TABLE VIII. *Wavelengths of satellites.*

ELEMENT	ORDER		λ
		β' LINE	
		$\lambda\beta'-\lambda\beta_{1,3}$	
Mn	1−2	4.0 X.U.	
	1+1	4.2	1.9101A
	1+2	3.2	
Fe	1−2	3.0	1.7560
	1+1	3.1	
Co	1−2	2.2	
	1+1	2.2	1.6197
	1+2	2.2	
Ni	None		
Cu	1−2	1.4	1.3907
	1+1	1.3	
	1+2	1.3	
Zn	1+1	1.2	1.2938
	1+2	1.2	
Ga	1+1	1.0	1.2064
	1+2	1.0	
		β'' LINE	
		$\lambda\beta_{1,3}-\lambda\beta''$	
Cu	1+2	1.0 X.U.	1.3884A
Zn	1+1	.62	1.2920
	1+2	.59	
Ga	1+1	.59	1.2048
	1+2	.63	
Ge	1−2	.62	1.1260
	1+1	.61	
	1+2	.59	
		β''' LINE	
		$\lambda\beta_5-\lambda\beta'''$	
Mn	1+1	2.0 X.U.	1.8912A
Fe		1.9	1.7386
Co		1.8	1.6038
Ni		1.2	1.4843
Cu		1.5	1.3768
		$\lambda\beta_2-\lambda\beta'''$	
Zn		1.6	1.2795
Ga		None	
Ge		1.4	1.1133

fact that these lines are very asymmetrical and hence on a photographic plate the center of gravity may not coincide with the position of maximum intensity. The $K\beta_2$ line disappears abruptly at Cu 29, at least so far as these experiments show. In any event, it would be so close to $K\beta_5$ for atomic numbers less than that of Cu that the two could not be resolved.

Because of the large width and the fact that they are usually superimposed on the side of a stronger line the positions of the various satellites were not determined very accurately. Some check is possible where the satellites were observed in more than one order. Table VIII gives the wavelength data for these satellites in decreasing order of wavelength.

We are indebted to Drs. L. G. Parratt and L. P. Smith for helpful discussions and to Dr. Lewis V. Judson of the Bureau of Standards for valuable aid in the calibration of the divided circle. The Geneva Society spectrometer was a generous donation by Dr. A. L. Loomis of Tuxedo Park. The samples of pure Va and Ti metals were kindly supplied by the Vanadium Corporation of America and the Titanium Alloy Manufacturing Company.

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A Measurement of the Absolute Probability of *K*-Electron Ionization of Silver by Cathode Rays

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The method of Ross's balanced foils was used to separate the $K\alpha$ quanta from the thin silver foil target radiation. The x-ray energy was absorbed in CH_3Br and SO_2 in a standard ionization chamber and the ionization currents measured by means of a Compton electrometer and calibrated ionization system. Stockmeyer's value for the energy per ion pair for these gases was used to compute the number of quanta absorbed in the chamber. The number of Ag K ionizations per bombarding electron was then calculated, after the necessary target and absorption corrections. Measurements were made on 10 thin targets of average thickness 0.17 micron, and the K-electron ionization cross section for silver for 70.0 kv electrons was determined to be $\Phi(U)=(4.80\pm0.43)\times10^{-23}\ \mathrm{cm}^2$. The measurements of relative probabilities of K electron ionization for silver by Webster, Hansen and Duveneck are normalized at 70.0 kv, ($U=2.75$) by the above value of $\Phi(U)$, and these experimental data compared with classical quantum theory and the wave mechanics theories of Massey, Mohr and Burhop, Soden and Wetzel. The theories of Soden and Wetzel compare more favorably with the experimental data than those of Massey, Mohr and Burhop, or indeed with the classical theory.

I. INTRODUCTION

THE problem of determining the probability of K-electron ionization by electron impact has been studied both theoretically and experimentally by numerous authors. Theoretical solutions have been given for hydrogen,[1] helium,[1, 2] silver[3, 4] and mercury,[3] and the experimental determinations cover hydrogen,[5] helium[6] and silver.[7] The experimental work on silver by

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[1] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. **A140**, 613 (1933).

[2] W. W. Wetzel, Phys. Rev. **44**, 25 (1933).

[3] D. Graf Soden, Ann. d. Physik **19**, 409 (1934).

[4] E. H. S. Burhop and W. W. Wetzel have given solutions as yet unpublished, but discussed in the theoretical section of this paper.

[5] J. T. Tate and P. T. Smith, Phys. Rev. **39**, 270 (1932).

[6] P. T. Smith, Phys. Rev. **36**, 1293 (1930).

[7] D. L. Webster, W. W. Hansen and F. B. Duveneck, Phys. Rev. **43**, 851 (1933).

Webster, Hansen and Duveneck covers measurements of the relative probabilities of K-electron ionization by cathode rays as a function of their kinetic energy. The experiments herewith reported have been performed to determine the absolute probability of K-electron ionization of silver by 70.0 kv cathode rays.

For the case where the probability determination is made by measuring the absolute intensity of characteristic x-rays emitted by a target of solid material, the absolute probability is defined[7] most conveniently as (the probability of ionization) ÷ (the number of atoms per unit area of the target). This identifies it as the cross-section area of each atom effective for ionization, denoted as $\Phi(U)$. Thus

$$\Phi(U)=(N_{KI}/N_E)/nX_0, \tag{1}$$

where N_{KI} is the number of K-electron ionizations produced by N_E electrons striking a target of thickness X_0, the target containing n atoms per cubic centimeter. The number of K ionizations is determined in this experiment in terms of the ionization current produced by absorbing a known fraction of the x-ray quanta in a standard ionization chamber, and the already measured[8] value of the energy required to produce a pair of ions in the absorbing gas used in the ionization chamber.

II. The Balanced Foil Method

When 70.0 kv electrons interact with silver atoms to produce x-rays, both characteristic and continuous x-rays are produced. For effectively separating the $K\alpha$ line energy from the series of higher terms, as well as from the continuous spectrum energy, the method suggested by Ross,[9] using balanced thin metallic foils, was used. This method utilizes thin foils of nearly adjacent elements in the periodic table, whose thicknesses are adjusted so that the absorption curves of each foil exactly coincide except in the region between their K absorption limits. The wide apertures allowable by the balanced foil method, makes possible the measurement of the small number of quanta obtained from the thin film targets which would otherwise be impossible by the usual methods of analysis. Since a large part of the energy of the K series is associated with the $K\alpha$ lines, a region of the spectrum including only these lines is measured and proper corrections are made to include all of the K series energy in the computation of $\Phi(u)$. The balanced foils most suitable for determining the $K\alpha$ line energy are rhodium (45), and masurium (43). However, masurium is not available, so it is necessary to use molybdenum (42). The K limit of Mo is 58 X.U. above, while the Rh K limit is 27 X.U. below that of the wavelength of the silver $K\alpha$ lines. This gives a band of continuous energy 85 X.U. in extent which is measured with the $K\alpha$ line energy. The correction for this continuous energy is treated separately, below.

