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Progress of the Directional Survey of Cosmic-Ray Intensities and Its Application to the Analysis of the Primary Cosmic Radiation

THOMAS H. JOHNSON, *Bartol Research Foundation of the Franklin Institute*

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Since the 1934 report, asymmetries have been studied by the coincidence counter method at several stations in Mexico and in Colorado so that the completed survey now includes data at high (4300 m), low (sea level) and intermediate elevations at the geomagnetic equator, in the intermediate latitudes λ29°, and in the high latitude λ50° besides additional data at an intermediate elevation in latitude λ36°. For the measurements an automatic multidirectional intensity comparator has been developed. Using the results of the theoretical investigations of Lemaitre, Vallarta and Bouckaert the measured asymmetries have been reduced to relative specific intensities (per unit range of x) of the unbalanced positive component and it is found that this varies with the atmospheric path length h sec z, according to an exponential law. The observed intensities, taking account of absorption, have been used in conjunction with the results of LVB to compute the longitude and latitude effects to be expected from the positives alone. The latitude effect thus computed is greater than that observed between Peru and Panama but the computed longitude effect is, in general, too small. It is possible that the anomaly is to be explained by the departures of the horizontal intensity from the values which would pertain if the earth's field were accurately that of a simple doublet. The work has been done under grants from the Carnegie Institution of Washington, administered by its Cosmic-Ray Committee.

INTRODUCTION

THE survey, undertaken in 1933 for the study of directional distributions of the cosmic radiation in various latitudes and elevations, resulted[1] in reasonably accurate values of the east-west asymmetries for the elevations 3340 m and 4300 m in Peru on the geomagnetic equator, at sea level in Panama, and at sea level in the United States. Less accurate results were also obtained at sea level in Peru, at three elevations in Mexico ranging from sea level to 3300 m, and at 3300 m in Colorado. Asymmetries in these and in other locations have also been measured by other investigators, confirming these results and adding to the comprehensiveness of the general survey.

Because of the importance of the asymmetry measurements for the analysis of the primary cosmic radiation, further studies were undertaken in 1934 with the view to improving the accuracy and to extending the range of elevations of the measurements in and between the latitudes λ29° and λ50° where most of the change of intensity takes place. For the earth-magnetic analysis of the cosmic radiation, measurements of the intensities and east-west asymmetries as functions of the latitude determine the distribution with respect to the magnetic rigidity, $H\rho$, of both positive and negative components but the earth-magnetic mass spectrograph has not been provided with the customary electric field by which it would have been possible to separate the velocity and e/m factors in the expression $H\rho = mv/e$. For the final identification of the particles, therefore, it is necessary to rely upon absorption measure-

[1] T. H. Johnson, Phys. Rev. **45**, 569 (1934).

ments. In order to provide data for this purpose, as high an accuracy as possible was desired in regard to the variation of the asymmetry with elevation and zenith angle.

During the survey several other related investigations were made, some of which have already been reported. For further estimates of the role of the soft secondary radiations in relation to the asymmetry, east and west intensities were compared at the high elevation station in Mexico with an absorbing block of lead between the counters. Further studies were also made of the north-south asymmetries[2] at two stations in Mexico. Shower intensities were measured, in relation to the vertical cosmic-ray intensity as a standard, at all of the 1934 stations. At the high elevation station in Mexico the east-west asymmetry of the shower-producing radiation was studied.[3]

Apparatus for Determination of the Directional Distribution of the Rays which Produce Coincidence Discharges of Collinear Counters

Considerable experience with coincidence counter measurements has shown that cosmic-ray intensities from two directions can be compared with an accuracy depending only upon the statistical fluctuations of the total number of rays counted, provided the apparatus is shifted back and forth between the two directions at frequent intervals. In the present apparatus this shifting of direction, as well as the taking of readings at the end of each interval, were wholly automatic. In order to count a large number of rays in limited time, seven independent trains of counters were mounted on a single cylindrical chassis, Figs. 1 and 2. Around its periphery 42 Geiger-Müller tube counters, with cylinders 1.6 cm in diameter and 15 cm long, were densely spaced and were parallel connected, as indicated in a typical case by the brackets of Fig. 1, in fourteen groups of three each. These constituted the outside members of the seven triple-coincidence trains. The central member of each train was a common, axial group of five counters. Each group was connected to the grid of a type 77 vacuum tube and the amplified pulses from diametrically opposite groups were combined with those from the central group for the selection and recording of the coincidences. The circuit for this purpose embodied the principles outlined by Fussell and Johnson[4] and is shown diagrammatically in Fig. 3. The distance between the outside groups of each train was 28 cm and the solid angle for which half or more of the

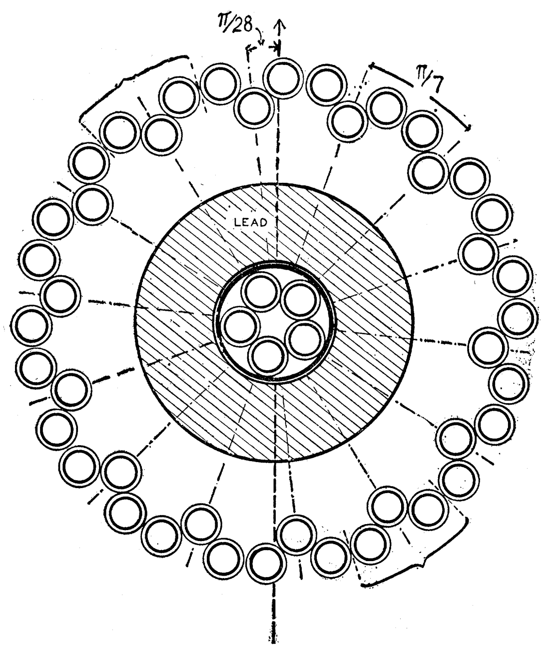

Fig. 1. Diagram of the arrangement of counters in the multidirectional cosmic-ray intensity comparator.

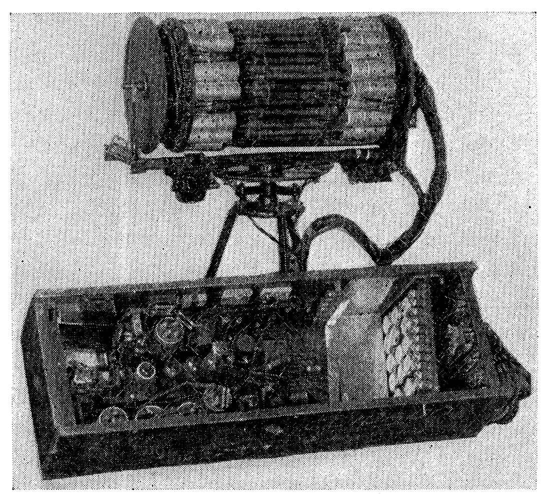

Fig. 2. The multidirectional cosmic-ray intensity comparator and its recording equipment.

