Simplified Theory of the Michelson-Morley Experiment

ROY J. KENNEDY, *University of Washington*

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It is shown that a correct application of Huygens' principle in the theory of the experiment leads to the same expression for the expected result as is derived in the simple classical theory. The effect due to path difference is shown to be the same as the effect derivable from the relative rotation of the interfering beams. Critics of the classical theory have mistakenly regarded the latter value as a compensating factor almost exactly offsetting the first.

THE usually accepted theory of the Michelson-Morley experiment has been adversely criticized in a number of papers, the first of which apparently was that of Hicks.[1] Since then Righi[2] and Hedrick[3] have concluded that the effect is not the accepted one, at least not for the so-called "ideal" adjustment of the mirrors, while Woempner[4] and Cartmel[5] derive a third-order expression for the fringe-shift. (By the order of the effect is meant the degree of the ratio β of the velocity of the system through the ether to the velocity of light.) The formulas derived by all these writers involve factors which depend on the adjustment of the apparatus. Righi, Hedrick and Woempner suppose that the fringe-shift to be expected in view of the difference in the times required to traverse the two paths in the interferometer is compensated for by the relative rotation of the two recombining beams, at least in special cases. Their procedure seems to be essentially that of computing the same quantity approximately in two different ways, and then mistakenly subtracting one result from the other. They find that the position of the central fringe is practically unaffected by rotation of the apparatus, but wrongly infer from this fact a null (or very small) effect for the experiment. Oddly enough the third-order effects derived by Woempner and by Cartmel seem to be in good agreement with Miller's experimental data if the linear velocity of the solar system due to rotation of the galaxy is used in evaluating the ratio β.

It was long ago demonstrated by Lorentz[6] that a rotation of the interfering beams of the magnitude actually occurring could not offset the phase difference produced by the relative lengthening of one path as compared to the other. Contrary results have been reached by so many other investigators, however, that it has seemed worth while to attack the problem by a variation of their detailed method with a view to reconciling it with Lorentz's beautifully simple treatment. The present discussion is based almost entirely on the careful application of Huygens' principle to the reflection from the moving mirrors. By this means the directions of the rays in a reference system supposed fixed in the ether are computed; from these directions it is a simple matter to infer the courses of the two beams with respect to the apparatus, and the fringe-shift (or relative phase change) which would result from rotation of the apparatus. It turns out to be unnecessary to compute the lengths of the actual paths in the ether.

In the first place a simple demonstration will be given of the relation of the angle of reflection ϕ to the angle of incidence θ (glancing angles) of a beam of light falling on a mirror moving with velocity v in a direction at an angle α with the normal to the back of the mirror. In Fig. 1 is represented an element of the mirror of length $\delta s(=\overline{op})$. During the time δt between the arrival of the wave front at o and its arrival at p the point p of the mirror will have moved a distance $v\delta t=\overline{pp'}$ to the position p'. Hence by the usual argument it is evident that the reflected wave front will be along $\overline{gp'}$, which is tangent to the

[1] Hicks, Phil. Mag. **6**, 3, 32, 555 (1902).

[2] Righi, Comptes rendus, 1917, several papers. These are summarized in English by Stein, Memori della Societa Astronomica Italiana **1**, 283 (1920).

[3] Hedrick, Conference on the Michelson-Morley Expt., Astrophys. J. **68** (1928).

[4] Woempner, unpublished manuscript.

[5] Cartmel, paper presented at the Pittsburgh Meeting of the Am. Phys. Soc., Dec. 27–29, 1934. Phys. Rev. **47**, 333A (1935).

[6] Lorentz, demonstration restated at Conference, reference 3.

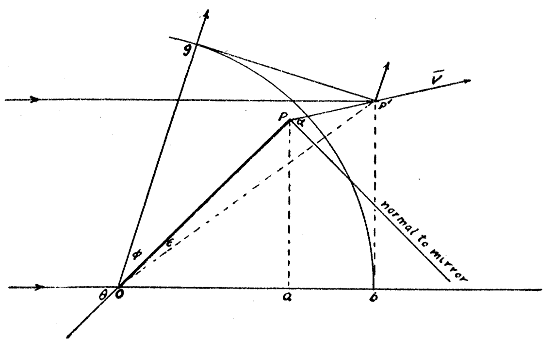

FIG. 1. Mirror OP moving with velocity $\bar{v}$. Incident ray at a glancing angle θ.