III. Apparatus

The general assembly of apparatus is shown diagrammatically in Fig. 1. The x-ray tube with removable anode and cathode is entirely enclosed in a lead house, and the x-rays to be measured allowed to escape only through an opening controlled by an electromagnetic shutter. This shutter is actuated by a selector switch and standard clock circuit which permits duplication of exposure-time intervals estimated to be accurate to ±0.05 second. After passing through two thin aluminum windows in the tube, one being 0.040 mm thick on the side of the target to shield electrostatically the thin silver foil, and the other being the tube window 0.075 mm thick, the x-rays pass through air until they encounter the balanced foils. These foils are mounted on a holder which allows either the Mo or the Rh foil to be placed in the x-ray path. The beam is next limited by a circular aperture in a thick lead plate, this opening defining the solid angle of x-ray energy entering the standard ionization chamber from the target. In the chamber the quanta are partially absorbed by either methyl bromide (used during measurements for seven of the ten silver targets), or sulphur dioxide, (used for the remaining three targets), and their intensity measured in terms of the ionization produced. The ionization current is measured by means of a Compton electrometer.

The second ionization chamber shown in Fig. 1 is used with the calcite crystal to determine the

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[8] W. Stockmeyer, Ann. d. Physik **404**, 71 (1932).

[9] P. A. Ross, Phys. Rev. **28**, 425 (1926).

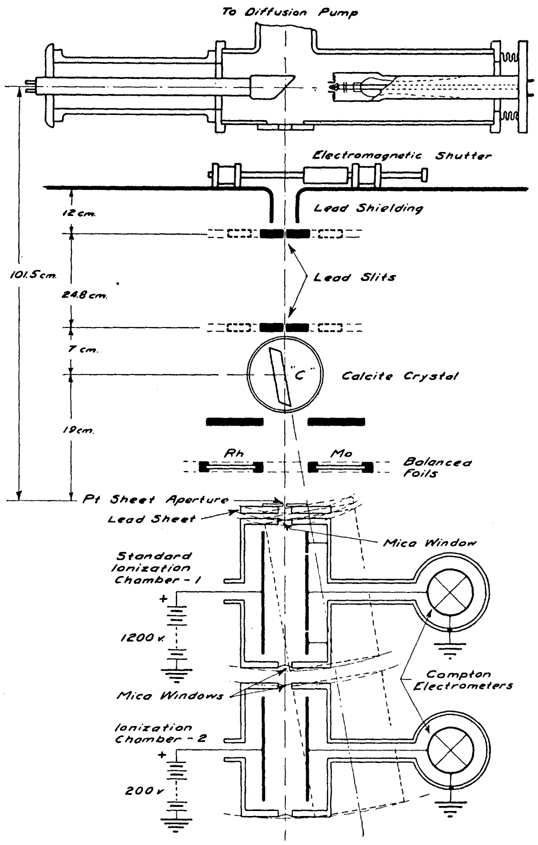

FIG. 1. General assembly of apparatus.

absorption coefficient of the gas in the standard chamber as a function of wavelength. Both chambers are therefore mounted on a movable carrier which rotates about the axis of the crystal "*C*." The crystal is placed in the path only when spectrometer measurements are being made. When the crystal is removed, the standard chamber axis is properly aligned with the x-ray tube window and target.

The high voltage power source used during these experiments[9] utilizes a 500-cycle current, the output voltage of the transformer being rectified and filtered so that the ripple voltage is only 4 volts at 10 milliamperes. The high voltage was measured by means of an electrostatic voltmeter[10] which had just recently been calibrated[11] by means of standard cells and resistors checked by the U. S. Bureau of Standards. During all exposures of x-rays into the standard ionization chamber, as well as for the various correction determinations, the x-ray tube voltage and current were maintained very constant by means of manually operated controls in the high voltage source circuit.

The x-ray tube cathode heater circuit, consisting of the electrical wiring, batteries, meters and various controls, is entirely electrostatically shielded.[11] This shielding prevents the possibility of corona currents being included with the *x*-ray tube current. This precaution is absolutely necessary for this experiment because the corona currents may be many times greater than the feeble tube currents generally employed, namely, 25–60 microamperes. The microammeter used to measure these currents was calibrated in its operating position during each run by means of a standard potentiometer circuit and a standardized 10,000-ohm resistor.

The x-ray tube is shown diagrammatically in Fig. 2a. It will be noticed that the principal section of the tube is made of brass, kept at cathode potential. This central brass tube is insulated from the anode and grounded diffusion pump by means of standard sections of Pyrex glass flanged pipes, two inches in internal diameter. The cathode holder is adjustable from the exterior by means of flexible metal bellows. Pinhole pictures were taken to make sure that the focal spot was in the middle of the target, and that it was sufficiently broad to allow maximum tube currents to be used. Ionization chamber measurements were frequently made to make sure that no x-rays were produced from field currents within the tube. Similar measurements were made to determine the magnitude of the extraneous radiation emitted from places on the target assembly other than the focal spot. These latter measurements were made with normal tube currents flowing, but without the thin silver film mounted on the holder.

The target of the x-ray tube is shown diagrammatically in Fig. 2b. The silver foils used for the target were commercial thin silver leaf, very much like gold leaf, and of average thickness 0.17 micron. These films were strong enough to support their own weight when placed across the end of the aluminum supporting tube, thus elim-

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[10] H. Clark, Rev. Sci. Inst. **1**, 615 (1930).

[11] P. Kirkpatrick and P. A. Ross, Phys. Rev. **45**, 456 (1934).

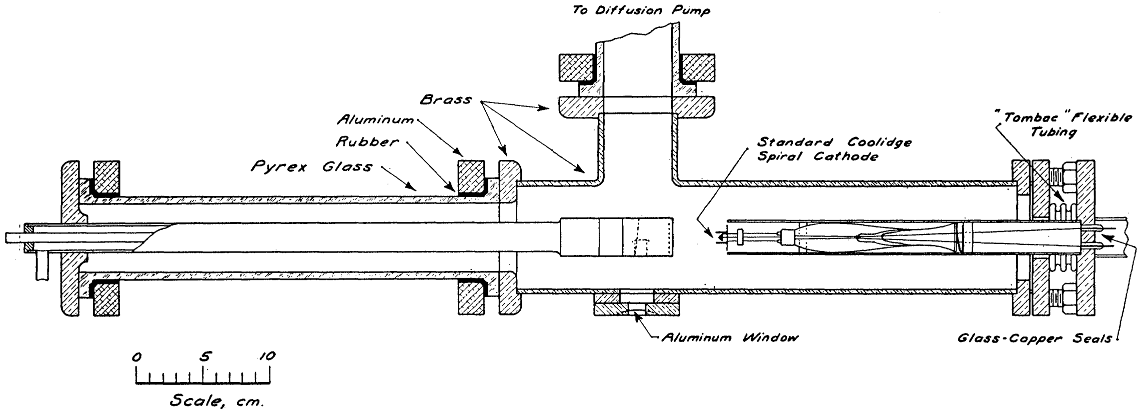

FIG. 2a. X-ray tube.