[2] T. H. Johnson, Phys. Rev. **47**, 91 (1935).
[3] T. H. Johnson, Phys. Rev. **47**, 318 (1935).
[4] L. Fussel, Jr. and T. H. Johnson, J. Frank. Inst. **217**, 517 (1934).

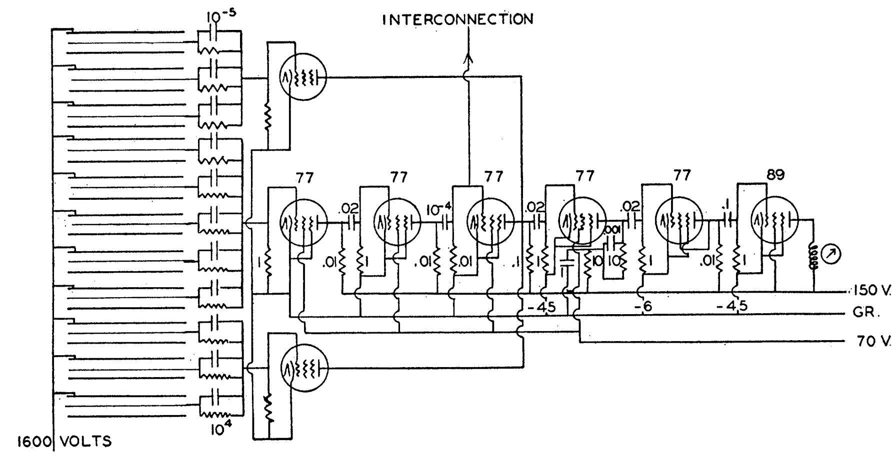

FIG. 3. The circuit for selecting and recording coincidences. Resistances in megohms, capacities in microfarads, and potentials in volts.

counter areas were effective extended to 4.5° in zenith angle and to 14° measured in the plane of the counter wires on both sides of the midline of the train. The central counters were inside a steel tube of 3.2 mm wall thickness and, when additional absorbing material was desired, lead rings were slipped over this as indicated in Fig. 1. Thicknesses up to 10 cm could thus be introduced into the paths of the coincidence producing rays. The elements of each of the coincidence-selecting circuits were mounted on the chassis but the out-put pulses from the last stage amplifiers were led out on a cable running through the axis of the chassis to an auxiliary box containing the seven recording dials and a photographic device. The dials, a watch for time records, and a series of lamp bulbs, certain of which flashed at the instant of photographing to indicate the position of the chassis, were all mounted together on a panel and this was photographed at regular intervals with a Simplex Pockette 16-mm movie camera. The exposures were timed by weight-driven clock works and during the field work they were made at fifteen minute intervals. Simultaneously with the operation of the camera, small motors geared to the chassis were set in operation and these continued to run until a determined shift in position had been effected. On the even intervals, shifts in azimuth were made by rotation about a vertical axis and on the odd intervals the chassis was rotated in zenith angle about a horizontal axis. The two fixed azimuth positions were adjustable by a pair of stops clamped to a circle but the successive zenith angle positions were spaced at fixed angles, $\pi/7$ apart, and at these positions the midlines of the counter trains pointed at angles 6.4°, 32.1°, 57.9° and 83.5° from the zenith on one side and at 19.3°, 45° and 70.7° on the other side. A shift in zenith angle carried each counter train from one of these positions to the next in the order of rotation. The sequence of readings from a particular train thus contained pairs of successive data at each zenith angle with one datum for each of two azimuths. Each pair of data resulted in a value of the azimuthal asymmetry, α, defined below, which was independent of long time variations of counter sensitivity or of relative sensitivities of different trains.

In the determination of asymmetries by this method it is essential that the axis of azimuthal rotation be exactly vertical. If the axis is out of vertical by an angle δ an apparent asymmetry at zenith angle z can be introduced of maximum amount $\Delta j/j = 4\delta \tan z$. To guard against an error of this nature the carriage supporting the chassis was mounted on leveling screws at the top of an iron tripod and the legs of the tripod were usually set in concrete on a stone pier or other firm foundation. An 8-inch level, reading to one minute of arc, was fixed to the carriage and constant checking during each run showed variations due to varying tensions on the cable