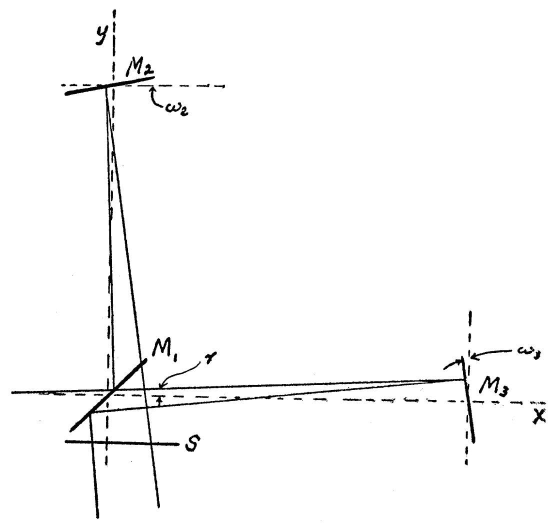

FIG. 2. Arrangement of mirrors.

elementary wavelet centered at o. The point b, which is the projection of p' on the direction of the incident light is at a distance $c\delta t$ from o, c being the velocity of light. Hence from symmetry $\phi=\theta-2\epsilon$. Also

$$\sin\epsilon=(\overline{pp'}/\overline{po})\sin\angle pp'o=v(\delta t/\delta s)\cos(\alpha+\epsilon);$$

$$\therefore\qquad \tan\epsilon=\frac{(v\delta t\cos\alpha)/\delta s}{1+(v\delta t/\delta s)\sin\alpha}.$$

Now $c\delta t=\overline{ob}=\overline{oa}+\overline{ab}$

$$=\delta s\cos\theta+v\delta t\cos(\pi/2-\theta-\alpha)$$

$$\therefore\qquad \delta t=(1/c)\delta s\cos\theta/[1-\beta\sin(\theta+\alpha)].$$

Substituting in above expression we get

$$\tan\epsilon=(\beta\cos\alpha\cos\theta)/(1-\beta\sin\theta\cos\alpha);$$

$$\therefore\quad \varphi=\theta-2\beta\cos\alpha\cos\theta-\beta^2\cos^2\alpha\sin 2\theta. \qquad (1)$$

Here and throughout the discussion terms involving higher powers than the second in β are disregarded, as are also terms such as $\omega^3\beta$, $\gamma_0{}^3\beta$, $\gamma_0{}^2\omega\beta$ and $\gamma_0\omega^2\beta$; γ_0 and ω are defined hereafter.

Following the plan of others, we consider the position of interference bands formed on a screen S due to a plane-parallel beam incident from the left on the diagonal mirror M_1 (Fig. 2). This of course is an idealization of the actual experiment, but it will be shown that similar effects are to be expected. Hence the angle of incidence on M_1 of a ray inclined at a small angle γ to the x axis (fixed in the apparatus as in the figure) will be

$$\theta_1=\pi/4-\gamma=\pi/4-[\gamma_0-\beta\sin(\gamma_0-\psi)],$$

$$\text{where}\quad \psi=\alpha-\pi/4.$$

This expression is got by adding the aberration, $-\beta\sin(\gamma_0-\psi)$, to the angle of incidence that would exist if the system were at rest. This angle is taken as slightly different from $\pi/4$ for the sake of generality.

Applying Eq. (1) to the beam B initially reflected from M_1 and then from M_2 (which is inclined at a small angle ω_2) and to the beam B' initially traversing M_1, then reflected from $\boldsymbol{M_3}$ (inclined at angle ω_3) and again from M_1 we obtain, after correcting for the aberration, the inclinations of the emergent beams to the y axis fixed in the apparatus as, respectively

$$\rho=2\omega_2+\gamma_0+\beta^2\gamma_0+(\beta^2/2)\sin 2\psi+\beta^2/2\cos 2\psi$$

and

$$\rho'=2\omega_3+\gamma_0-\beta^2\gamma_0+(\beta^2/2)\sin 2\psi-\beta^2/2\cos 2\psi.$$

The computation is long and tedious and will not be given here. The list below contains a few details, the subscripts referring to the particular mirror involved; the primed values are for reflection of the beam B' from M_1.

