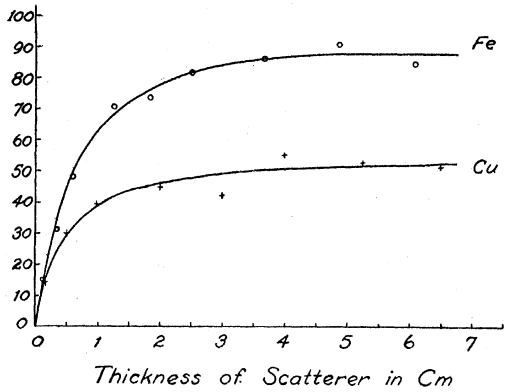

FIG. 2. Activity of Ag + scattering minus activity of Ag divided by activity of Ag alone (in percentage) plotted against thickness of scatterer.

particles were counted in the four minutes when silver alone was present.

The results for iron and copper are shown plotted in Fig. 2. It will be seen that a thickness of about 3.5 cm of either is sufficient to give maximum scattering, which was about 52 percent for copper and 88 percent for iron. It should be noted that the number of atoms per square centimeter is the same in either case, since the atomic densities of iron and copper are about equal. The results are, of course, corrected for the decay of radon.

An attempt was made to get an approximate measurement of the relative number of slow and fast neutrons scattered back by copper. A sheet of cadmium 1 mm thick shielded the neutron source from the silver. With this arrangement about 25 percent as many counts were obtained with Cd as without, indicating that this fraction is due to fast neutrons. A thickness of 4.3 cm of copper placed on top of the silver caused an increase of about 50 per cent in the count.

In another series of experiments the silver foil was not shielded by cadmium from the source, but a sheet of cadmium 1 mm thick, and the same size as the sample, was placed on top of the foil. With this arrangement no change in the activity of the sample was noted. With 4.3 cm of copper placed above the silver and cadmium there was only an 11 percent increase instead of the 52 percent increase obtained without cadmium. This experiment, taken in conjunction with the one just described, indicates that the scattering of fast neutrons by copper is comparable to that of the slow ones.

We should like to thank Dr. Leo Szilard for many suggestions and much helpful discussion. We are indebted to Dr. C. B. Braestrup of the Physical Laboratory of the Department of Hospitals of the City of New York for many favors and also to the American Association for the Advancement of Science for a grant to one of us (A. C. G. M.) with the help of which apparatus has been purchased.

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[1] J. R. Dunning, G. B. Pegram, G. A. Fink and D. P. Mitchell, Phys. Rev. **47**, 416 (1935).

Precision X-Ray Wavelength Measurements

Precision measurements of the diffraction angles (or wavelengths) of x-ray lines are being made with the double crystal spectrometer with an observational error of 0.0002 to 0.001 percent.[1] The correction due to the divergence of the x-ray beam in the vertical plane which is to be applied to these measured angles is of the order of 0.002 percent.

The most recent and generally used correction formula is[2]

$$\delta\theta = [(a^2+b^2)/24L^2] \tan\theta, \quad (1)$$

where a and b are the effective slit heights, L the distance between the slits and θ the Bragg angle. In view of the demands for increasing precision in wavelength determinations it should be pointed out that the numerator of Eq. (1) is in error and that the correct expression is

$$\delta\theta = [(a+b)^2/24L^2] \tan\theta \quad (2)$$

or, in case $\theta_A \neq \theta_B$, as in different orders or reflection or with dissimilar crystals,

$$\delta\theta = [(a+b)^2/48L^2]\lambda D, \quad (3)$$

where D, the dispersion, is given by

$$D \equiv (1/\lambda)(\tan\theta_A \pm \tan\theta_B). \quad (4)$$

For slits of equal heights Eq. (2) differs from Eq. (1) by a factor of two. This angular shift due to the vertical divergence is toward larger values of θ and consequently $\delta\theta$ must be subtracted from the observed angles. Expressions (2) and (3) are derived from a simplification of Schwarzschild's treatment[3] assuming the instrument is in proper adjustment.

If the diffraction patterns of the crystals are asymmetrical, as predicted by the Darwin-Ewald-Prins theory,[4] a correction for this asymmetry should also be applied to the angle observed in any antiparallel position of the double spectrometer. The center of area of the asymmetrical pattern is shifted to a smaller value of θ and the correction would be added to the observed angle. As mentioned by Compton and Allison,[5] this correction in the (1, +1) position, based on the theoretical diffraction patterns of calcite crystals, at the wavelength 1.54A (Cu $K\alpha_1$) is of the order of magnitude of the correction for the vertical divergence of the beam and the two corrections approximately offset each other. However, the degree of asymmetry of the theoretical patterns varies markedly with wavelength. At the wavelength of, and less than, 0.71A (Mo $K\alpha_1$) (which wavelength has been measured most accurately) the theoretical diffraction pattern of calcite is practically symmetrical. The center of area shift due to asymmetry increases with wavelength to 3.06A, the K absorption limit of calcium. At wavelengths greater than 3.06A the theoretical patterns are again more nearly symmetrical until the region of 5 to 6A is reached where the asymmetry is about the same as, or slightly greater than, at 1.54A.

No positive experimental information has been reported as to the supposed asymmetries of the crystal patterns but indications that such asymmetries do exist are apparent from recent measurements of the shapes of x-ray lines in various antiparallel positions of the spectrometer with various crystals.[6]

Then, unless we assume the validity of the Darwin-Ewald-Prins theory, we are at a loss to correct for the asymmetry of the crystal patterns and this state of affairs reduces the relative importance of an accurate slit-height correction. On the other hand, certain features of the theoretical patterns (but not asymmetries, which as yet cannot be directly checked by experiment) have been shown to give surprisingly good agreement with experimental measurements[4] in the (1, −1) positions and it is likely that the actual pattern asymmetries are given qualitatively by the theory as indicated above. One hesitates to say the experimental crystal effects could be evaluated quantitatively because the theory is based on the assumption of a "perfect" crystal which does not actually exist. Perhaps it is best to apply the slit-height correction and await results of future researches to decide the crystal effects.

