$\epsilon$ is the difference of energy between the beginning and end states of the nucleus in units $m c^{2}$. $A_{1}$ and $A_{2}$ are given by (8); they contain the dependence of the process on the energy levels of the intermediate nucleus. It turns out, however, that the values of those energies do not greatly influence the probability. Roughly one can put

$$
A_{1}+A_{2} \simeq\left|\frac{H_{n k} H_{k m}}{W_{k}-W_{n}}\right|^{2} \sim \frac{g^{4}}{m^{2} c^{4}}
$$

with $g$ given by (5). Numerical evaluation for $Z=31$ leads to

$$
\begin{align*}
P & =1.15 \times 10^{-35} F(\epsilon-2) \mathrm{sec}^{-1} \\
& =3.6 \times 10^{-28} F(\epsilon-2) \mathrm{year}^{-1} \tag{12}
\end{align*}
$$

As seen from the general formula (11) $P$ is almost independent of $Z$.

The value of $F$ for some arguments is given in the following table:
$\left.\begin{array}{lcccc} & \stackrel{4}{\epsilon} & \stackrel{6}{6} & \stackrel{8}{4} & 10 \\ F(\epsilon+2) & = & 0.37 \times 10^{2} & 9.2 \times 10^{4} & 3.4 \times 10^{6}\end{array}\right) 4.2 \times 10^{7}$

The author wishes to express her gratitude to Professor E. Wigner for suggesting this problem, and for the interest taken in it.

# The Infrared Absorption Spectra of the Linear Molecules Carbonyl Sulphide and Deuterium Cyanide 

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#### Abstract

Carbonyl sulphide. The discovery of new vibrationrotation bands in the infrared absorption spectrum of carbonyl sulphide has made it possible to determine the vibrational energy level scheme of the molecule. The agreement between theory and experiment is quite satisfactory. Deuterium cyanide. The fundamental bands of deuterium cyanide $\omega_{2}$ at $570 \mathrm{~cm}^{-1}$ and $\omega_{3}$ at $2630 \mathrm{~cm}^{-1}$ have been measured with grating spectrometers. The former has a strong zero branch at $570.16 \mathrm{~cm}^{-1}$, and the fine structure


THE infrared absorption spectrum of carbonyl sulphide (COS) has been investigated by Cassie and Bailey ${ }^{1}$ who found ten unresolved bands in the region $1-20 \mu$ using a prism spectrometer. Vegard ${ }^{2}$ has deduced from x-ray measurements that the molecule is linear with interatomic separations $\mathrm{C}-\mathrm{O}$ of 1.10 Angstrom units and C-S of 1.96 Angstrom units. The calculated moment of inertia is $178 \times 10^{-40}$ gram $\mathrm{cm}^{2}$.

The perturbed expression for the vibrational energy is

$$
\begin{aligned}
& E(\text { vibration })=\nu_{1} V_{1}+\nu_{2} V_{2}+\nu_{3} V_{3}+X_{11} V_{1}^{2} \\
& \quad+X_{22} V_{2}^{2}+X_{33} V_{3}^{2}+X_{12} V_{1} V_{2}+X_{13} V_{1} V_{3} \\
& \quad+X_{23} V_{2} V_{3}+X_{l l} l^{2}+\text { constant }
\end{aligned}
$$

${ }^{1}$ Bailey and Cassie, Proc. Roy. Soc. A135, 375 (1932).
${ }^{2}$ Vegard, Zeits. f. Krist. 77, 411 (1931).
lines on each side are well separated. From the line spacing the moment of inertia of the molecule is found to be $22.92 \times 10^{-40} \mathrm{gram} \mathrm{cm}^{2}$. A comparison of this value with the corresponding one for HCN , i.e., $18.72 \times 10^{-40} \mathrm{gram}$ $\mathrm{cm}^{2}$, permits the calculation of the internuclear distances. From the positions of these two bands, together with those of the observed fundamentals of HCN , the zeroth order quadratic potential energy expression is computed.
where the $V$ 's are the vibrational quantum numbers, $l$ is the azimuthal quantum number, and $\nu_{1}$ to $X_{l l}$ are constants. If ten bands which involve these constants in an independent way are located experimentally the ten constants may be calculated and the energy level system for the molecule determined. This is of fundamental importance because it provides the correlating network and serves to predict new absorption bands.