TABLE I. *East-west asymmetry measurements of the 1933, 1934 surveys.*

Zenith angle z	Asymmetry A	No. of data N	Total counts west C_w	Total counts east C_e	Probable error of A R
Cerro de Pasco, Peru; λ0°; mean barometer, b, 46 cm.; atmospheric depth, h, 6.24 m of water; elevation 4300 m					
15°	0.084	183	59,295	55,049	0.0042
30°	.125	160	39,601	35,418	.0054
45°	.139	72	11,024	9,764	.0085
60°	.174	98	6,760	6,028	.015
*45°	.188	46	4,260	3,595	.019
* 3.8 cm lead between counters					
Huancayo, Peru; λ0°; b, 51.8 cm; h, 7.03 m; elev. 3340 m					
15°	.063	118	25,866	24,332	.0060
30°	.117	143	24,874	21,963	.0065
45°	.140	218	24,819	21,284	.0061
60°	.156	151	8,039	7,056	.011
75°	.061	87	1,666	1,560	.023
Lima, Peru; λ0°; b, 76 cm; h, 10.3 m; sea level					
15°	.064	94	10,120	9,930	.0093
30°	.092	144	12,358	10,865	.0082
45°	.145	63	3,541	2,937	.016
Barro Colorado Island, Panama Canal Zone; λ20°; b, 76 cm; h, 10.3 m					
15°	.036	157	17,669	17,386	.0077
30°	.047	328	30,758	29,513	.0056
45°	.066	350	23,146	21,190	.0072
60°	.035	195	6,415	6,229	.012
*75°	−.011	155	2,077	2,322	.019
* western horizon possibly obscured by forest					
El Pico Nevado de Toluca; λ29°; b, 64.2 cm; h, 6.25 m; elev. 4300 m					
6.4°	.0304	245	30,175	29,343	.0054
19.3°	.0654	243	23,191	21,814	.0055
32.1°	.1012	242	20,260	18,354	.0074
45.0°	.1162	248	14,911	13,282	.0085
57.8°	.1288	239	7,419	6,564	.0110
70.7°	.051	243	3,834	3,646	.015
*83.5°	−.062	245	1,628	1,736	.025
* western horizon obscured up to 12° by mountain ridge					
El Pico Nevado de Toluca; 8.6 cm of lead between counters					
6.4°	.0092	179	12,122	11,942	.0092
19.3°	.0496	173	11,105	10,564	.0087
32.1°	.084	176	9,016	8,232	.0097
45°	.088	176	7,040	6,445	.0112
57.8°	.081	169	3,684	3,392	.0144
70.7	.030	177	2,322	2,282	.021
*83.5	.116	176	1,368	1,206	.029
* See above.					
Vera Cruz, Mexico; λ28°; b, 76 cm; h, 10.3 m; sea level; 1934					
6.4°	.0149	271	10,331	10,365	.0097
19.3°	.021	269	9,549	9,385	.010
32.1°	.050	270	7,608	7,184	.011
45	.023	266	6,163	6,131	.023
57.8°	.062	237	3,859	3,520	.0123
70.7°	.028	264	2,633	2,611	.021
83.5°	.111	258	1,542	1,475	.032
San Rafael, Mexico; λ29°; b, 50 cm; h, 6.8 m; elev. 3300 m; 1933					
20°	.062	73	14,976	16,181	.0095
30°	.078	54	10,522	11,058	.0099
50°	.081	41	4,763	4,774	.018
Mexico City, Mexico; λ29°; b, 55 cm; h, 7.5 m; 2200 m elev.; 1933					
30°	.048	59	15,702	14,098	.0084
40°	.078	65	21,424	13,621	.008
50°	.086	38	7,080	6,221	.013
*50°	.12	19	2,554	2,505	.019
65°	.037	23	1,533	1,747	.037
* 3.8 cm lead between counters					
Parral, Chihuahua, Mexico; λ36°; b, 62.4 cm; h, 8.45; elev. 1730 m; 1934					
6.4°	.015	376	19,367	19,217	.0073
19.3°	.027	374	18,439	17,973	.0072
32.1°	.053	370	13,862	13,148	.0085
45°	.031	370	10,504	10,218	.0095
57.8°	.027	370	6,072	5,975	.014
70.7°	.006	373	4,089	4,013	.016
83.5°	.011	367	2,322	2,297	.024
*60.0°	.039	139	14,177	13,616	.008
* Large counters used.					
Mt. Evans, Colorado; λ49°; b, 46.2 cm; h, 6.25 m; elevation 4300 m; 1934					
6.4°	−.003	206	62,999	62,761	.0052
19.3°	.016	209	59,563	59,068	.0054
32.1	.020	203	44,599	43,831	.0067
45°	.016	209	38,865	38,387	.0073
57.8°	.005	204	24,511	23,980	.0097
70.7°	.028	208	21,180	20,946	.011
*83.5°	.022	207	17,360	16,788	.014
* western horizon obscured by mountain ridge up to 8°					
Echo Lake, Colorado; λ49°; b, 52.4 cm; h, 7.10 m; elev. 3180 m; 1934					
6.4°	.021	240	36,520	35,977	.0054
19.3°	−.007	240	31,924	32,759	.0061
32.1	.023	237	25,493	25,105	.0073
45°	.019	232	18,293	17,870	.0076
57.8°	.035	229	11,323	11,069	.012
70.7°	.044	237	7,341	6,962	.016
83.5°	.015	237	4,897	4,759	.022
Swarthmore, Pennsylvania; λ51° 27′; b, 76 cm; h, 10.3 m; elev. 100 m 1934					
6.4°	.021	563	37,343	36,620	.0052
19.3°	.004	563	35,176	34,927	.0050
32.1°	.005	562	27,331	27,156	.0057
45°	−.005	563	19,660	19,814	.0082
57.8°	.022	557	12,307	12,054	.0089
70.7°	.059	558	7,585	7,354	.0129
83.5°	.049	453	3,670	3,517	.016

of not more than five minutes. The average error in leveling could not have exceeded two minutes and the false asymmetry from this source could not have been over a tenth of a percent.

Sites for the measurements were always selected for open horizons and the apparatus was covered only by a canvas tent for weather protection. Azimuthal orientations were made with respect to the local magnetic fields as determined by compass readings at several nearby points.

The apparatus operated on 110 volts 60 cycles, supplied either from commercial lines or from a Cushman gasoline-electric generator of 500 watts capacity. For the shower studies, supplementary equipment consisting of three parallel groups of large counters was also used.

In the period from August first to November tenth, 1934, the instruments were operated at the following stations in the order given, Echo Lake (Colorado), Mt. Evans (Colorado), Copilco (Mexico), El Pico Nevado de Toluca (Mexico), Vera Cruz (Mexico), Parral (Mexico). Following these an extensive run was made on the roof of the Bartol Laboratory in Swarthmore, Pa. These stations as well as those of the 1933 survey are indicated on the map of Fig. 4.

Results of the East-West Asymmetry Measurements

The results of the east-west asymmetry measurements of the 1933 and 1934 surveys are contained in Table I. In the previous reports the 1933 measurements had been computed in terms of the ratio of the east and west counts, j_w/j_e. The statistical fluctuations of this quantity are not symmetric with respect to the value unity when $\bar{j}_w=\bar{j}_e$ and in the case of low counting rates and short time intervals, an appreciable false asymmetry favoring the west can be introduced in the computation. For this reason the 1933 data have been recomputed and the 1934 data computed in terms of the quantity $\alpha=2(j_w-j_e)/(j_w+j_e)$, (1), for which the fluctuations are symmetric with respect to the value zero when $\bar{j}_w=\bar{j}_e$. Table I therefore supersedes Table I, reference (1). The asymmetry, A, listed in the second column is the average of the individual determinations of α. $A=\Sigma\alpha/N$, (2), where N is the total number of pairs of data, given in the third column. The total number of rays counted in the east and west directions are also listed under C_e and C_w. The probable error of A has been computed from the dispersion of the α's and is listed under R. Instead of the usual method of averaging the squares of the deviations, R has been computed by a shorter method avoiding the extra summations. The short method agrees with the usual method if the dispersion follows a Gaussian distribution and an approximate check on this condition was made in each case by plotting the frequency curves for α. If S^+ and S^- represent the sums of the positive and negative values of α, respectively, then $A=(S^+-S^-)/N$ and, in the case of the Gaussian distribution, is easy to show that the standard deviation of α is

$$\sigma=(\pi/2)^{\frac{1}{2}}[(S^+ + S^-)/N - A\phi(A/\sigma)] \times \exp(A^2/2\sigma^2), \quad (3)$$

where $\phi(x)=(2/\pi^{\frac{1}{2}})\int_0^x \exp(-x^2)dx$. Usually $A\ll\sigma$ and it was easy to solve (3) for σ by successive approximations. The standard deviation of the average asymmetry A is $\Sigma=\sigma/(N)^{\frac{1}{2}}$ and the probable error of A is $R=0.6745\Sigma$.

The only systematic error which merits discussion is that due to accidental counts. Since each counter train was periodically pointed at the low angle of 83.5°, the counting rate in that direction gave frequent check on the possible number of counts due to this cause. Except for some of the readings on Mt. Evans, where it was discovered that the selecting bias had been adjusted too closely, the horizontal rate was less than 10 percent of the vertical rate and since most of these counts must have been due to cosmic rays, no correction for accidentals was needed. At higher zenith angles where natural double coincidences are more frequent, the accidental triple coincidences would also be more frequent, but these would be proportional to the double coincidence rates and hence subject to the directional asymmetries. Correction for this effect would be of a second order and would involve only the lower angular resolution of the double coincidences. In the case of the Mt. Evans data accidental counts uninfluenced by the direction of the instrument were not more than twenty percent of the vertical counting rate and this would call for a correction to be added to the tabulated values of A, not to exceed 40 percent at $z=45°$.