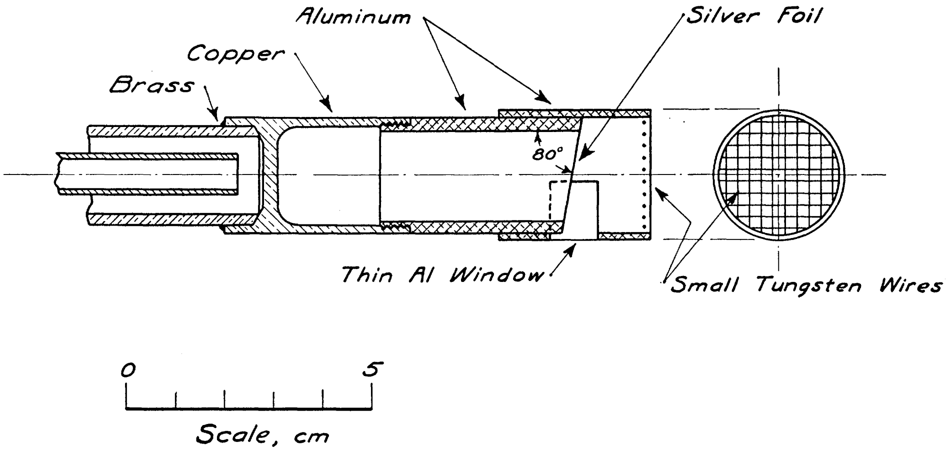

FIG. 2b. Detail of target.

inating the necessity of a backing target. The thin wall aluminum cylinder used to support the film is threaded to a copper rod, which furnishes the back end of the target and which is soldered to a brass supporting tube extending to the anode end of the x-ray tube. The aluminum supporting tube is made long enough so those electrons which pass straight through the film will not experience collisions with any material which is in the optical path of the measuring apparatus. The front end of the aluminum tube is ground off to make an angle very nearly equal to 80 degrees with its longitudinal axis, thus placing the silver foil at the same angle with respect to the direction of the incident electrons from the cathode.

The ionization chamber is shown diagrammatically in Fig. 3. The main cylinder and end plates are made of brass, and the seals are designed so that the gas pressure within the chamber may be greater or less than atmospheric. The standard collector plate, guard plates and potential plate are made of 1/16-inch sheet aluminum. The collector plate is supported between the guard plates by the terminal used for making connection to the electrometer circuit. The terminal is held rigidly in place by an amber bushing, which also insulates it electrically from the grounded chamber walls. The guard electrodes are connected to the grounded chamber, and are mounted in the same plane as the collector electrode. The potential electrode is bent so the electric field between the plates is very nearly uniform, thus causing all of the ions which are produced by the x-rays in the column of gas defined by the collector plate

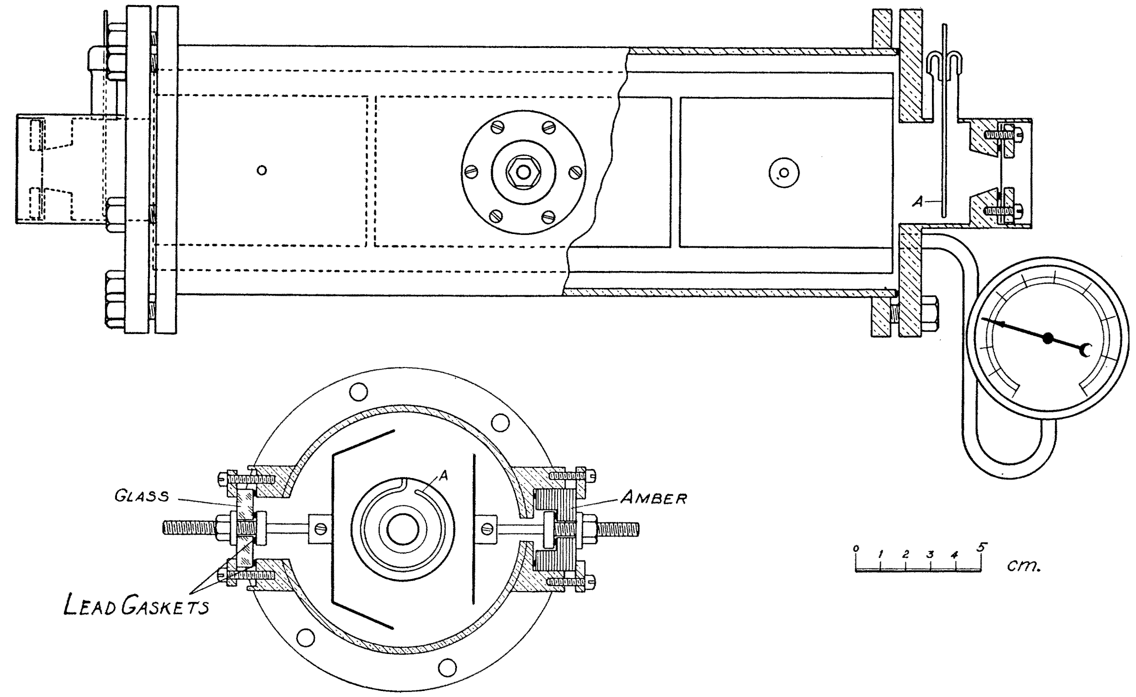

FIG. 3. Standard ionization chamber.

to be carried to electrodes. The nature of the electric field in the region of the collector plate, at right angles to, and along the chamber axis was determined by mapping the equipotential lines. Mapping the field was accomplished by means of a dimensional model of the chamber placed in a tray of water, connecting the proper elements to a 1000-cycle oscillator, and probing the field produced with a telephone head set. The potential and collector plates are placed far enough apart so that photoelectrons produced along the path of the x-rays lose all of their energy to the gas before reaching the plates.

The windows of the ionization chamber are thin sheet aluminum 0.078 mm thick. The ring electrodes, marked A in Fig. 3, at each end of the chamber were maintained at a potential of 200 volts above that of the chamber to collect photoelectrons produced by x-rays passing through the windows. These electrodes were not necessary for this experiment since relatively high pressures were used for both gases. Fuse wire gaskets were used throughout in the assembly of this apparatus.

The ionization system consists of the ionization chamber, an electrically shielded connection between the collector electrode and the electrometer, and the Compton electrometer. The electrometer sensitivity was approximately 4000 mm/volt. The electrical capacity of the ionization system was determined in terms of two standardized cylindrical condensers, using the method described by R. Rinkel.[12] The capacities of the cylindrical condensers, which were exactly similar except for their length, were calculated, and a correction due to the distortion of the electric field at ends of the condensers was determined by a series of measurements as outlined in the above method. The capacity of the ionization system as a function of the electrometer deflection was then determined in terms of the corrected capacities of the standard condensers. A Leeds and Northrup type K potentiometer and a U. S. Bureau of Standards certified standard cell was used during all ionization system calibrations.

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[12] R. Rinkel, Physik. Zeits. **33**, 544 (1932).