This problem is similar to that of HCN which has been developed through a number of experimental researches to a fairly complete solution. ${ }^{3,4,5,6,7,8}$ Recently Herzberg and Spinks ${ }^{9}$

[^0]have photographed two HCN bands at $\lambda 10,385$ and $\lambda 11,645$ under high resolution. The moment of inertia obtained from the fine structure observed is $18.70 \times 10^{-40}$ gram $\mathrm{cm}^{2}$. By assuming the $\mathrm{C}-\mathrm{H}$ distance to be the same as in acetylene, i.e., 1.06 Angstrom units, they found the atomic distance $\mathrm{C}-\mathrm{N}$ to be 1.15 Angstrom units. These dimensions may be determined in a more reliable manner by comparing the spectra of HCN and DCN. We have observed two of the fundamental bands of the latter molecule, and have completely resolved one of them, thus obtaining a precise measure of the moment of inertia.

Adel ${ }^{10}$ gives an analysis of vibrational isotope effects in three particle systems showing how the infrared spectra of two or more isotopic molecules can be applied to the analysis of their quadratic potential energy function making the assumption that the molecular force field is invariant under an isotopic change of mass. We are now able to compute these potential constants for HCN and DCN.

Three infrared spectrometers were available. The first, which is equipped with a rocksalt prism and gratings having 2400 and 4800 lines
${ }^{10}$ Adel, Phys. Rev. 46, 222 (1934).
per inch has been described by Barker and Meyer. ${ }^{11}$ It was used for the region $1-10 \mu$. The second is a recording instrument with a KBr prism set up by Randall and Strong ${ }^{12}$ and is particularly adapted for rapid exploration out to $20 \mu$. The third spectrometer has a grating with 1200 lines per inch and a KBr foreprism, the optical system having been described by Hardy. ${ }^{13}$ It was used for analyzing the bands at $17.5 \mu$ and $19 \mu$.
Two absorption cells were used. The first is twenty cm in length with KBr windows, and the second is twenty-five cm in length with windows of rocksalt.

Carbonyl sulphide was prepared by treating $50 \mathrm{~cm}^{3}$ of concentrated water solution of KSCN with a cold mixture of 520 grams of concentrated $\mathrm{H}_{2} \mathrm{SO}_{4}$ and $400 \mathrm{~cm}^{3}$ of water. The gas COS is given off at $25^{\circ} \mathrm{C}$ (contaminated with a few impurities in slight amount namely $\mathrm{CO}_{2}, \mathrm{CS}_{2}$ and $\mathrm{H}_{2} \mathrm{~S}$ ) according to the reaction; ; ${ }^{14} \mathrm{KSCN}$ $+\mathrm{H}_{2} \mathrm{O}+2 \mathrm{H}_{2} \mathrm{SO}_{4} \rightarrow \mathrm{COS}+\mathrm{KHSO}_{4}+\mathrm{NH}_{4} \mathrm{HSO}_{4}$.

[^1]Table I. Observed bands of carbonyl sulphide.