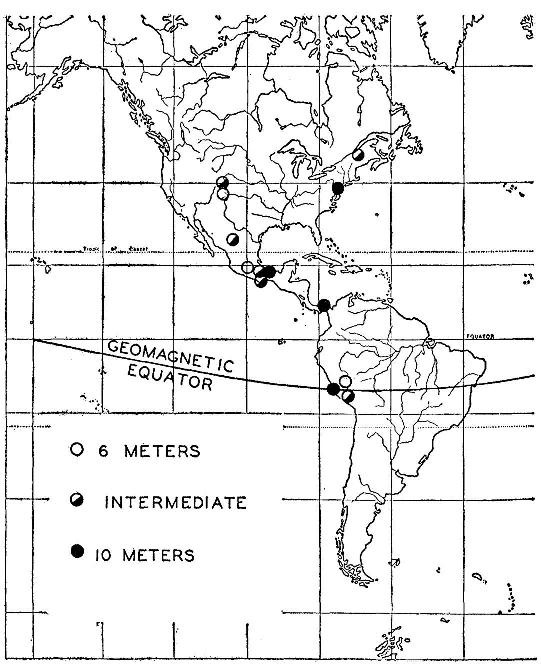

FIG. 4. Map indicating the scope of the directional survey.

The results of the two surveys as well as those of all other observers who have reported quanti-

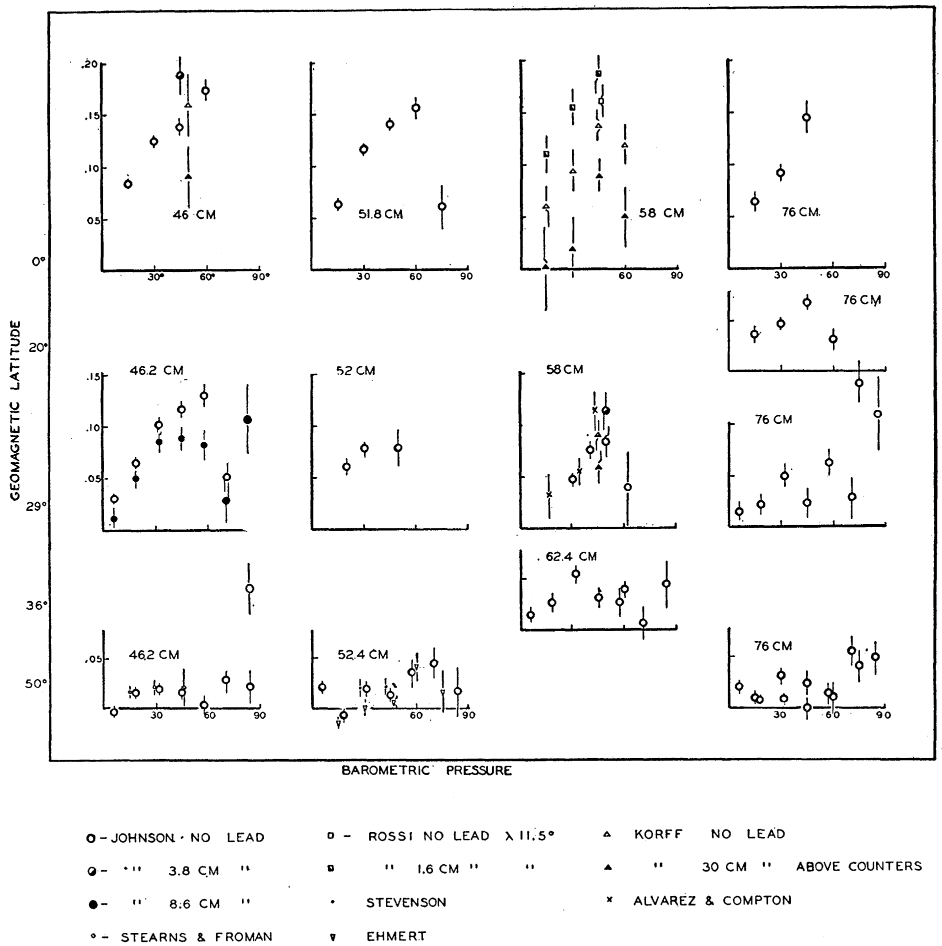

FIG. 5. Compilation of east-west asymmetry measurements. For each station the asymmetry, A, is plotted against zenith angle.

tative measurements which have come to the writer's attention are plotted in Fig. 5. The probable errors are indicated by the vertical lines. Data obtained with lead absorbers are represented by shaded points as noted. Rossi and the writer have placed the lead between the counters whereas Korff placed his lead above the train.

The new data have not changed the salient features of the asymmetry effect but serve to bring out more strongly than before the increase of the asymmetry at the equator over that of higher latitudes. The new results for high and low elevations in Mexico confirm very definitely the effect reported in reference (1) that the asymmetry increases with elevation. In the latitude λ50° the Mt. Evans asymmetries agree with those found at the same location by Stearns and Froman[5] whereas the new Swarthmore asymmetries are in general lower than those found previously by Johnson and Stevenson.[6] It is also noted that for many of the stations the low angles of 70.7° and 83.5° seem to show consistently higher asymmetries than would have been expected from simple extrapolation of the higher angle values. This effect does not appear in the Nevado de Toluca and Mt. Evans results at the 83.5° angle, possibly because of shielding on the west by mountain ridges but of the data for 70.7° and 83.5° at Vera Cruz, Parral, Echo

[5] J. C. Stearns and D. K. Froman, Phys. Rev. **46**, 535 (1934).

[6] T. H. Johnson and E. C. Stevenson, Phys. Rev. **44**, 125 (1933).

Lake and Swarthmore and for 70.7° on Mt. Evans and Nevado de Toluca, eight show asymmetries greater than three percent compared with three showing asymmetries less than three percent. Since the high angle data seem to extrapolate to a zero effect at the horizon, there thus seems to be indication for abnormal asymmetries at low angles. These could possibly be explained by the deflection of primary positive rays after they have been slowed down by atmospheric energy losses.

It is again noted that no significant part of the observed asymmetry can be accounted for by the deflection of secondaries generated in the atmosphere. This is clearly brought out by the theory of the secondary effect given by Bowen[7] and was simply demonstrated from the experimental point of view by the reasoning given in reference (1). It may be, and probably is, true that most of the rays actually counted, whether they belong to the field sensitive component or not, are secondaries, but these, in general, should have very nearly the same direction as the primaries which produced them. Conservation of direction during the production process is insured, in the case of energies large compared with the rest mass energy, by the conservation of energy and momentum, and the magnetic deflection by the earth's field, before the ray has lost too much energy to allow it to pass through the instrument, cannot exceed a few degrees at the elevations included in the survey. Since the evidence of Anderson[8] and others indicates equal numbers of positive and negative secondaries such magnetic deflections, even if of significant magnitude, could produce no asymmetries.