IV. Corrections

Target corrections

The electrons, upon entering the target, experience many minor deflections from their straight paths, thus increasing their path length through the film target. Some of these electrons may bend[13] more than 90° and thus be rediffused through the target film, while others may go straight through the film and pass along the hollow supporting tube to the copper base. A part of those striking the copper base may also experience rediffusion and find their way back through the silver foil target. These three diffusion effects will increase the number of K ionizations per electron striking the target. Another effect is that due to electron retardation while passing through the target, which reduces the energy of the electron and thus reduce the number of K ionizations. There is also the effect of fluorescence within the target. That is, continuous spectrum x-rays of wavelength shorter than those of the K series may be absorbed on their way out of the target by atoms which emit K series quanta, thus increasing the number of K ionizations per electron. Of the above effects the corrections for the rediffusion of the electrons from the copper end of the target, the electron retardation, and the fluorescence effect, have been calculated by the methods used by Webster, Hansen and Duveneck[7] and found to be negligible. The corrections for the remaining two effects are now discussed following the procedure applied by the above authors.

Diffusion within the target. If the observed intensity of the K quanta is i and the intensity corrected for diffusion is i_d, then the diffusion correction k_d is $i_d \div i$. This is shown to be[7] given by $k_d = 1/(1+\lambda_0/2)$. In this expression λ_0 is the value of λ at $x = X_0$ where X_0 is the thickness of the silver foil target, and λ is the most probable angle of deflection of an electron deflected by multiple scattering when passing through a substance. As given by Bothe,[14]

$$\lambda = \frac{8.0}{V}\,\frac{V+511}{V+1022}\,Z\left(\frac{\rho x}{A}\right)^{\frac{1}{2}},$$

where λ is expressed in radians if the electron is at a distance x microns from the forward side of the target, Z, A and ρ are the atomic number, atomic weight in grams, and density in grams per cubic centimeter, respectively, and V is the accelerating potential of the electron, in kilovolts. In this experiment 70.0 kv electrons were used with silver foil targets of mean thickness 0.17 micron, making $\lambda = 0.373$ and $k_d = 0.935$. From the assumptions used in Bothe's formula together with those involved in deducing the expression for k_d, the probable error in k_d is estimated to be ± 1.0 percent.

Rediffusion within the target. Bothe[14] gives for the ratio of electrons which rediffuse from a target of thickness x to the number incident,

$$p_x = p(1-e^{-2\alpha x})/(1-p^2 e^{-2\alpha x}),$$

where α is Lenard's "practical absorption coefficient" for the incident electrons, and p is the ratio of electrons which rediffuse from a massive target to the number incident. Schonland[15] gives $p = 0.39$ for Ag and shows it is independent of V. The absorption coefficient α for silver and 70.0 kv electrons is about[16] 4.2×10^3 cm^{-1}. Accordingly, for 0.18 micron Ag foils $p_x = 0.05$, and for $x = 0.09$ micron, $p_x = 0.03$. Thus three percent of the electrons entering the target penetrate to a depth equal to one-half the target thickness and then return to the entering surface. If the rediffused electrons pass straight into the target, reversed their direction and returned in straight paths out, the fact that approximately half of the rediffused electrons penetrate to only one-half the total target thickness eliminates any rediffusion correction for this case. However, the electrons experience multiple collisions within the target and have very crooked paths, which means that those electrons which do penetrate half-way through the target before being reversed undoubtedly travel considerably farther than one target thickness while they are in the target. In any event, the rediffusion correction is small and because there is not enough known about the paths of the electrons while in the target, an evaluation of this correction cannot be made.

Auger effect correction. The ratio of the number

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[13] Bothe[14] gives const.$\times\theta e^{-\mathrm{const.}\times\theta^2}d\theta$ for the angular distribution of diffused electrons.

[14] W. Bothe, *Handbuch der Physik*, 1927, Vol. 24, Chap. I, Sec. 9.

[15] B. F. J. Schonland, Proc. Roy. Soc. **A108**, 187 (1925).

[16] Andrade, *The Structure of the Atom*, C. Bell and Sons, London, 3rd ed., p. 15.

of K quanta produced (N_{KQ}) to the number of K ionizations (N_{KI}), defined as the fluorescence efficiency, w_K, has been measured by several observers[17] for various elements. From these data the value $0.72 = w_K$ for silver will be here used. The number of K quanta may be calculated in terms of the measured number of $K\alpha$ quanta from the relation

$$N_{KQ}=N_{K\alpha}\frac{(I\lambda)_{K\alpha}+(I\lambda)_{K\beta}+(I\lambda)_{K\gamma}}{(I\lambda)_{K\alpha}}. \qquad (2)$$

In this equation $N_{K\alpha}$ denotes the number of $K\alpha_1$ and $K\alpha_2$ quanta measured, and $(I\lambda)_{K\alpha}$, etc., are quantities proportional to the number of $K\alpha$ quanta, and I and λ denote the intensity and wavelength, respectively, of the lines. Using the relative intensities of the Ag K series lines as given by Siegbahn,[18] we obtain N_{KI}

$$N_{KI}=(1.16/w_K)N_{K\alpha}. \qquad (3)$$

Ratio of line to continuous energies

In this experiment N_{KI} is determined in terms of $N_{K\alpha}$ as shown above. However, since the method of balanced thin foils does not completely separate the $K\alpha$ line energy from the band of continuous energy, it is necessary to obtain a measure of the ratio of the $K\alpha$ line energy to the energy of the continuous spectrum between the K limits of the foils for the thin silver target radiation. In this case, the area is taken to mean $\int_{\nu_2}^{\nu_1} i(\nu)d\nu$, where $i(\nu)d\nu$ denotes the intensity of the radiation for a frequency range $d\nu$ at the frequency ν. Numerical values of this quantity were obtained from planimeter measurements on spectral curves obtained by means of a Bragg spectrometer. The continuous radiation from a thin silver target is very weak, but spectral curves have been obtained throughout the spectral region necessary for this ratio determination. These curves were run at the voltage for which the ionization cross section was determined, namely 70.0 kv, and a correction for the second order continuous spectrum was necessary. Kirkpatrick[19] has outlined a method for obtaining the second order correction. The procedure involves determining the intensity of x-rays of wavelength λ and $\lambda/2$ reflected from the crystal with and without a film of material of known thickness and absorption coefficients for the two wavelengths, intercepting the x-ray beam. As previously stated the directly reflected beam of continuous energy is very weak (an electrometer deflection of approximately 2 mm is obtained during a 60-sec. exposure with the tube current and voltage 50 microamperes and 70.0 kv, and the silver foil target 0.17 micron thick), and it becomes impossible to accurately measure this radiation when an absorbing film is placed in the path. The intensity of the radiation of wavelength $\lambda/2$ can, however, be determined by multiplying the intensity of the thin target continuous energy for the first-order crystal reflection by the ratio of the "second-order crystal reflection" to the "first-order crystal reflection." In this case, "first-order crystal reflection" denotes the energy of wavelength λ reflected from the crystal in unit time when the crystal is adjusted to reflect the wavelength λ. Likewise "second-order crystal reflection" denotes one-half of the energy of wavelength $\lambda/2$ reflected per unit time when the crystal is set to reflect the wavelength $\lambda/2$, one-half the energy being taken in order to correct for the crystal dispersion. Measurements of the ratio of crystal reflections were made using radiation from a thick silver target, and the above mentioned method given by Kirkpatrick. A molybdenum foil 0.04 mm thick was used for the absorbing film, and Richtmyer's[20] values for μ/ρ at λ_M and $\lambda_M/2$ were used, λ_M being the wavelength midway between the Rh and Mo K limits.