| Observed by <br> Barker \& Bartunek |  |  |  | Observed by Cassie \& Bailey |  |  |  |  |  | Raman lines observed by Dadieu \& Kohlrausch |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Band } \\ & \text { No. } \end{aligned}$ | $\left(\mathrm{cm}^{-1}\right)$ | $\begin{aligned} & \text { Band } \\ & \text { No. } \end{aligned}$ | $\left(\mathrm{cm}^{-1}\right)$ | $\begin{aligned} & \text { Band } \\ & \text { No. } \end{aligned}$ | ( $\mathrm{cm}^{-1}$ ) | Assignment | $\begin{aligned} & \text { BAND } \\ & \text { No. } \end{aligned}$ | $\left(\mathrm{cm}^{-1}\right)$ | Assignment | $\begin{gathered} \text { Line } \\ \text { No. } \end{gathered}$ | ( $\mathrm{cm}^{-1}$ ) | Assignment |
| 1 | 4101 | 9 | 1558.6 | 1 | 4084 | $2 \omega_{3}$ |  | - | $3 \omega_{2}$ | 1 | 2233 | $2 \omega_{1}+\omega_{2}$ |
| 2 | 3739 | 10 | 1530.4 | 2 | 3742 | $2 \omega_{1}+\omega_{3}$ | 10 |  | $\omega_{3}-\omega_{2}$ | 2 | 2055 |  |
| 3 | 3096 | 11 | 1047.4 | 3 | 3095 | $2 \omega_{2}+\omega_{3}$ | 11 | 1051 | $2 \omega_{2}$ | 3 | 1383 | $\omega_{1}+\omega_{2}$ |
| 4 | 2919 | 12 | 859.2 | 4 | 2904 | $\omega_{1}+\omega_{3}$ | 12 | 859 |  | 4 | 1041 | $2 \omega_{2}$ |
| 5 | 2575 | 13 | 526.6 | 5 |  | $\omega_{2}+\omega_{3}$ | 13 |  | $1_{1} \rightarrow 22$ | 5 | 859 |  |
| 6 | 2050.5 | 14 | 521.5 | 6 | 2079 |  | 14 | 527 |  | 6 | 678 | $2 \omega_{1}-2 \omega_{2}$ |
| 7 | 1892 | 15 | 514.3 | 8 | 1898 | ${ }_{2} \omega_{2}+\omega_{1}$ | 15 | - | $1_{1} \rightarrow 20$ | 7 | 524 | $\omega_{2}$ |
| 8 | 1710 |  |  | 8 | 1718 | $2 \omega_{1}$ |  |  |  |  |  |  |

Table II. Carbonyl sulphide bands.

| Observed | Calculated | Assignment | Observed . | Calculated | Assignment |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4101* | 4101 | $2 \omega_{3}$ | 514.3 | 516 | $1_{1} \rightarrow 20$ |
| 3739 | 3779 | $2 \omega_{1}+\omega_{3}$ |  | Raman lines |  |
| 3096 | 3094 | $2 \omega_{2}+\omega_{3}$ | 2233 | 2227 | $2 \omega_{1}+\omega_{2}$ |
| 2919** | 2919 | $\omega_{1}+\omega_{3}$ | 2055 | 2050.5 |  |
| 2575* | 2575 | $\omega_{2}+\omega_{3}$ | 1383 | 1380.7 | $\omega_{1}+\omega_{2}$ |
| 2050.5* | 2050.5 | $\omega_{3}$ | 1041 | 1037.5 |  |
| 1892* | 1892 | $2 \omega_{2}+\omega_{1}$ | 859 | 859.2 | ${ }_{2}^{\omega_{1}}{ }_{1}-2 \omega_{2}$ |
| ${ }^{1710} 158.6^{*}$ | 1710 1558.6 | ${ }_{3}^{2} \omega_{1}$ | 678 524 | 672.5 521.5 | ${ }_{\omega} \omega_{1}-2 \omega_{2}$ |
| 1530.4 | 1529 | $\omega_{3}-\omega_{2}$ |  | Constants |  |
| 1047.4 | 1037.5 | $2 \omega_{2}$ | $\nu_{1}=863.4$ | $X_{11}=-4.2 \quad X_{12}=-2.35$ | $X_{l l}=2.65$ |
| 859.2* | 859.2 | $\omega_{1}$ | $\nu_{2}=518.95$ | $X_{22}=-.10 \quad X_{13}=9.3$ |  |
| 526.6* | 526.6 | $1_{1} \rightarrow 2{ }_{2}$ | $\nu_{3}=2050.5$ | $X_{33}=0 \quad X_{23}=3$. |  |
| 521.5* | 521.5 |  |  |  |  |













Fig. 1. Carbonyl sulphide bands and the deuterium cyanide band $\omega^{3}$.

