. **43**, 945 (1933); also Phys. Rev. **46**, 432 (1934).