The results of five determinations of the ratio of line to continuous areas for 70.0 kv x-rays from two thin silver targets of average thickness 0.17 micron, corrections being made for the second order continuous energy gives.

$$k=A^L/A^C=3.41\pm0.13.$$

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[17] P. Auger, Comptes rendus **182**, 1215 (1926); G. Locher, Phys. Rev. **40**, 484 (1932); D. K. Berkey, Phys. Rev. **45**, 437 (1934); A. H. Compton, Phil. Mag. **8**, 961 (1929); L. H. Martin, Proc. Roy. Soc. **A115**, 420 (1927); D. K. Berkey, Phys. Rev. **46**, 74 (1934); F. Harms, Ann. d. Physik **82**, 87 (1926); W. S. Stockmeyer, Ann. d. Physik **105**, 71 (1932); R. J. Stephenson, Phys. Rev. **46**, 73 (1934); M. Haas, Ann. d. Physik **16**, 473 (1932).

[18] M. Siegbahn, *Spektroskopie der Röntgenstrahlen*, 2nd ed., Julius Springer, Berlin, 1931, p. 355.

[19] P. Kirkpatrick, Phys. Rev. **22**, 37 (1923).

[20] F. K. Richtmyer, Phys. Rev. **30**, 759 (1927).

Correction for absorption in balanced foils

From the definition of the preceding paragraph we may write

$$A=A^L+A^C \quad \text{and} \quad A^L=Ak/(k+1).$$

However, it is not the energy proportional to A which is measured in the standard ionization chamber, but only that received by the chamber after the balanced foil absorption, which shall here be denoted by A_F. That is

$$A_F=\int_{\nu_1}^{\nu_2} i(\nu)\{e^{-[\mu_2(\nu)x_2]}-e^{-[\mu_1(\nu)x_1]}\}d\nu,$$

where ν_1 and ν_2 are the frequencies of the K limits, $\mu_1(\nu)$ and $\mu_2(\nu)$ the absorption coefficients, and x_1 and x_2 the thicknesses, of the Rh and Mo foils, respectively. By setting $R=A_F/A$, we have

$$A^L=(A_F/R)k/(k+1). \tag{4}$$

In this equation the value of k is already available and R may be obtained from

$$R=\frac{A^L[e^{-\mu_2(\nu)x_2}-e^{-\mu_1(\nu)x_1}]_{\nu=K\alpha}+A^C[e^{-\mu_2(\nu)x_2}-e^{-\mu_1(\nu)x_1}]_{\nu=\lambda_M}}{A^L+A^C},$$

where $\nu=K\alpha$ denotes the frequency of the Ag $K\alpha$ lines, and $\nu=\lambda_M$ is the frequency midway between the Rh and Mo K limits. For the balanced Mo and Rh foils used in this experiment Eq. (4) becomes

$$A^L=1.51A_F. \tag{5}$$

If we now denote the number of $K\alpha$ quanta which get through the foils during the exposure time interval by $N_{K\alpha}'$, and likewise let N_C' be the number of continuous quanta in the frequency range between the foil K limits which get through the foils in the same time interval, and further set $N_F=N_C'+N_{K\alpha}'$, then since the spectral areas previously defined are proportional to energy, we have

$$A^L/A_F=N_{K\alpha}\nu_{K\alpha}/N_F\nu_n=1.51, \quad \text{from (5).}$$

In this equation ν_n is defined by

$$\begin{aligned}A_F\nu_n=A_L\nu_{K\alpha}&[e^{-\mu_2(\nu)x_2}-e^{-\mu_1(\nu)x_1}]_{\nu=K\alpha}\\&+A_C\nu_M[e^{-\mu_2(\nu)x_2}-e^{\mu_1(\nu)x_1}]_{\nu=M}.\end{aligned} \tag{6}$$

Thus, from Eqs. (5) and (3) we have

$$N_{KI}=(1.75/w_K)(\nu_n/\nu_K)N_F. \tag{7}$$

General absorption corrections

It is seen from Fig. 1 that after leaving the target the x-rays travel through two thin aluminum films in the x-ray tube, one 0.040 mm thick and the other 0.075 mm thick. The rays next pass through 97.0 cm air, the balanced films and the front window of the ionization chamber which is 0.075 mm aluminum. Inside the chamber the rays first pass through the column of absorbing gas in the region of the front guard electrode before entering the region defined by the collector electrode. It is only those quanta absorbed in the collector electrode region which are measured. To obtain the fraction absorbed in this region, we define the following terms: let i_0 denote the intensity of the $K\alpha$ rays leaving the target i_1 and i_2 the intensity of the $K\alpha$ rays at the front and back e.m.f. of the collector electrode region, respectively, μ_1 the absorption coefficient for aluminum at $\lambda=$ Ag $K\alpha$, and y_1 the thickness of all the aluminum in the x-ray path, μ_2 and y_2 the absorption coefficient and thickness of the air from tube to chamber, μ_3 and y_3 the absorption coefficient and length of the absorbing gas in the chamber, from the front window to the front end of the collector plate, and y_4 the length of the standard collector plate. Then, neglecting the absorption due to the Mo and Rh foils (treated separately above) and calling the target intensity i_0' until this connection is added, we have

$$\begin{aligned}i_2&=i_0'\{e^{-[\mu_1y_1+\mu_2y_2+\mu_3(y_3+y_4)]}\}\\i_1&=i_0'\{e^{-[\mu_1y_1+\mu_2y_2+\mu_3y_3]}\}.\end{aligned} \tag{8}$$

Denoting the powers of the exponential terms involved in i_2 and i_1 by m_2 and m_1, respectively, and also denoting N_F' the number of quanta which get through the foils and are absorbed by the gas in the collector electrode region, we have

$$\frac{N_F}{N_F'}=\frac{i_0}{i_1-i_2}=\frac{1}{e^{-m_1}-e^{-m_2}}. \tag{9}$$

This last equation is correct only if all of the energy extracted from the x-ray beam in the standard collector electrode region is used to produce ions. Actually, this is not true when CH_3Br is the absorbing gas, since in this case fluorescent bromine rays are produced and some of these escape from the collector electrode region unabsorbed. This correction is discussed in the following paragraph.