The products were passed from the generating flask through a trap immersed in a mixture of ice and salt which condenses the greater part of the $\mathrm{CS}_{2}$, then through a 33 percent solution of KOH to precipitate $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{~S}$, and finally through traps containing $\mathrm{CaCl}_{2}$ and $\mathrm{P}_{2} \mathrm{O}_{5}$. The dry gas was then condensed with liquid air and stored in a five liter reservoir.
The deuterium cyanide was supplied by the California Isotope Company in a small glass capsule containing a mixture of about 40 percent DCN and 60 percent HCN.
The readings were taken in the usual in and out method for all the bands with the exception of the $17.5 \mu$ band of DCN, the $14 \mu$ band of HCN and the $19 \mu$ region of COS. In these cases the cell was in the beam permanently during the time deflections were taken. It was then removed and the "background" mapped on the same day. Care was always taken of course to insure a pure spectrum. The Nernst glower was used as a source of continuous radiation. The calibration of the spectrometers is in terms of the water vapor lines in the region of $6 \mu$.

The results for carbonyl sulphide are shown in Fig. 1 and Tables I and II. Table I gives the observed bands in terms of wave numbers with
the bands observed by Cassie and Bailey ${ }^{1}$ for comparison. At the end of the table the Raman lines observed by Dadieu and Kohlrausch ${ }^{15}$ are listed.
The $2.67 \mu$ band is not reproduced. It is very irregular due to superposition of the $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{~S}$ bands in the same region since it is impossible to remove every trace of these substances from the gas. It is extremely difficult to judge the position of the center but it seems to be at $3739 \mathrm{~cm}^{-1}$. Cassie and Bailey ${ }^{1}$ found a band in this region which they list as $3742 \mathrm{~cm}^{-1}$. This band was not used in calculating the constants. The two bands $1558.5 \mathrm{~cm}^{-1}$ and $1530 \mathrm{~cm}^{-1}$ are also not reproduced. Both have sharp zero branches similar to the $2575 \mathrm{~cm}^{-1}$ band. Two of the three zero branches in the $19 \mu$ region are upper stage transitions from the first excited state to the second excited states. These are clearly shown on the photographic record, Fig. 1. The proportion of molecules in the first excited state at room temperature is 17 percent which is sufficient to account for the upper stage absorption.
Table II gives the observed and calculated positions of the infrared bands and Raman lines,

[^2]

Fig. 2. Rotation lines of the $17.5 \mu$ region of deuterium cyanide.
the calculated values of the ten constants being obtained from the bands marked with asterisks. These constants are given at the end of the table.

The results for deuterium cyanide are shown in Fig. 2 indicating the fine structure of the $17.5 \mu$ region; in Fig. 1 where the fundamental band $\omega_{3}$ appears; and in Tables III and V. Table III gives the observed and calculated positions of the rotation lines of DCN while Table IV gives similar results for HCN. The
calculated values in these tables were obtained from the formula $\nu=\nu_{0}+A m+B m^{2}+C m^{3}$ the constants $\nu_{0}, A, B$ and $C$ being obtained empirically. The ordinal number " $m$ " is counted from the band center. The constants are for DCN $\nu_{0}=569.10 ; A=2.425 ; B=0.0013 ; C=-0.00003$ and for $\mathrm{HCN} \nu_{0}=712.07 ; A=2.952 ; B=0 ; C=0$.

The part of the band immediately to the low frequency side of the zero branch is somewhat irregular due to the superposition of the upper stage bands of DCN. The rather sharp intense

Table III. Rotation lines of the $17.5 \mu$ band of $D C N$ zero branch $570.16 \mathrm{~cm}^{-1}$.

