

curves in Figs. 1 and 2 have been drawn according to this formula, fitting the data as well as possible, which necessitated taking as upper limits E_0 , 10.5 and 5.25 M.E.V., respectively. It is seen that these fit the experimental points well within the probable statistical fluctuations, although, from the standpoint of the data alone, one might be inclined to place the upper limits slightly lower than 10.5 and 5.25 in consideration of the possibility of a little straggling. The lifetimes of these substances, on the other hand, lie too near those to be expected on the basis of Fermi's original formulation to permit the inclusion of higher derivatives in the interaction energy.

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Interaction of Nuclear Particles

Heisenberg,¹ in an important set of papers, attributed nuclear binding entirely to proton-neutron attractions. Supposing these to be of a classical nature it is easy to show (if the proton-neutron potential energy is a simple monotonic function of the inter-particle distance) that the nuclear binding energy is

$$W \sim -Z(A - Z). \quad (1)$$

This is essentially what is obtained if the Coulomb repulsion between protons is neglected. Inclusion of Coulomb forces would, however, still give a binding energy varying quadratically with the number of nuclear particles. If this were right we could not understand natural radioactivity and the ending of the periodic system of elements. The failure of a classical interaction law is not apparent for the lightest nuclei and Wigner² was able to show that the mass defects of the deuteron and the α -particle were compatible with classical forces. From the experimental law that mass defects are roughly proportional to the number of nuclear particles we see that our theory must give instead of Eq. (1)

$$W \sim -A. \quad (2)$$

Now $Z(A - Z)$ is just the number of interactions of Z protons with $A - Z$ neutrons. Heisenberg, and later Majorana,³ saw that the simplest way of obtaining something like Eq. (2) instead of Eq. (1) was the introduction of interactions of such a nature that a given proton or neutron feels only those neutrons or protons in the same quantum state. They introduced such an interaction and obtained instead of Eq. (2)

$$W \sim -Z. \quad (3)$$

Heisenberg's proposed interaction gave saturation with one-proton-one-neutron, but the proposal of Majorana allowed two-protons-two-neutrons to interact in an effective way. The latter is much more satisfactory.

These were steps in the right direction but, for heavy nuclei, Eqs. (2) and (3) are not equivalent and experiment favors Eq. (2). To improve the theory we must depart from the original Heisenberg hypothesis that only neutron-proton bonds are important. With neutron-neutron and proton-proton bonds of the type suggested by Majorana for the proton-neutron bond, it is possible to develop a theory giving binding energies $\sim -A$ and nuclear radii $\sim A^{\frac{1}{3}}$. These bonds are taken to be the same without regard to the kind of interacting particles. To make this statement more definite we will give, for comparison, expressions for the nuclear potential energy in the theories discussed in this letter.

$$\begin{aligned} \text{(Classical)} \quad V &= -\int \rho_\pi(\mathbf{r}_1) I(r_{12}) \rho_\nu(\mathbf{r}_2) d\tau_1 d\tau_2, \\ \text{(Majorana)} \quad V &= -\int \rho_\pi(\mathbf{r}_1, \mathbf{r}_2) I(r_{12}) \rho_\nu(\mathbf{r}_2, \mathbf{r}_1) d\tau_1 d\tau_2, \\ \text{(Proposed)} \quad V &= -\int \rho(\mathbf{r}_1, \mathbf{r}_2) I(r_{12}) \rho(\mathbf{r}_2, \mathbf{r}_1) d\tau_1 d\tau_2 + S. \end{aligned}$$

In these expressions ρ_π and ρ_ν are the density functions for protons and neutrons, respectively. Where one argument is written, these are just the usual functions but if two arguments are indicated the density matrices of Dirac are to be used. In the last equation $\rho = \rho_\pi + \rho_\nu$ and S is such a function that this expression does not include the interaction of a particle with itself.

The assumption that $I(r)$ has the form of a simple potential hole of depth αmc^2 and radius $\beta \epsilon^2 / mc^2$ has been made. To get agreement with experimental binding energies of nuclei it was found that the parameters α and β must be

$$\alpha \sim 40, \quad \beta \sim 1.00.$$

The calculations on which these results rest are not accurate enough to allow one to decide whether or not all the bonds are of the same strength. It is probable that they are not, so the above numbers must be a sort of average.

Independent arguments for the existence of strong neutron-neutron and proton-proton binding can be found in the number theory properties of stable nuclei. For Z odd and $A > 2Z$ the number of neutrons $A - Z$ is always even. The most simple and unforced explanation of this seems to be that strong attractive interactions exist between paired neutrons. In order that the balance between protons and neutrons, within the nucleus, be preserved, similar forces must exist between paired protons.

White⁴ has just published preliminary results on proton-proton scattering which indicate just such large deviations from the Coulomb law at distances of the order 10^{-13} cm. It is to be hoped that such experiments and furthermore accurate theoretical work will teach us a great deal about these fundamental interactions.

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¹ W. Heisenberg, Zeits. f. Physik **77**, 1 (1932); **78**, 156 (1933); **80**, 587 (1933).

² E. Wigner, Phys. Rev. **43**, 252 (1933).

³ E. Majorana, Zeits. f. Physik **82**, 137 (1933).

⁴ M. G. White, Phys. Rev. **47**, 573 (1935).