Correction for unabsorbed fluorescent rays produced in the ionization chamber

When a beam of Ag $K\alpha$ rays pass through the column of gas a fraction of the incident energy is extracted according to the usual law of mass absorption. Part of the absorbed quanta are scattered by the absorbing gas atoms and the remainder interact with the gas atoms to produce photoelectrons and fluorescent x-rays of energy smaller than the incident quanta. The fluorescent rays are partially absorbed by the surrounding gas, producing photoelectrons. If the gas is maintained at sufficient pressure, and the ionization chamber electrodes are far enough apart, all of the energy of the photoelectrons, except those produced within a millimeter or two from the electrodes, will be absorbed by the gas to produce ions. In this experiment for the cases when CH_3Br is used as the absorbing gas, the relatively short fluorescent bromine K rays produced along the axis of the standard ionization chamber are not totally absorbed before they reach the collecting electrodes. A correction is therefore made to account for the energy contained in the unabsorbed Br K quanta. Practically all of the fluorescent bromine L and M quanta produced will be absorbed. A similar correction for the unabsorbed scattered Ag K quanta is negligibly small.

From the geometry of the ionization chamber it is seen that the correction for the unabsorbed fluorescent Br K quanta may be considered in the five following parts: I, the Br K quanta which pass through the boundary of an imaginary cylinder of mean radius "a" and length l, equal to 1/2 the distance between the chamber electrodes and the length of the collector electrode, respectively; II and III, the Br K quanta which leave the collector electrode region through the front and back ends of the above-mentioned imaginary cylinder; IV and V the Br K quanta which enter the collector electrode region through the front and back ends of the cylinder, originating from the front and back guard plate regions. If μ'_{Br} denotes the total absorption coefficient Br $K\alpha$ rays in CH_3Br, w the fluorescence efficiency of bromine, τ the photoelectric absorption coefficient for Ag K rays in CH_3Br, i_1 the intensity of the Ag $K\alpha$ rays at the front end of the collector electrode region, i_x the intensity of the Ag $K\alpha$ rays along the axis of the collector electrode region, i_f the intensity of the unabsorbed Br K rays escaping from the absorption region, then $di_x = -\mu_{\mathrm{Br}} i e^{-\mu'_{\mathrm{Br}} x} dx$, and the intensity of the Br K rays on a ring dz wide along the cylinder and at a distance r from their source along the axis of the cylinder, will be

$$di_f = di_x \left(\frac{\nu_{\mathrm{Br}}}{\nu_{\mathrm{Ag}}} \right) w \frac{\tau}{\mu_{\mathrm{Br}}} e^{-\mu'_{\mathrm{Br}} r} \frac{2\pi r \sin\theta}{4\pi r^2} dz \sin\theta,$$

where $r = [(z-x)^2 + a^2]^{\frac{1}{2}}$ and $\sin\theta = a/r$. For each of the five mentioned cases the above expression leads to an integral for i_f which must be calculated by graphical integration, as indeed was found by Stockmeyer[8] for similar calculations in his measurements on the energy per ion pair of various substances. Evaluation of the integrals for the above-mentioned five parts of the correction, leads to a final value of

$$(1 + k_f) = 1.119 \pm 0.006, \qquad (10)$$

where $k_f = i_f / i_1$. Thus for the ionization measurements on Targets 1–8, during which CH_3Br was used in the ionization chamber, approximately 18 percent of the absorbed Ag $K\alpha$ energy escaped from the collector electrode region as fluorescent Br K quanta without producing ions in the chamber.

Saturation current correction in the ionization chamber

Webster and Yeatman[21] and Stockmeyer[8] have measured the electric field strength required to remove nearly all of the ions produced in the absorbing gases used here before recombination occurs. Their saturation curves show that a field strength of approximately 250 volts/cm is necessary to be sure that all but one percent of the ions formed will be collected before recom-

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[21] D. L. Webster and R. M. Yeatman, J. Opt. Soc. Am. **17**, 254 (1928).

bination takes place. The electrodes of the ionization chamber used in this experiment were 5.0 cm apart, and a source of 1300 volts of Burgess radio "B" batteries were used. Saturation curves were made with x-ray intensities sufficient to produce ion currents twice as great as any measured during the actual tests to make sure that no more than one percent of the ions formed failed to reach the collecting plates. A one-percent correction is added to the ionization currents to correct for this lack of complete saturation.

Ionization cross section in terms of measured quantities

The method of balanced foils as used in this experiment requires two measurements of charge produced in the ionization chamber, the first being the charge accumulated on the collector plate of the ionization chamber during the exposure time interval and with the Rh foil in the x-ray path, denoted by Q_{Rh}, and second the similar charge (Q_{Mo}) collected when the Mo foil is in the path. The difference between these two charges is proportional to the number of quanta N_F'. Actually N_F' consists of $K\alpha$ quanta as well as the continuous quanta contained between the balanced foil K-limits. If all of these quanta are considered to be of one frequency only, namely, ν_n, as defined by Eq. (6), then the difference between the two charge measurements is

$$Q_{\mathrm{Rh}}-Q_{\mathrm{Mo}}=N_F'h\nu_n e/\epsilon, \tag{11}$$

where h is Planck's constant, e the electronic charge and ϵ the energy required to produce a pair of ions in the absorbing gas. Combining Eqs. (3), (5), (9), (10), (11) we obtain for the total number of K ionizations produced by N_{CR} cathode rays in the time interval t,

$$N_{KI}=\frac{1.75}{W_K}\,\frac{4\pi}{\Delta\Omega}\,\frac{k_d(k_f+1)\epsilon}{eh\nu_K}\,\frac{Q_{\mathrm{Rh}}-Q_{\mathrm{Mo}}}{e^{-m_1}-e^{-m_2}}(1.01). \tag{12}$$

In this equation the factor $\Delta\Omega$ is the solid angle subtended by the lead aperture directly in front of the ionization chamber with the focal spot on the target as the center, and 1.01 is the saturation current correction.

The number of electrons which strike the target during the exposure time interval t is $N_E=it/e$, where i is the tube current and e the electronic charge.

The number of atoms per unit cross section of the target is given by $nX_0=(L\rho/A)X_0$, where L represents Avogadro's number, ρ the density of the target material, A its atomic weight and X_0 the thickness of the target. In this case the thin silver foil is inclined at an angle of 80° with respect to the direction of the incident electrons, so the thickness becomes X_0 sec. 80°.