| $R$ Branch $\quad \Delta J=+1$ |  |  |  | $P$ Branch $\quad \Delta J=-1$ |  |  | $R$ Branch $\quad \Delta J=+1$ |  |  |  | $P$ Branch $\quad \Delta J=-1$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Obs. | Calc. | Res. | $m$ | Овs. | Calc. | Res. | Obs. | Calc. | Res. | $m$ | Obs. | Calc. | Res. |
|  | 571.53 |  | 1 |  |  |  | 600.76 | 600.78 | -. 02 | 13 | 537.86 | 537.87 | -. 01 |
| 573.74 | 573.96 | -0.22 | 2 | 564.52 | 564.26 | 0.26 | 603.17 | 603.22 | -. 05 | 14 | 535.53 | 535.48 | . 05 |
| 576.28 | 576.39 | -. 11 | 3 | 561.81 | 561.83 | -. 02 | 605.66 | 605.67 | -. 01 | 15 | 533.17 | 533.11 | . 06 |
| 578.84 | 578.82 | . 02 | 4 | 559.41 | 559.42 | -. 01 | 608.05 | 608.11 | -. 06 | 16 | 530.83 | 530.75 | . 08 |
| 581.06 | 581.26 | -. 20 | 5 | 556.98 | 557.00 | -. 02 | 610.45 | 610.54 | -. 09 | 17 | 528.36 | 528.40 | -. 04 |
| 583.67 | 583.69 | -. 02 | 6 | 554.62 | 554.61 | . 01 | 612.94 | 613.00 | -. 06 | 18 | 526.02 | 526.04 | -. 02 |
| 586.25 | 586.13 | . 12 | 7 | 552.29 | 552.19 | . 10 | 615.60 | 615.44 | . 16 | 19 | 523.86 | 523.71 | . 15 |
| 588.59 | 588.56 | . 03 | 8 | 549.98 | 549.80 | . 18 | 617.99 | 617.88 | . 11 | 20 | 521.51 | 521.36 | . 15 |
| 591.15 | 591.02 | . 13 | 9 | 547.69 | 547.41 | . 18 | 620.56 | 620.32 | . 24 | 21 |  |  |  |
| 593.60 | 593.45 | . 15 | 10 | 545.10 | 545.01 | . 09 | 622.71 | 622.76 | -. 05 | 22 |  |  |  |
| 595.94 | 595.90 | . 04 | 11 | 542.59 | 542.63 | -. 04 | 624.88 | 625.21 | -. 33 | 23 | - | - | - |
| 598.31 | 598.34 | -. 03 | 12 | 540.37 | 540.24 | . 13 |  |  |  |  |  |  |  |

Table IV. Rotation lines of the $14 \mu$ band of $H C N$ zero branch $712.28 \mathrm{~cm}^{-1}$.

| $R$ Branch $\quad \Delta J=+1$ |  |  |  | $P$ Branch $\quad \Delta J=-1$ |  |  | $R$ Branch $\quad \Delta J=+1$ |  |  |  | $P$ Branch $\quad \Delta J=-1$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Obs. | Calc. | Res. | $m$ | Obs. | Calc. | Res. | Obs. | Calc. | Res. | $m$ | Obs. | Calc. | Res. |
| - | 715.02 | - | 1. |  |  | - | 753.35 | 753.40 | -. 05 | 14 | 670.69 | 670.74 | -. 05 |
| 717.86 | 717.97 | -0.11 | 2 | 706.33 | 706.17 | 0.16 | 756.29 | 756.35 | $-.06$ | 15 | 667.66 | 667.79 | $-.13$ |
| 720.81 | 720.93 | -. 12 | 3 | 703.23 | 703.21 | . 02 | 759.25 | 759.30 | -. 05 | 16 | 664.66 | 664.84 | $-.18$ |
| 723.78 | 723.88 | $-.10$ | 4 | 699.60 | 700.26 | -. 66 | 762.23 | 762.25 | -. 02 | 17 | 661.69 | 661.89 | -. 20 |
| 726.77 | 726.83 | -. 06 | 5 | 697.25 | 697.31 | -. 06 | 765.24 | 765.21 | . 03 | 18 | 658.75 | 658.93 | $-.18$ |
| 729.80 | 729.78 | . 02 | 6 | 694.24 | 694.36 | -. 12 | 768.28 | 768.16 | . 12 | 19 | 656.08 | 655.98 | . 10 |
| 732.85 | 732.73 | . 12 | 7 | 691.38 | 691.41 | $-.03$ | 770.99 | 771.11 | $-.12$ | 20 | 653.19 | 653.03 | . 16 |
| 735.93 | 735.69 | . 24 | 8 | 688.42 | 688.45 | $-.03$ | 774.07 | 774.06 | . 01 | 21 | 650.33 | 650.08 | . 25 |
| 738.72 | 738.64 | . 08 | 9 | 685.48 | 685.50 | -. 02 | 777.18 | 777.01 | . 17 | 22 | 647.02 | 647.13 | -. 11 |
| 741.85 | 741.59 | . 26 | 10 | 682.57 | 682.55 | . 02 | 779.97 | 779.97 | . 00 | 23 | - | 644.17 | - |
| 744.69 | 744.54 | . 15 | 11 | 679.69 | 679.60 | . 09 | 782.78 | 782.92 | -. 14 | 24 | - | 641.22 | - |
| 747.55 | 747.49 | . 06 | 12 | 676.57 | 676.65 | $-.08$ | 785.84 | 785.87 | $-.03$ | 25 | - | 638.27 | - |
| 750.44 | 750.45 | $-.01$ | 13 | 673.49 | 673.69 | $-.20$ | 788.81 | 788.82 | $-.01$ | 26 | - | - | - |