$$\theta_1=\pi/4-[\gamma_0-\beta\sin(\gamma_0-\psi)],\qquad \alpha_1=\psi+\pi/4,$$

$$\theta_2=\varphi_1+\pi/4-\omega_2,\qquad \alpha_2=\tfrac{3}{2}\pi+\psi-\omega_2,$$

$$\theta_3=\pi/2-\omega_3-[\gamma_0-\beta\sin(\gamma_0-\psi)],\qquad \alpha_3=\psi+\omega_3,$$

$$\theta_1'=-\varphi_3+\tfrac{3}{4}\pi+\omega_3,\qquad \alpha_1'=\psi+\tfrac{3}{4}\pi.$$

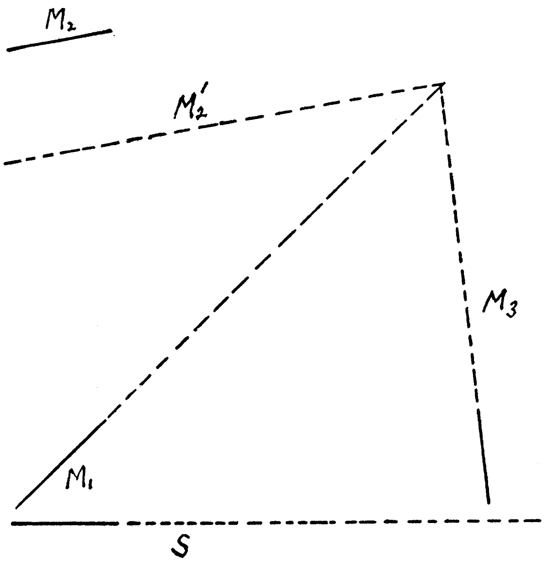

FIG. 3. Positions of mirrors extended.

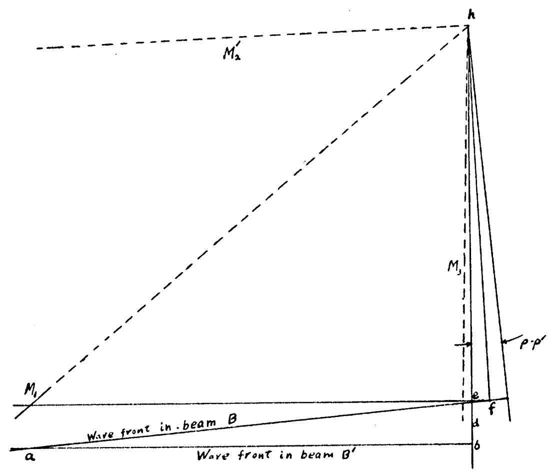

FIG. 4. Positions of interfering wave fronts.

The angle between the wave fronts, then, is simply

$$\rho-\rho'=2(\omega_2-\omega_3)+\beta^2\cos 2\psi.$$

We resort now to a simple device. Let us imagine the mirror M_3, say, extended as indicated by the dotted lines in Fig. 3 to intersect M_1, also extended. From the intersection we imagine a mirror M_2' drawn parallel to M_2. In such an arrangement the angles characterizing the rays would be the same as in the actual case. Now the central fringe, defined as that resulting from the interference of rays having a difference in path in space equal to zero, would evidently be produced by rays incident on the diagonal mirror in the neighborhood of the intersection of the mirrors. Hence this fringe would be formed on the extended screen in the neighborhood of M_3. Furthermore, the expressions for ρ and ρ' indicate that the fringe is practically stationary since, but for the effect of the terms $(\beta^2/2)\sin 2\psi$, the wavefronts oscillate symmetrically about a line (hf, Fig. 4) fixed in the moving system; i.e., they are at equal angles on opposite sides of hf which vary with the orientation of the apparatus.

Now the fringe of interest is not the central fringe defined above, but rather the fringe in the neighborhood of the y axis, in this case a distance from the central fringe approximately equal to the separation l of M_3 and M_1. By referring to Fig. 4 it is seen that a wave front (af) in the beam B, instantaneously passing through the fringe f, produces a fringe at a (in the neighborhood of the y axis) by interfering with a wavefront (ab) in B' with respect to which it is retarded by a constant number n of waves, such that

$$n\lambda=d=l\tan 2(\omega_2-\omega_3).$$

As the apparatus rotates, the fringe formed at this intersection will move about the y axis, its x coordinate being (except for a small constant) evidently

$$x=l-d\cot[2(\omega_2-\omega_3)+\beta^2\cos 2\psi]$$

$$=l\left[1-\frac{\tan 2(\omega_2-\omega_3)}{\tan 2(\omega_2-\omega_3)+\beta^2\cos 2\psi}\right]$$

$$=\frac{l\beta^2\cos 2\psi}{\tan 2(\omega_2-\omega_3)}\text{ approximately.}\quad(2)$$

The approximation is justified, since in the usual performance of the experiment $\omega_2-\omega_3$ is of the order 10^{-4} while β^2 is 10^{-8} for the earth's orbital velocity and 10^{-6} for its velocity due to galactic rotation.