The expression for the ionization cross section (Eq. (1)) may now be written in the final form involving the quantities determined by the measurements. Thus

$$\Phi(U)=\frac{\dfrac{1.75}{it}\,\dfrac{4\pi}{\Delta\Omega}\,\dfrac{k_d(k_f+1)}{h\nu_{K\alpha}}\,\dfrac{Q_{\mathrm{Rh}}-Q_{\mathrm{Mo}}}{e^{-m_1}-e^{-m_2}}(1.01)}{(L\rho/A)X_0\ \text{sec. }80^\circ}. \tag{13}$$

In the above expression $\Phi(U)$ has the units of cm^2 if it is measured in ampere-seconds, $Q_{\mathrm{Rh}}-Q_{\mathrm{Mo}}$ in coulombs, ϵ and $h\nu_{K\alpha}$ in electron volts, ρ in $\mathrm{g/cm}^3$ and X_0 in cm. The *International Critical Tables* (1926) values for the atomic weight and density of silver were used and the I.C.T. values of Ag $K\alpha_1K\alpha_2$ were corrected according to Bearden's[22] results, making $h\nu_{K\alpha}=22{,}068$ electron volts. The value of the linear absorption coefficient for aluminum for $\lambda=559.4$ X.U. was computed from Jönsson's[23] empirical formula to be $\mu_{\mathrm{Al}}=7.81\ \mathrm{cm}^{-1}$. The linear absorption for air was taken from a curve of data represented by Siegbahn.* For air at 760 mm and 20°C at $\lambda=559.4$ X.U., $\mu_{\mathrm{air}}=0.00082\ \mathrm{cm}^{-1}$. The lead aperture used to define the x-ray beam was 4.69 ± 0.02 mm in diameter. This measurement was very kindly furnished by Mr. M. S. Hugo of the U. S. Government Gauge Laboratory at Stanford University. The distance from the focal spot to the lead aperture varied for various target measurements. For targets 1–4 this distance was 101.5 ± 0.3 cm for Targets 4–10 the x-ray tube was moved a small amount and this distance was 107.6 ± 0.3 cm. The error introduced in $4\pi/\Delta\Omega$ by the uncertainties in aperture diameter and focal spot to aperture distance is negligible. The values of the energy per ion pair for $\mathrm{CH_3Br}$ and

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[22] J. A. Bearden, Phys. Rev. **43**, 92 (1933).

[23] E. Jönsson, Inaugural Dissertation, Upsala University, Sweden, 1928.

* M. Siegbahn, *Spektroskopie der Röntgenstrahlen*, 2nd ed., Table 227.

TABLE I. *Thickness of silver foils for targets.*

Method	Number of foils tested	Thickness per foil X_0
X-ray abs.	8	0.171 micron
" "	7	0.180
" "	6	0.153
Weight	1	0.171
		0.169 ± 0.004 micron

SO_2 are those given by Stockmeyer[8, 24] as $\epsilon_{CH_3Br} = 25.4$ electron volts, and $\epsilon_{SO_2} = 29.7$ electron volts. These values are based on Eisel's[25] determination of ϵ for air to be 32.2 electron volts. Eisel gives his probable error to be ±1.5 percent, and allowing for an error in the relative measurements of Stockmeyer, the above values for ϵ are here taken to be correct to within ±2.5 percent.

V. TARGET THICKNESS

The thin silver foils used for targets were taken from the same lot as those used by Wesbter, Hansen and Duveneck.[7] These foils were obtained commercially under the name of pure silver leaf, and look and behave very much the same as gold leaf. They are prepared by a "beating" process and their thickness is not uniform.

Measurement of thickness by x-ray absorption

The absorption of x-rays of wavelength where the mass absorption coefficient of silver is accurately known is too small to measure for a single foil of such a small thickness (mean thickness 0.17 micron). In order therefore to use absorption methods several foils are placed in the x-ray path. Foils of this thickness are difficult to handle, and it is best not to remove them from the tissue paper booklet in which they are transported. In this experiment the absorption measurements were made at the Ag K limit, in which case the effect of the paper is eliminated from the thickness determinations. That is, if μ_a and μ_b denote the minimum and maximum absorption coefficient of silver at the K limit, respectively, I_a and I_b the corresponding intensities of the transmitted x-ray beam, and μ_p the absorption coefficient of the paper at this wavelength, then

$$I_a = I_0 e^{-\mu_a N X_0 - \mu_p x}; \quad I_b = I_0 e^{-\mu_b N X_0 - \mu_p x}$$

and

$$X_0 = \frac{1}{N} \frac{\log (I_a/I_b)}{\mu_b - \mu_a},$$

where NX_0 represents the thickness of N silver foils. Measurements of x-ray intensities were made over a wavelength region of 100 X.U. above and below the K limit with the foils in the path. A spectral curve was plotted, and because of the width of the discontinuity the upper and lower portions of the curve were extended to the middle point of the discontinuity, in order to obtain the magnitude of the discontinuity accurately. This extrapolation was done by the λ^3 law, and Richtmyer's[20] values for μ/ρ at the K limit were used. The results of the thickness

TABLE II. *Film 1, test 1.* Tube voltage 70.0 kv, exposure time t 18.0 sec., tube current i 30.15 microamperes.

Balanced film	Electrometer deflection, mean of 5 expos.	Vltg. on ioniz. system to cause deflection	Capac. of ionizat'n syst. at this deflect'n	Charge collected
Rh	249.0 mm	0.0702 volt	118.5$\mu\mu f$	8.32×10^{-12} coul.
Mo	134.6	0.0398	114.5	4.56×10^{-12}
			uncorrected $Q_{Rh} - Q_{Mo}$ =	3.76×10^{-12}
	Extraneous radiation from places on target other than focal spot			
Rh	7.85 mm	0.0020 volt	110.6$\mu\mu f$	0.221×10^{-12}
Mo	6.00	0.0015	110.6	0.166×10^{-12}
			extraneous rdn. correction $Q_{Rh} - Q_{Mo}$ =	0.055×10^{-12}
			Resultant charge collected $Q_{Rh} - Q_{Mo}$ =	3.70×10^{-12} coul.

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[24] This work was completed in January, 1935, and since that time Gaertner, Ann. d. Physik **21**, 564 (1934), reports experiments on the determination of the energy per ion pair for air and gives the value $\mu_{air} = 35.2$ electron volts. Also E. Arends, Ann. d. Physik **22**, 281 (1935), reports the fluorescence efficiency for silver $w_K = 0.795$. These results would increase the value of $\Phi(U)$ reported here by one percent.

[25] A. Eisel, Ann. d. Physik **395**, 277 (1929).

TABLE III. *Values of* $\Phi(U)$ *for all silver films.*

Target	No. of tests	Mean $\Phi(U)$ ($\times 10^{23}$ cm^2)
1	2	4.58
2	2	5.04
3	1	5.13
4	9	3.68
5	5	5.51
6	6	4.28
7	3	4.40
8	3	6.37
9	3	4.52
10	3	4.46
	Average	4.80±0.16

measurements, together with a thickness determination by "*weighing*" are shown in Table I.

Measurement of thickness by weight measurements

The area and weight of one of the foils 89.094 cm^2 and 16.005 g was determined by a series of measurements with a Starrett steel scale and microscope and an F. Sartorious beam balance. The weights used were the laboratory standards of this department.

VI. RESULTS

A set of sample data are presented in Table II. From formula (13) the value of the ionization cross section for this test is therefore $\Phi(U) = 4.57 \times 10^{-20}$ cm^2. In the following Table III is shown the results of tests made on 10 thin silver foil targets. The ionization currents for Films 1–7 were produced by absorbing the x-rays in CH_3Br at approximately 680 mm Hg, while SO_2 at about 20 lbs./in.^{2} was used for Films 7–10. Only one test was obtained on Film 3 before it was punctured, either by melting or a sudden release of gas in the x-ray tube. Tube currents varying between 30 and 60 microamperes were used throughout the measurements, although it was found that these silver foils would not last more than an hour or so with currents over 50 microamperes. The probable error of ±0.16 is due to variations in the measurements of $(Q_{\mathrm{Rh}} - Q_{\mathrm{Mo}})$ and the variations in film thickness X_0.