The observations made with lead absorbing screens are of interest in this connection. Those of the writer in Peru[1] and of Rossi[9] in Asmara showed an increase of asymmetry due to the effect of the lead. Later measurements by Korff[10] with lead placed above, instead of between, the counters and with thicknesses up to 30 cm, showed a reverse effect. The present measurements on Nevado de Toluca with 8.6 cm of lead between the counters also show a reduction in the asymmetry, agreeing with Korff. The results of all observers made at a zenith angle of 45° are compiled in Fig. 6. Although the three points showing an increase due to lead are accompanied by rather high probable errors, the combined evidence seems sufficient to show that the effect of lead placed between counters is of the character indicated by the curve. The negative result of Korff for two centimeters of lead can possibly be explained by the increased count due to poorly directed showers generated in the lead above. Any undirected background of this character would tend to reduce the percentage asymmetry. For lead between counters, the first thicknesses reduce the diffuse background of softest radiations but further thicknesses cut into the asymmetric component more than into the average radiation. This accords with the view that the background radiation consists of higher energy corpuscles. Present indications are that a slight correction should be added to the measured asymmetries to take account of the diffusion of softest secondaries, though it is felt that the effects should be studied further before such corrections can be applied with certainty.

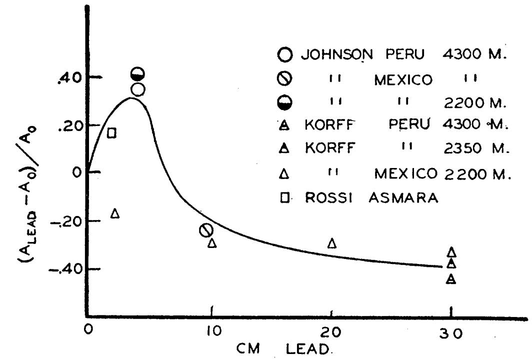

FIG. 6. Effect of lead absorbing screens of varying thickness on the east-west asymmetry at a zenith angle of 45°.

SIGNIFICANCE OF EAST-WEST ASYMMETRIES

The theory of Størmer,[11] as was first clearly stated by Rossi,[12] gives a simple interpretation

[7] I. S. Bowen, Phys. Rev. **45**, 349 (1934).
[8] C. D. Anderson and S. H. Neddermeyer Report at the Int. Cong. on Physics, London (1934).
[9] B. Rossi, Ricerca Scient. **5**, 594 (1934).
[10] S. A. Korff, Phys. Rev. **46**, 74 (1934).
[11] C. Størmer, University Observatory, Oslo, Pub. No. 10 (1934); Ergebnisse der Kosmischen Physik **1**, 1 (1931). Størmer has also made estimates of the cones for infinite orbits but for convenience his basic first approximation will be referred to as the Størmer theory.
[12] B. Rossi, Nuovo Cimento (3) **8**, 3 (1931); Rend. Lencei **13**, 47 (1931).

to the east-west asymmetry. Positive rays must have a higher rigidity if they are to reach an observer on the surface of the earth from an eastern direction than is required from the same zenith angle to the west. Rays of rigidity between the two limiting values corresponding to eastern and western directions will illuminate the western but not the eastern direction. In the case of negative rays the directions are reversed, but if there is an excess of positives, unbalanced in this sense by an equal intensity of negatives in the same range of rigidity, the western intensity will be the greater. The difference of the western and eastern intensities gives a measure of the unbalanced positives included within the range of rigidity between the minimum values for the two directions. In the Størmer theory the value of the rigidity, $H\rho$, of rays incident in latitude λ from meridian angle θ (between the vertical plane of the magnetic meridian and the direction from which the ray has come, measured positively towards the west if the rays are positive) is given by

$$H\rho = Mx^2/r^2, \tag{4}$$

where M is the magnetic moment of the earth (8.04×10^{25} e.m.u.), r is the radius of the earth (6.37×10^8 cm) and

$$1/x = -\gamma/\cos^2\lambda + (\gamma^2/\cos^4\lambda + \sin\theta/\cos\lambda)^{\frac{1}{2}}. \tag{5}$$

Henceforth for convenience the term *rigidity** will apply to the quantity x instead of to $H\rho$.

In the east-west plane at the equator the orbits for which the constant γ has the value -1 are those along which rays of lowest rigidity reach the earth from infinity. At higher latitudes and from other directions at the equator the values of γ for infinite orbits of minimum rigidity are greater than -1 and depend upon the direction. This question has been extensively investigated by Lemaitre and Vallarta (LV)[13] and by Lemaitre, Vallarta and Bouckaert (LVB)[14] who have made precise determinations of the minimum rigidities of orbits from infinity as a function of the angle of incidence in latitudes $\lambda 0°$, $\lambda 10°$ and $\lambda 20°$, but for the higher latitudes the theory is not yet completed. The application of Liouville's theorem, as shown by LV and by Swann[15] assures that all positions and directions will be illuminated with uniform intensity (except for the effect of atmospheric absorption) by radiation whose rigidity is above the minimum value corresponding to those positions and directions. Hence the intensity of corpuscular radiation from any direction can be expressed as the sum of two integrals, one for the positive the other for the negative radiation, the integrands of which are the specific intensities (per unit range of x) at the point and from the direction of observation, and the integrations extend to infinity from the minimum values of x corresponding to the latitude, the direction, and the sign of charge. The observed specific intensity is determined by the specific intensity in external space and by absorption along the atmospheric path $h\sec z$. The difference of eastern and western intensities at the same zenith angle is the difference of the same two integrals, or, since the lower limit of x for negative rays from the east is the same as that for positive rays at the same zenith angle from the west, these reduce to a single integral whose integrand is the specific intensity of unbalanced positives, $k(x)$, and the limits are the minimum values of x for positive rays from the two directions. Thus the asymmetry, defined as in (1) and (2) is

$$A(\lambda, h\sec z) = [1/j_\lambda(h\sec z)]\int_{x_w(\lambda,\ z)}^{x_e(\lambda,\ z)} k(x, h\sec z)dx. \tag{7}$$

Over small ranges of x corresponding to the minimum values for eastern and western directions near the zenith a uniform distribution with respect to x of the specific intensity, k_λ, may be assumed and to this approximation (7) reduces to

$$(k/j)_\lambda(h\sec z) = A/(x_e - x_w). \tag{8}$$

(k/j) may be called the specific relative intensity of unbalanced positive radiation after filtering

* x is the quantity sometimes loosely referred to as the energy. The energy in electron volts is given by

$$v = (300\, m_0c^2/e)[(Z^2e^2M^2x^4/m_0^2c^4r^4+1)^{\frac{1}{2}} - 1], \tag{6}$$

where m_0 and Ze are rest mass and charge of the ray and c is the velocity of light.