Table V. Average separations of the rotation lines.

| $\begin{gathered} \text { DCN } 17.5 \mu \text { BAND } \\ \underline{R(m)-P(m)} \end{gathered}$ |  | $\begin{gathered} \mathrm{HCN} 14 \mu \text { BAND } \\ R(m)-P(m) \end{gathered}$ | $\begin{gathered} \text { DCN } 17.5 \mu \text { Band } \\ \underline{R(m)-P(m)} \end{gathered}$ |  | $\begin{gathered} \text { HCN } 14 \mu \text { BaND } \\ \underline{R(m)-P(m)} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $2 m$ | $m$ | $2 m$ | $2 m$ | $m$ | $2 m$ |
| $2.305 \mathrm{~cm}^{-1}$ | 2 | $2.883 \mathrm{~cm}^{-1}$ | 2.416 | 14 | 2.952 |
| 2.412 | 3 | 2.930 | 2.416 | 15 | 2.954 |
| 2.429 | 4 | 3.023 | 2.413 | 16 | 2.956 |
| 2.408 | 5 | 2.952 | 2.414 | 17 | 2.957 |
| 2.421 | 6 | 2.973 | 2.414 | 18 | 2.958 |
| 2.426 | 7 | 2.962 | 2.414 | 19 | 2.953 |
| 2.413 | 8 | 2.969 | 2.412 | 20 | 2.954 |
| 2.414 | 9 | 2.958 |  | 21 | 2.946 |
| 2.425 | 10 | 2.964 |  | 22 | 2.958 |
| 2.425 | 11 | 2.955 | Average $2.414 \mathrm{~cm}^{-1}$ |  | Average $2.955 \mathrm{~cm}^{-1}$ |
| 2.414 2.419 | ${ }_{13}^{12}$ | 2.958 2.959 | $I=22.92 \times 10^{-40} \mathrm{~g} \mathrm{~cm}^{2}$ |  | $I=18.72 \times 10^{-40} \mathrm{~g} \mathrm{~cm}^{2}$ |
| 2.419 | 13 | 2.959 |  |  |  |

line at $561.07 \mathrm{~cm}^{-1}$ is probably the zero branch of the transition $1_{1} \rightarrow 2_{0}$ where the subscript indicates the $l$ value. The upper stage transition $1_{1} \rightarrow 2_{2}$ probably lies somewhere within the zero branch of that from the ground state $0_{0} \rightarrow 1_{1}$. At room temperature about 14 percent of the molecules are in the first excited state. This is sufficient to account for the presence of upper stage zero branches.

Table V gives the average separations of the lines from which the moment of inertia about the center of gravity is calculated. Values of $(R(m)-P(m)) / 2 m$ are given for both HCN and DCN, with the calculated moments of inertia.

A search was made for the other two fundamentals $\omega_{1}$ and $\omega_{3}$ as well as for harmonics. Because of the rather small amount of gas available the only other band observed was $\omega_{3}$ at $2630 \mathrm{~cm}^{-1}$.

In the case of HCN because of the strong absorption of carbon dioxide in the background, readings could not be taken for a short space as the deflections dropped to zero. Because of the regularity of the lines the missing line was interpolated and found to agree very well with the calculated value.