Accuracy of results: It is seen from the report of this work that the measurements involved in the determination of $\Phi(U)$ are not direct, but that the final value is obtained from a number of very different physical measurements. Some of the measurements are those made by the author, while those involved in the determinations of certain constants and coefficients such as w_K, μ_{Al}, μ_{air}, ϵ, $I_{K\alpha} : I_{K\beta}$, etc., are the results of work by others. The errors in the measurements by the author are well known, and in the cases where the errors of other workers are not stated, a fair estimate is here assumed for the purpose of calculating the error of this work.

In the factor $1.75/w_K$, w_K was obtained from a graph drawn from the data of several observers,[16, 24] and estimated to be 0.72±3.0 percent while the term 1.75 is the result of two numerical values obtained from formula (2) and (4). In (2) the errors of each of $(I\lambda)$ are estimated to be correct to one percent and in (4) the factors k and R are computed to be correct to ±3.8 and ±4.96 percent, respectively. This makes the probable error in the fraction $1.75/w_K$ equal to ±7.6 percent. The probable errors in the factors $4\pi/\Delta\Omega$, $h\nu_{K\alpha}$ and $(L\rho/A)$ sec. 80° are considered negligibly small. In the factor $e^{-m_1} - e^{-m_2}$ the terms m_1 and m_2 consists of terms of the form (μx), each of which are estimated to be in error by 1.0 percent, making the probable error of the total factor $e^{-m_1} - e^{-m_2}$ equal to ±3.0 percent. As previously discussed the probable error in ϵ is estimated to be ±2.5 percent. The factors remaining are $(Q_{\mathrm{Rh}} - Q_{\mathrm{Mo}})$ and X_0. A measure of the error in these terms is available in the data given in Table II. Here the variation in $\Phi(U)$ is due to the variation in both $(Q_{\mathrm{Rh}} - Q_{\mathrm{Mo}})$ and X_0 and the probable error is ±3.3 percent. The probable error in the variations of the Q's can be calculated from data available, but not represented in this report, and is ±0.3 percent.

The most probable value of the resultant error in $\Phi(U)$, obtained by combining the above outlined errors is found to be ±9.0 percent, making the most probable value of $\Phi(U)$ obtained from these measurements equal to[24]

$$\Phi(U) = (4.80 \pm 0.43) \times 10^{-23} \text{ cm}^2.$$

VII. THEORETICAL

There have been several theoretical attempts given to the problem of the determination of the probability of inelastic scattering of electrons by atoms. The earlier predictions are based on the classical quantum theory, while the more recent theories involve the wave mechanics. The classical quantum theories of Davis,[26] Rosse-

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[26] B. Davis, Phys. Rev. **11**, 433 (1918).

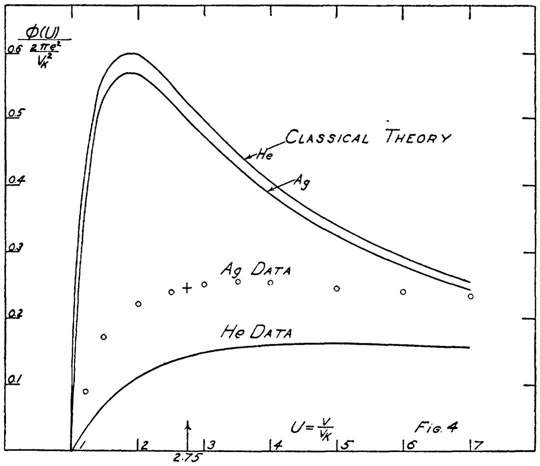

FIG. 4. Ionization cross sections *vs*. energy. Classical theory and an experiment compared for silver and helium.

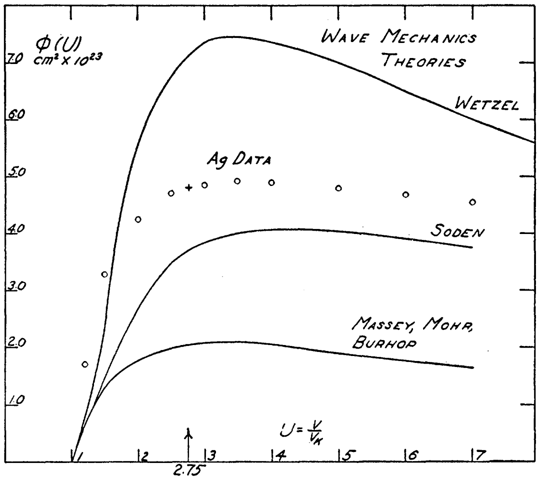

FIG. 5. Ionization cross section *vs*. energy. Wave-mechanical theories compared with data on silver.

land,[27] and Thomas[28] are fully discussed in the paper by Webster, Hansen and Duveneck.[7] Modifications of the nonrelativistic classical quantum theories to account for the deflection of the impinging electron before the ionizing impact are applied by the latter authors and the results, labeled "Classical theory," are shown in Fig. 4. In this figure P. T. Smith's[6] data on the absolute value of the ionization cross section for helium are shown together with data of Webster, Hansen and Duveneck on the relative probabilities of K electron ionization for silver. These relative measurements are normalized at $U=2.75$ by the absolute value of the ionization cross section for silver determined by the author. The curves show the classical theory represents a very rough approximation to the facts at high values of U, but that it is practically useless for $U<5$.

The wave mechanics theories appear to be leading to results which compare more favorably with the experimental data. However, all of the wave mechanics theories so far reported treat the interaction of the impinging electron with the K electron as a small perturbation in the same way as it is done in Born's theory of collisions. The approximation is valid only for electrons with kinetic energy large compared to the ionizing energy. It therefore seems quite certain that any theory based on Born's approximation cannot be expected to be accurate for a value of U as small as that used in this experiment, namely, $U=2.75$. Besides this most serious source of error, in all but Soden's paper, the orbits of the atomic electrons are treated as hydrogenic orbits; an assumption which is wrong and can only roughly be corrected by screening constants. It is probably due to the latter reason that the results of Burhop and Wetzel deviate more from the experimental values than those of Soden.

In Fig. 5 are shown Webster, Hansen, Duveneck and the present author's experimental data of the ionization cross section of silver as a function of U, compared with the wave mechanics theories. Three curves of Massey, Mohr and Burhop and Wetzel are taken from such curves contained in personal correspondence on this subject between these authors and Professor D. L. Webster during the interval between September, 1933 and May, 1934. It is evident from this correspondence that these curves represent only a first attempt at a solution of this problem for the case of silver.

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[27] G. Rosseland, Phil. Mag. **45**, 65 (1923).

[28] L. H. Thomas, Proc. Camb. Phil. Soc. **23**, 829 (1927).