[13] G. Lemaitre and M. S. Vallarta, Phys. Rev. **43**, 87 (1933).

[14] G. Lemaitre, M. S. Vallarta and L. Bouckaert, Phys. Rev. **47**, 434 (1935).

[15] W. F. G. Swann, Phys. Rev. **44**, 224 (1933).

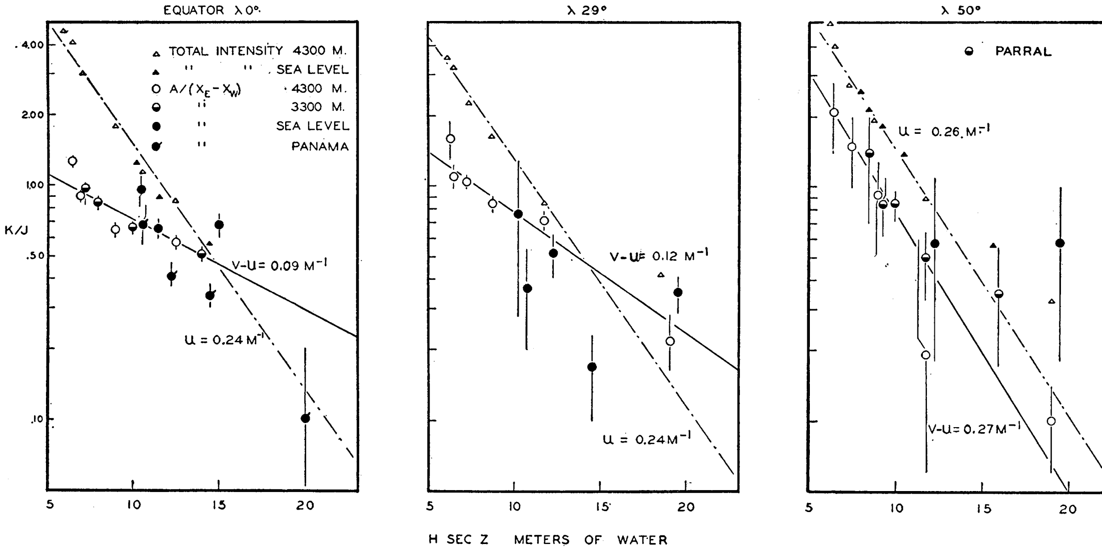

FIG. 7. The specific relative intensity of unbalanced positive radiation, and total intensity plotted against the atmospheric path length, $h \sec z$.

through atmospheric path length $h \sec z$. Values of $x_e - x_w$ are given by the theories and the values chosen are listed in the Table II. In this way each asymmetry value has been reduced to specific relative intensity of unbalanced positives and the results are plotted on logarithmic scale against $h \sec z$ in the three diagrams of Fig. 7 corresponding to the three latitudes λ0°, λ29° and λ50°. In each case the values deduced from measurements at 4300 meters are represented by the open circles, those from sea level measurements by solid circles and those from intermediate elevations by partially shaded circles. Since the energy values for λ20° are not appreciably different from those for the equator, the Panama results have been included in the plot for λ0° and with less justification the Parral data have been represented by the partially shaded circles in the λ50° plot. Data for zenith angles greater than 60° are not included in the range of the plots and if they were, the points would be somewhat erratic. In the case of the λ50° plot some of the small angle points for sea level are also erratic and fall off the vertical range. Therefore the analysis of the λ50° data cannot be given undue weight.

TABLE II. $(x_e - x_w)$ *for various values of* z *measured in the vertical east-west plane.*

Geomagnetic latitude	$z=15°$	30°	45°	60°	Theory
0°	0.068	0.139	0.216	0.305	LVB, Størmer
20°	.052	.112	.193	.300	LVB
29°	.020	.056	.095	.136	LV
36°	.025	.047	.064	.079	Størmer
50°	.006	.012	.018	.024	Størmer

Since the asymmetric component extends over but a short range of x it affords opportunity for the study of intensity variations in the atmosphere due to primaries of definite energy. It has been customary to think of the absorption of corpuscular radiation in terms of a definite range, R, depending upon the energy. If corpuscular rays of definite energy were characterized by a definite range, R, the absolute specific intensity of the unbalanced positive component would be uniform within path lengths from the top of the atmosphere $h \sec z < R$ and zero for $h \sec z > R$. Since the total intensity follows closely the exponential law, a logarithmic plot of k/j *vs.* $h \sec z$ would resemble Curve I Fig. 8. The upward slope of the first part of the curve is the positive value of the absorption coefficient u of the total radiation. Another type of absorption to be considered corresponds to processes of energy loss whereby the rays give up all of their energy at once, but the depth at which this happens is a matter of probability. This type is characterized by the exponential law. In this case the relative specific intensity

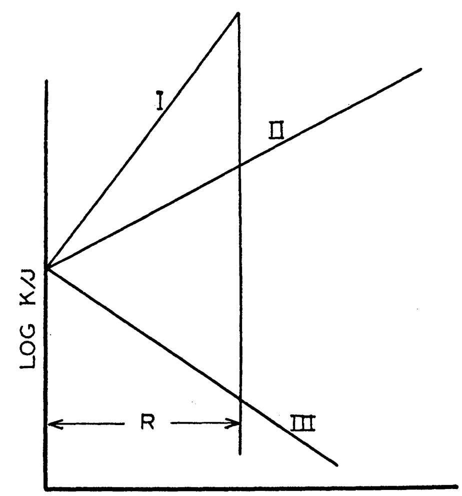

FIG. 8. Log k/j *vs.* h sec z. (I), assuming the positives are characterized by a definite range R; (II), assuming the positives are characterized by an exponential absorption coefficient $v<u$; and (III), assuming the positives are characterized by a coefficient $v>u$.

of unbalanced positives is represented by lines II or III accordingly as the positives are more or less penetrating than the average radiation. The slopes of II and III are the differences, $u-v$, of the absorption coefficients of the average and positive radiations, a quantity which may be called the filtering coefficient.

Although the probable errors are still high, it is evident from Fig. 7 that the results are best represented by curves of type III, Fig. 8, corresponding to exponential absorption with $v>u$. There is no indication of a range type of absorption such as has been usually assumed. It is also worthy of note that the intensity of the positives, within the present accuracy, is a single valued function of h sec z, regardless of the elevation, as must be expected for all types of absorption unless there is an appreciable scattering.