The expression for the moment of inertia of this linear triatomic molecule about its center of gravity is

$$
\begin{aligned}
& I=m_{1} x^{2}+m_{2} y^{2} \\
& \quad-\frac{1}{m_{1}+m_{2}+m_{3}}\left(m_{1}^{2} x^{2}+m_{2}^{2} y^{2}-2 m_{1} m_{2} x y\right)
\end{aligned}
$$

where $x$ is the $\mathrm{H}-\mathrm{C}$ distance and $y$ is the $\mathrm{C}-\mathrm{N}$ distance. $m_{1}, m_{2}$ and $m_{3}$ are the masses of the hydrogen, nitrogen and carbon atoms, respectively. Introducing the isotopic masses $m_{1}$ and
$m_{1}{ }^{\prime}$ and solving the two equations simultaneously we obtain the interatomic separations, $x=1.06 \times 10^{-8} \mathrm{~cm}$ and $y=1.15 \times 10^{-8} \mathrm{~cm}$.

The theory of small oscillations may be applied, considering the molecule as a group of uncoupled simple harmonic oscillators of which two are linear and one is plane isotropic in a plane perpendicular to the axis of the molecule. The normal modes of vibration are well known. We shall use coordinates somewhat different from those defined by Adel ${ }^{10}$ in order to preserve the same meaning of these coordinates for both isotopes. Let $r$ represent the distance from the central atom to the line joining the end atoms measured parallel to the displacement of the particles in the perpendicular vibration. Let $q$ measure the relative separation of the hydrogen and carbon atoms, and $z$ measure the relative displacement of the carbon and nitrogen atoms. In the perpendicular vibration let $y_{1}, y_{2}$ and $y_{3}$ represent the displacements of the hydrogen, nitrogen and carbon atoms. Then the kinetic energy is

$$
T=\frac{1}{2} m_{1} \dot{y}_{1}{ }^{2}+\frac{1}{2} m_{2} \dot{y}_{2}{ }^{2}+\frac{1}{2} m_{3} \dot{y}_{3}{ }^{2} .
$$

The expression for $r$ in terms of $y_{1}, y_{2}$ and $y_{3}$ and the internuclear distances is

$$
r=y_{1} y / r_{0}+y_{2} x / r_{0}-y_{3} \quad \text { where } \quad r_{0}=x+y .
$$

Since the momentum is conserved as well as the moments of momenta about any point one has two additional relations

$$
m_{1} \dot{y}_{1}+m_{2} \dot{y}_{2}+m_{3} \dot{y}_{3}=0 \text { and } m_{1} \dot{y}_{1} x-m_{2} \dot{y}_{2} y=0 .
$$

In terms of $r$ the kinetic energy becomes $T=\frac{1}{2} \mu_{r} \dot{r}^{2}$ where

$$
\mu_{r}=1 /\left[\frac{1}{m_{2}}\left(\frac{x}{r_{0}}\right)^{2}+\frac{1}{m_{1}}\left(\frac{y}{r_{0}}\right)^{2}+\frac{1}{m_{3}}\right]
$$

For the longitudinal vibrations $x_{1}, x_{2}$ and $x_{3}$ represent the displacements from equilibrium of the hydrogen, nitrogen, and carbon atoms, respectively. Then

$$
q=x_{3}-x_{1} ; \quad z=x_{2}-x_{3}
$$

and

$$
m_{1} \dot{x}_{1}+m_{2} \dot{x}_{2}+m_{3} \dot{x}_{3}=0
$$

In this case the kinetic energy is $T=\frac{1}{2} m_{1} \dot{x}_{1}{ }^{2}$ $+\frac{1}{2} m_{2} \dot{x}_{2}{ }^{2}+\frac{1}{2} m_{3} \dot{x}_{3}{ }^{2}$ or in terms of $\dot{q}$ and $\dot{z}, T=\frac{1}{2} \mu_{q} \dot{q}^{2}$ $+\frac{1}{2} \mu_{z} \dot{z}^{2}+\frac{1}{2} \cdot 2 \mu_{q z} \dot{q} \dot{z}$ where
$\mu_{q}=\frac{m_{1}\left(m_{2}+m_{3}\right)}{S} ; \quad \mu_{z}=\frac{m_{2}\left(m_{1}+m_{3}\right)}{S} ; \quad \mu_{q z}=\frac{m_{1} m_{2}}{S}$
and $S=m_{1}+m_{2}+m_{3}$, the total mass of the molecule. The potential energy is subject to the geometric symmetry of the molecule. It may be written $\quad V=\frac{1}{2}\left(k_{11} q^{2}+k_{22} r^{2}+2 k_{13} q z+k_{33} z^{2}\right)$. The secular determinant of the normal coordinate transformation factors into a linear and a quadratic expression, and yields the following set of equations,