The above discussion applies also to wavelength measurements made photographically with a single crystal spectrometer when the appropriate divergence correction is substituted for Eq. (3). The crystal effects in this case are, of course, those due to a single crystal instead of the compound effect of two crystals.

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Cornell University,
May 1, 1935.

[1] A. H. Compton, Rev. Sci. Inst. **2**, 365 (1931); J. H. Williams, Phys. Rev. **40**, 791 (1932); J. A. Bearden, Phys. Rev. **43**, 92 (1933).
[2] J. H. Williams, Phys. Rev. **40**, 636 (1932).
[3] M. M. Schwarzschild, Phys. Rev. **32**, 162 (1928).
[4] S. K. Allison, Phys. Rev. **41**, 1 (1932); L. G. Parratt, Phys. Rev. **41**, 561 (1932).
[5] Compton and Allison, *X-Rays in Theory and Experiment*, D. Van Nostrand Co., 1935, p. 737.
[6] L. G. Parratt and L. P. Smith, Phys. Rev. **47**, 805A (1935).
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The Scale of X-Ray Wavelengths

In a further effort to settle the question concerning the crystal and ruled grating scale of x-ray wavelengths the writer has carried out two additional experiments. First, the refraction method has been perfected by using a large diamond prism. Second, large ruled gratings have been used with a double crystal spectrometer to measure the wavelength of the copper $K\alpha_1$ line.

It has been shown[1] that the refraction of x-rays may be used as a means of determining the true scale of x-ray wavelengths. In the previous measurements a quartz prism was used and the wavelengths obtained agreed with the ruled grating values. However, there was some uncertainty in the results because of the difficulty of correctly making allowance for the effective number of electrons in the silicon. The present experiment was designed to eliminate this by using a prism of low atomic number. A diamond fulfills all the requirements better than any other substance.

Thus photographic refraction measurements by the method[1] previously described by the writer have been made for the copper $K\beta$ line using the 90° edge of a perfect diamond block 9 mm×9 mm×3 mm. Two surfaces were polished optically flat and intersected in a very perfect edge, though later it was found possible not to allow any part of the x-ray beam to strike the edge. Fig. 1 shows a typical plate. The average of the results from 25 of the best plates is $\delta=9.224\pm0.0005\times10^{-6}$. This gives for the wavelength on a $\lambda^{2.75}$ absorption law $\lambda=1.3924$A. This is 0.26 percent greater than the best crystal value and is in good agreement with the ruled grating wavelengths as is shown in Table I.

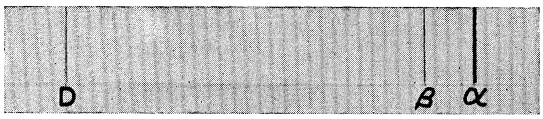

FIG. 1. D is the direct beam, β and α the refracted beams of the copper K series.

TABLE I. $(\lambda_g-\lambda_c)/\lambda_c$.

Observer	Grating	Cu $K\beta$	Cu $K\alpha$	Cr $K\beta$	Cr $K\alpha$	Al $K\alpha$
Bearden(1929)	a, b, c	0.24 (10)	0.25 (10)			
Bearden(1931)	1, c	.241(26)	.229(46)	0.239(16)	0.245(28)	
" "	4	.243(4)	.250(11)	.250(15)	.255(27)	
" "	4′	.264(30)	.257(49)	.253(3)	.254(5)	
" "	5	.246(41)	.234(73)	.235(32)	.239(51)	
" "	5′	.259(49)	.250(82)	.256(44)	.255(67)	
" "	6	.239(11)	.244(16)	.240(3)	.240(4)	
Bäcklin(1928)						0.17 (31)
" (1935)						.249(56)
Söderman(1935)			Cu $K\alpha_1$			.255(9)
Bearden(1935)	7		.253(8)			
" "	8		.247(4)			
" "	Refr.	.260(25)				
	Weighted average 0.248±0.0016%					

One of the objections to the method of using plane gratings for x-rays is that one uses only a small number of lines. Thus in the second experiment a plane grating 75 mm long was used. The entire length was used by placing the grating between the calcite crystals of a double crystal ionization spectrometer. The angles of incidence and diffraction were then measured by the angular displacement of the second crystal. The crystals were set in the (1, +1) position so that the reflected and diffracted lines were essentially in the (1, −1) position. By this method the two most important angular measurements were made with lines which were only 11 seconds to 16 seconds wide. Four carefully calibrated microscopes were used to read the precision circle and since two lines, ten minutes apart, were read in each microscope, eight angular readings were obtained for each individual setting of the circle. Different parts of the circle were used in order to eliminate any errors due to possible erratic rulings on the circle. Two gratings were used, the first (7, Table I) was ruled with 100 lines per mm and the second (8, Table I) was ruled with 300 lines per mm. The average of all 12 results gives for the copper $K\alpha_1$ line, $\lambda=1.5405$A. This is 0.25 percent greater than the corresponding crystal value and is in excellent agreement with the writer's previous results.[2]

Bäcklin[3] has recently repeated his earlier measurements on the Al $K\alpha$ line and now obtains results almost identical with the writer's 1929, 1931 and the present results. Also Söderman[4] has used a concave grating to compare a high order of the Al $K\alpha$ line with the first order of a known