Since the mean width of the fringes is $\lambda/\tan 2(\omega_2-\omega_3)$ where λ is the wavelength of the light employed, we find for the shift expressed in fringe-widths simply the ratio of x to this width, i.e.,

$$\frac{l\beta^2 \cos 2\psi/\tan 2(\omega_2-\omega_3)}{\lambda/\tan 2(\omega_2-\omega_3)}=\frac{l}{\lambda}\beta^2 \cos 2\psi. \quad (3)$$

Now the wave fronts from mirror M_2 are indistinguishable from those from M_2' except as to phase, and from the elementary theory of the experiment it turns out that the variable part of this difference of phase is the distance between M_2 and M_2' multiplied by $(\beta^2/\lambda) \cos 2\psi$. The distance is $l \tan \omega_2$ so the fringe-shift due to this path-difference is $(l/\lambda)\beta^2 \cos 2\psi \tan \omega_2$; because of the factor $\tan \omega_2$ this is evidently ignorable in comparison with the shift expressed in Eq. (3).

The errors of the writers referred to are of two kinds: they either confuse the central fringe defined above with the axial fringe, and so infer a null effect from the fact that the former is practically stationary, or else regard the effect computed above from the angular deviations of the rays as but one component of the whole effect, the other being the shift due to the difference in the lengths of the two paths. By computing the latter approximately (to the second order) and the former to the third order, one erroneously derives a third-order effect. It is readily shown, however, that the *total* effect is expressed in Eq. (3) which we have derived. For the wavefronts in each beam are parallel planes at fixed distances apart, two of which intersect in a line (the central fringe established by a hypothetical case) which is practically fixed in the axes moving with the apparatus. Hence their intersections, and the fringes formed at them, are completely determined in position by the angles between the wave fronts.

Hence the total effect to be expected in the experiment is expressed in Eq. (3), which is the same as results from the simple approximate theory.

The same variation in phase exists in the "ideal" case, i.e., that in which the end-mirrors are exactly perpendicular to the axes, although then the approximate expression in terms of fringe-width would not be valid and the fringes would become indefinitely broad. Nevertheless the method employed by the writer[7] and Illingworth,[8] in which the phase-shift would exhibit itself by unbalancing a split photometric field, would still be applicable. In other words the "ideal" case is special only in that the fringes produced are too broad to permit direct visual estimation of their positions.

The actual experiment, of course, deals with cones of rays brought finally to a focus in a telescope, instead of plane waves interfering on a screen. The latter have been considered here only because they have given rise to the whole confusion. It is much simpler, paraphrasing Lorentz, to treat the general case. The phase-differences may be computed by the elementary method in which the second-order differences in direction discussed above are ignored for the reason that such directional differences can only produce errors of order higher than the second in the result. For, if the approximate value for the length of either path be l_0 while the actual length is a function $l(\epsilon_1, \epsilon_2, \epsilon_3)$ where the arguments are the small angles (of the order of β^2) between corresponding segments of the actual and approximate paths, then on expansion of the function we find for the error

$$l(\epsilon_1, \epsilon_2, \epsilon_3)-l_0=\epsilon_1\frac{\partial l}{\partial \epsilon_1}+\epsilon_2\frac{\partial l}{\partial \epsilon_2}+\epsilon_3\frac{\partial l}{\partial \epsilon_3}$$

$$+\text{terms involving higher powers of the } \epsilon\text{'s}.$$

But Fermat's principle requires that the path be a minimum; hence the first derivatives vanish, showing that the error involves only powers of β higher than the third, and therefore ignorable.

[7] Kennedy, Proc. Nat. Acad. Sci. **12**, 621 (1926).
[8] Illingworth, Phys. Rev. **30**, 692 (1927).