TABLE III. *Results of graphical analyses of the specific intensity of positives and of the average total intensity.*

Geomagnetic latitude, λ	0°	29°	50°
Filtering coefficient, $(v-u)$ per m of water	0.09	0.12	0.27
Absorption coefficient of the average radiation, u, per m of water	.24	.24	.26
Absorption coefficient of the positives, v, per m of water	.33	.36	.53
Specific relative intensity of positives, k_0/j_0	1.73	2.5	12.5
Average x	.50	.43	.21
Average electron energy, 10^9 v	15	10.8	2.5
Average proton energy, 10^9 v	14.1	9.9	1.9

Though the x differences as derived from the LV and Stϕrmer theories are admittedly inaccurate, such inaccuracies should not affect the slopes of the lines representing the data as long as the x_0 differences are approximately linear functions of zenith angle over the short range near the zenith. The vertical displacements, however, would be so affected. There is an indication of a relative hardening of the positives with respect to the average radiation in the higher energy ranges occurring near the equator though the effect is by no means as great as would be expected if ranges $(1/v)$ were proportional to energies. In this connection the actual absorption coefficients of the positives have been derived from the filtering coefficients by adding the coefficients of the average radiation. The latter have been determined graphically in Fig. 7 from the coincidence counter measurements of average intensity *vs.* zenith angle. For the equator these were made in the north azimuth at two elevations, sea level and 4300 meters.[1] In the other latitudes east-west averages of the same data from which asymmetries have been derived have been used. In the case of the Mt. Evans data a somewhat uncertain correction has been made for the unusually high accidental rates experienced at that station. The results of these graphical analyses and data regarding the energies of the rays are collected in Table III.

By making use of the above results and of the values of the cone openings given by LVB the intensity of the unbalanced positive component can be integrated over all directions and over the ranges of x included in the asymmetrical components up to that which comes in with full intensity from the eastern horizon at the equator. A similar integration was made in reference (1) using the values of the cones published in 1933 by Vallarta.[16] Vallarta[17] has also made a somewhat similar calculation in his paper on the longitude effect, using an arbitrarily chosen energy distribution function and without considering the effect of atmospheric absorption. In the following calculations the observed intensity of the unbalanced positives and its variation with atmospheric path are used, so that a comparison of the calculated and observed

[16] M. S. Vallarta, Phys. Rev. **44**, 1 (1933).
[17] M. S. Vallarta, Phys. Rev. **47**, 647 (1935).

latitude and longitude effects may be regarded as a criterion for the existence or nonexistence of an additional mixed component, and as a test of the underlying assumptions. According to Fig. 7 the specific relative intensity of unbalanced positives in the range of x corresponding to the asymmetry in latitude λ may be expressed by

$$(k/j)=(k_0/j_0)_\lambda \exp\left[-(v-u)h \sec z\right] \quad (9)$$

and the total or average intensity by

$$j=j_0 \exp\left[-uh \sec z\right], \quad (10)$$

where, in both cases, the constants are given in Table III. If $f(x,z)$ is the fraction of the elementary solid angle $2\pi \sin zdz$, illuminated by rays of rigidity x, then the total intensity at depth h due to unbalanced positives of rigidity in the range dx, integrated over all angles is

$$\bar{k}(x)dx=2\pi dx\int_0^{\pi/2} k(x, h \sec z)f \sin zdz \quad (11)$$

and the fraction of the total intensity, J, due to rays of rigidity within the observable range below x_{0e} which cuts off at the eastern equatorial horizon is

$$K/J=1/J\int_{x_w}^{x_{0e}} \bar{k}dx, \quad \text{where}$$

$$J=2\pi\int_0^{\pi/2} j_0 \exp\left(-uh \sec z\right) \sin zdz. \quad (12)$$

It is noted that for $x>x_e$, $f(x,z)=1$ and for $x<x_w$, $f=0$. For $x_w<x<x_e$, the values of f are obtained from the results of LVB and are reproduced for $\lambda 0°$ and $\lambda 20°$ in Fig. 9. The integration of (11) and (12) has been made graphically, and with $x_{0e}=0.7$ as an upper limit it is found that at the equator $K/J=10.4$ percent at sea level and 14.2 percent at 4300 meters. Since the edge of the cone for $x=0.7$ is 55° below the zenith, the corpuscular intensity having this rigidity is greatly reduced by absorption and the experiments are somewhat insensitive for determining k/j values out to this limit. A more conservative estimate based upon integrations extending to the limit $x_{0e}=0.6$, which cuts off 30° below the zenith towards the east, is that at the equator at least 4.9 percent

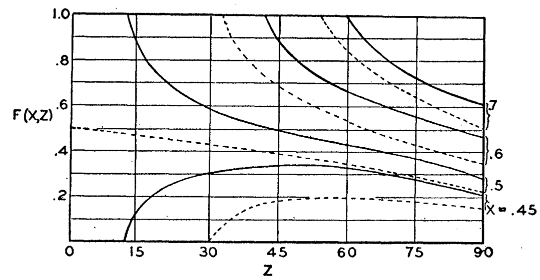

FIG. 9. $f(x, z)$ *vs.* z for various values of x taken from LVB theory. Dotted curves are for $\lambda 0°$ and solid curves for $\lambda 20°$.

at sea level and 6.8 percent at 4300 meters is unbalanced positive radiation.

Since this component is over and above the corpuscular intensity coming into the latitude effect, it may be added to the latter to give the lower limit of the total known corpuscular intensity in high latitudes. Using the results of Compton[18] and the above values corresponding to integration to $x_{0e}=0.6$, it appears that at least 30 percent of the high latitude intensity at 4300 meters and 16.6 percent of the intensity at sea level is due to corpuscular primaries of both signs of charge.

At the latitude $\lambda 20°$ for which f values are also given by LVB the integration of (12) to the upper limit 0.7 gives the values $K/J=12.0$ percent at sea level and 16.9 percent at 4300 meters. Thus the latitude effect between Panama and Peru, accountable by positives alone, is $12.0-10.4=1.6$ percent at sea level and $16.9-14.2=2.7$ percent at 4300 meters. If the values for $x_{0e}=0.6$ are used the predicted latitude effects are 1.4 percent at sea level and 2.0 percent at 4300 meters. Comparing these figures with the sea-level measurements of Millikan and Neher[19] and the airplane measurements of Bowen, Millikan and Neher[20] it appears, anomalously, that the unbalanced positives are more than enough to account for the whole of the latitude effect between these two latitudes. The observed effect at sea level is only 0.5 percent and at 4300 meters no effect was found within an uncertainty which may be estimated at about three percent. In the case of the sea-level

[18] A. H. Compton, Phys. Rev. **43**, 387 (1933).
[19] R. A. Millikan and H. V. Neher, Phys. Rev. **47**, 205 (1935).
[20] I. S. Bowen, R. A. Millikan and H. V. Neher, Phys. Rev. **46**, 641 (1934).

data there thus seems to be a definite discrepancy between the theory and the observations and there is a probable discrepancy at the higher elevations.