$$
\begin{aligned}
\lambda_{1}+\lambda_{3} & =\frac{\mu_{q} k_{33}+\mu_{z} k_{11}-2 \mu_{q z} k_{13}}{\mu_{q} \mu_{z}-\mu_{q z}^{2}} \\
\lambda_{1} \lambda_{3} & =\frac{k_{11} k_{33}-k_{13}^{2}}{\mu_{q} \mu_{z}-\mu_{q z}^{2}} \\
\lambda_{2} & =k_{22} / \mu_{r}
\end{aligned}
$$

In normal coordinates the kinetic and potential energies assume the forms

$$
T=\frac{1}{2}\left(\dot{Q}^{2}+\dot{R}^{2}+\dot{Z}^{2}\right), \quad V=\frac{1}{2}\left(\lambda_{1} Q^{2}+\lambda_{2} R^{2}+\lambda_{3} Z^{2}\right)
$$

The normal vibrations expressed in wave numbers are $\nu_{i}=(1 / 2 \pi c)\left(\lambda_{i}\right)^{\frac{1}{2}}$. The constant $k_{22}$ may be computed from $\omega_{2}$ of HCN and DCN and is nearly the same for both as would be expected. Using the fundamentals $\omega_{1}$ and $\omega_{3}$ of HCN and the fundamental $\omega_{3}{ }^{\prime}$ of DCN the unobserved fundamental $\omega_{1}^{\prime}$ of DCN can be calculated as well as the other three constants $k_{11}, k_{13}$ and $k_{33}$. There are two sets of constants because the equations are quadratic. These are

| $k_{22}=0.8071 \times 10^{5}$ |  | $k_{22}=$ | $0.8071 \times$ | $\times 10^{5}$ | dynes/cm |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $k_{11}=5.6987$ " | Or | $k_{11}=$ | 5.6981 | " |  |
| $k_{13}=6.443$ |  | $k_{13}=$ | -0.30491 | " |  |
| $k_{33}=25.34$ |  | $k_{33}=$ | 18.065 | " |  |

$k_{13}$ represents the interaction between the extreme atoms, and should be small. Hence the second set is the more plausible.

The observed and calculated values of the wave numbers for the fundamental bands of HCN and DCN are as follows:

$$
\begin{aligned}
& \text { Observed wave numbers } \\
& \qquad \begin{array}{l}
\omega_{1}=2089.0 \mathrm{~cm}^{-1} \\
\omega_{2}=712.1 \\
\omega_{3}=3312.9 \\
\omega_{1}^{1}=- \\
\omega_{2}{ }^{1}=569.1 \\
\omega_{3}{ }^{1}=2630.0
\end{array}
\end{aligned}
$$

## Absorption Spectrum of Tin Sulphide

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#### Abstract

The absorption spectrum of tin sulphide between 3200A and 4600 A has been photographed at moderate dispersion and between 4000 A and 4600 A at high dispersion. The system in the near ultraviolet may be represented by the formula: $$
\begin{aligned} \nu=28,336.2+332.77\left(v^{\prime}+\frac{1}{2}\right) & -1.42\left(v^{\prime}+\frac{1}{2}\right)^{2} \\ & -488.25\left(v^{\prime \prime}+\frac{1}{2}\right)+1.47\left(v^{\prime \prime}+\frac{1}{2}\right)^{2} . \end{aligned}
$$


This formula is not in agreement with that given by

Butkow and Tschassowenny who seem to have incorrectly assigned the system origin. A new system has been located in the visible which is limited by predissociation to two $v^{\prime \prime}$ progressions. These bands are given by the formula:
$\nu=23,287.67+367.5\left(v^{\prime}+\frac{1}{2}\right)-488.25\left(v^{\prime \prime}+\frac{1}{2}\right)+1.47\left(v^{\prime \prime}+\frac{1}{2}\right)^{2}$.
The observed isotope structure of the strong bands in this system confirms their vibrational assignment.


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