A criterion for the existence of a mixed component which should be more satisfactory from the theoretical standpoint is the comparison of observed longitude effects with those to be expected from the unbalanced positive intensity. This calculation involves only the cone openings at the equator. Following the method of Vallarta, it is assumed that the longitude effect is caused by the displacement of the magnetic center of the earth from its geometric center. Distances from the magnetic center of several points on the earth's surface and a compilation of observed intensities have been copied directly from Vallarta's paper.

To determine the longitude effect expected from a given intensity of corpuscular radiation integrations similar to (12) must be performed between limits chosen to correspond with the different magnetic radii. Since the curves of LVB, analogously with Eq. (5), fix the values of x as a function of λ and the direction, independently of r, the limits of integration expressed as x values are also fixed independently of r although the absolute rigidity $H\rho$, corresponding to a given value of x varies with r according to (4). It is therefore convenient to define $x=r(H\rho/M)^{\frac{1}{2}}$ as the value of x corresponding to a given $H\rho$ at a distance from the magnetic center equal to the standard earth radius, $r=6370$ km, and to define $x'=r'(H\rho/M)^{\frac{1}{2}}$ as the value of x corresponding to the same value of $H\rho$ at a point where the magnetic radius is r'. The LVB theory then defines the same cone openings at all points on the magnetic equator for the same values of x' though for a particular value of x the cones vary from place to place with r'. To integrate (12) over a definite band of radiation the limits must correspond to fixed values of x. Thus to determine the intensity due to rays of rigidity less than x_e, the integration must extend to $x'=x_e'r/r'$. For $x'>x_e'$, $f(x',z)=1$, and it is convenient to separate (12) into two parts

$$K/J=(K/J)_0+\Delta K/J,$$

in which the longitude effect is (13)

$$\Delta K/J=1/J\int_{x_e'}^{x_e'r/r'} k(x')\sin z dz \tag{14}$$

and $(K/J)_0$ is the relative corpuscular intensity at the standard earth radius, r.

To an approximation consistent with present experimental accuracies it appears unnecessary to take account of a possible functional relation between k and x either for the range of x included in any particular asymmetrical component or, even less so, for ranges involved in the longitude effect. Thus the second term of (13) reduces to the simple form

$$\Delta K/J=x_e'k_0(r'-r)G(vh)/j_0rG(uh), \tag{15}$$

where the Gold integral,

$$G(y)=2\pi\int_0^{\pi/2}\exp(-y\sec z)\sin z dz.$$

With the values of the constants given in Table III and with $x_{0e}'=0.7$, (15) reduces, for sea level at the equator to

$$\Delta K/J=0.35(r'-r)/r. \tag{16}$$

Values of the longitude effect calculated from (16) are given in Table IV where they are compared with the observed values, $\Delta I/I$. The agreement is not as good as could be hoped for, and in general, the positive component seems insufficient to account for as large a variation with longitude as is observed. Since this same component was more than sufficient to account for the latitude effect, it seems impossible to reconcile the two conclusions, except through

TABLE IV. *Comparison of observed longitude effect with that expected from the unbalanced positive intensity at the equator. All points are on the geomagnetic equator.*

Location	Longitude	$(r'-r)/r$	$\Delta K/J$ Calc.	$\Delta I/I$ Obs.
Lima, Peru	76W	0.023	0.008	0.012
SS Lane, Honolulu to Sydney	160W	−.039	−.014	.008
SS Lane, Bombay to Colombo	75E	.0031	.001	.000
SS Lane, S. Hampton to Cape Town	15W	.044	.015	.042
SS Lane, Hong Kong to Singapore	110E	−.0236	−.008	−.038

some modification of the theory. Possibly consideration of the real field of the earth would provide a satisfactory interpretation of the situation. It may be noted that the latitude effect between the equator and 20°N for the voyage of the Millikan-Neher electroscope between Los Angeles and Melbourne was about 2.5 percent and if this is taken as a better value for comparison with the calculations based upon the equatorial asymmetry, both latitude and longitude effects demand the existence of an additional mixed component. On the other hand it must be borne in mind that certain corrections for diffusion of secondary radiations and for accidental counts are suggested as being applicable to the asymmetry measurements and these tend to raise the calculated latitude and longitude effects, so that it is quite possible that the positives alone can account for the whole of the field sensitive component.

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The Infrared Absorption Spectrum of Solid Hydrogen Chloride

PAUL E. SHEARIN, *Mendenhall Laboratory of Physics, Ohio State University*

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The infrared absorption spectrum of solid hydrogen chloride has been studied in the region of 3.7 μ with the aid of an echellette grating. The band is composed of a series of fairly broad lines, and if the center is taken to be the line at 2669 cm^{-1} the remaining lines taken in pairs can be grouped about this line with approximately equal spacings. Thus it appears that the band differs from the band in gaseous hydrogen chloride in that it has a zero line.

THE study of the absorption bands in the infrared of gas molecules under high dispersion has revealed many facts enabling one quite clearly to understand their behavior and structure. Many crystals show fully as well-developed bands as do gas molecules when measured with prism spectrometers, but only in few cases have measurements under high dispersion been made. Hardy and Silverman[1] and Silverman[2] have made measurements on quartz crystals and on the carbonates using an echellette grating and have found these bands to have a fine structure far more complex than that of gas molecules. The bands may as in gas molecules be ascribed to vibrations of the atoms comprising the ionic configurations in the crystal lattice, but no mechanism adequately serves to explain the fine structure. Of the proposed explanations perhaps the most plausible is the theory that the ionic configurations may be thought of as rotating under certain constraints. Such theories have been advanced by chemists to explain specific heat curves for crystals. Attempts quantitatively to describe these phenomena have been made by Pauling,[3] Stern[4] and recently more

[1] Hardy and Silverman, Phys. Rev. **37**, 176 (1931).
[2] Silverman, Phys. Rev. **39**, 72 (1932); Phys. Rev. **45**, 158 (1934).
[3] L. Pauling, Phys. Rev. **A36**, 430 (1930).
[4] Stern, Proc. Roy. Soc. **130**, 551 (1930).

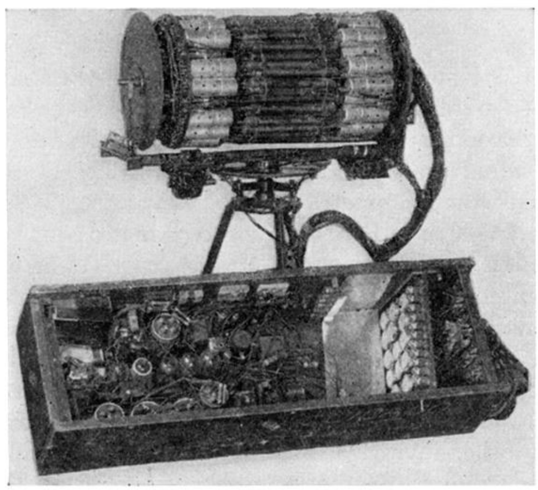

FIG. 2. The multidirectional cosmic-ray intensity comparator and its recording equipment.