Neutron-Proton Interaction

Part II. The Scattering of Neutrons by Protons

EUGENE FEENBERG, *Research Laboratory of Physics, Harvard University*
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It may be that also for free particles, as well as in the case of binding, the neutron-proton interaction operator can be represented by a potential $J_1(r)$. Because the conditions of binding and of scattering are so different there is no reason to expect J_1 to be identical with J, the potential for binding. The inequalities $J_1<J$ and $\partial J_1/\partial W<0$ (W denotes the kinetic energy) should hold if some part of the potential results from a polarization of each particle in the field of the other or from a high frequency exchange process. For slow neutrons a thirty percent *decrease* in the magnitude of the interaction in going from binding to free particles yields an *increase* in the cross section for scattering by protons of several hundred percent over the value given by J. This larger value is required by recent measurements. The rapid fall of the experimental cross section with increasing W requires that J_1 decrease steadily as W is made larger in accordance with the relation $J_1(r, v) = e^{-g(\beta_0+v/c)}J(r)$. Here g is a positive constant, v is the relative velocity of the colliding particles and $\beta_0 c$ is identified with the average relative velocity of the particles in the deuteron.

In Part I of this paper[1] it was shown that the binding energies of the hydrogen and helium isotopes can be understood in terms of a neutron-proton interaction potential, $J(r)=Ae^{-\alpha r^2}$, with $A=170\ m_ec^2$, $1/\alpha^{\frac{1}{2}}=1.3\times10^{-13}$ cm. These values are averages of those obtained from the theories of Wigner and of Majorana. The differences between the theories are not relevant to the present discussion.

There appears to be no good reason for believing that the representation of the interaction operator as a potential function is in any sense exact or fundamental. In fact, any potential function which is small except for distances of separation less than 10^{-12} cm and gives the correct value for the binding energy of the deuteron will yield for the scattering cross section of neutrons by protons values in striking disagreement with the experimental facts.[2] It seems necessary to suppose that the interaction operator involves both the separation of the particles and their momenta and also (in scattering problems) in some way the collision time. In the absence of a fundamental theory the only means of bringing to light the properties of the interaction operator is to make simple rather naive assumptions and compare the consequences with experiment. We need not expect any one simple assumption to be adequate for all problems. By comparing the different assumptions required to explain different phenomena, for example, binding and scattering, something may be learned about the fundamental interaction operator. From this point of view the potential $J(r)$ is an approximate representation of the unknown fundamental operator, suitable for the description of binding.

It may be that for the scattering of neutrons by protons as well as in the case of binding, the interaction operator can be represented approximately by a potential $J_1(r)$ (Wigner theory) or $J_1(r)P_{np}$ (Majorana theory). Now since the conditions of scattering and of binding are so very different, we need not be surprised if J_1 differs from J. A value for J_1 greater than J or a J_1 increasing with increasing relative velocity of the colliding particles would be difficult to understand. But the inequalities

$$J_1(r)<J(r),\quad \partial J_1/\partial W<0, \tag{1}$$

relating J_1 to J and expressing the trend of the dependence of J_1 on the relative kinetic energy, W, of the colliding particles, are acceptable. Indeed, (1) must hold if a mutual polarization of each particle in the field of the other contributes to the interaction potential or if some part of the interaction arises from a high frequency exchange process such as that suggested by Heisenberg.[3] To insure that (1) is satisfied we write

$$J_1(r)=e^{-f(W)}J(r), \tag{2}$$

[1] See preceding paper.
[2] Massey and Mohr, Proc. Roy. Soc. **A148**, 206 (1935).
[3] Heisenberg, Zeits. f. Physik **77**, 1 (1932).

in which $f(W)$ is a positive valued monotonic increasing function of W. This involves the assumption that the effective radius of interaction changes little in passing from conditions of binding to those of free particles. The simple qualitative picture of the relation between the binding and the scattering interactions expressed by (1) and (2) is consistent with the facts.

The essential experimental facts are these:[4] For very slow neutrons the cross section for scattering by protons has the large value 13.3×10^{-24} cm^2. The cross section falls rapidly with increasing velocity coming down to 2.53×10^{-24} cm^2 ($v=1.3\times10^9$ cm/sec.), 1.41×10^{-24} cm^2 ($v=2\times10^9$ cm/sec.), and 0.73×10^{-24} cm^2 ($v=3\times10^9$ cm/sec.).

We proceed to the determination of $f(W)$ by a comparison of computed cross sections with the experimental values using first the simple model defined by the equations[5]

$$\begin{aligned} &J(r)=D, \quad 0\leqq r\leqq a=0.15,\\ &J_1(r)=D_1(W)\equiv De^{-f(W)},\\ &J_1(r)=J(r)=0, \quad r>a. \end{aligned} \tag{3}$$

Here D has the value 139 computed from the relation[6]

$$\begin{aligned} D=(\pi/2a)^2+2|E|^{\frac{1}{2}}/a+|E|(1-4/\pi^2)\\ -(4/\pi-\pi/3)|E|^{\frac{3}{2}}(8/\pi^3)a+\cdots \end{aligned} \tag{4}$$

connecting the depth and breadth of the hole with the energy eigenvalue of H^2.

The cross section for scattering by the potential J is[6]

$$\sigma=10.1\{1+a|E|^{\frac{1}{2}}\}/(W+|E|), \tag{5}$$

in units of 10^{-24} cm^2, subject to the condition $a|E|^{\frac{1}{2}}\ll\pi^2/8$. In our case $a|E|^{\frac{1}{2}}=0.15\times2.0 \doteq 0.30$ which is small enough. This yields about one-fourth of the experimental cross section for slow neutrons and double the experimental value for fast neutrons. It is not possible to obtain agreement for *either* high or low velocities by changing a within reasonable limits.

The cross section given by J_1 is most simply expressed in terms of the quantities $p=(W)^{\frac{1}{2}}$ and $x=\pi/2-a(D_1+W)^{\frac{1}{2}}$. The scattering cross section is easily shown to be

$$\sigma=10.1a^2\frac{\{\cos pa-(\pi/2-x)\tan x\cdot\sin pa/pa\}^2}{(\pi/2-x)^2\tan^2 x+(pa)^2} \tag{6}$$

in units of 10^{-24} cm^2. Only the partial wave with zero angular momentum is here considered.[7] With the aid of (6) and the experimental cross sections $f(W)$ was first determined at $p=0.0$ and $p=2.0$. The assumption that $f(W)$ is a linear function of p then yields perfect agreement with the experimental cross sections for other values of p. In this way it was found that

$$\begin{aligned} J_1(r)&=e^{-0.09(3.74+p)}J(r),\\ &=e^{-1.93(0.18+v/c)}J(r), \end{aligned} \tag{7}$$

in the case of the simple model defined by (3).

We have used (3) because of its mathematical simplicity and because the cross section depends only slightly on the precise form of the potential. The analysis of the data, by means of numerical integrations, in terms of the Gaussian error potential leads to essentially the same functional dependence on W with slightly different numerical coefficients:

$$\begin{aligned} J_1(r)&=e^{-0.105(4.26+p)}Ae^{-\alpha r^2}\\ &=e^{-2.26(0.20+v/c)}Ae^{-\alpha r^2} \end{aligned} \tag{8}$$

Eqs. (7) and (8) show that the transition from binding to free particles is marked by a thirty to thirty-five percent decrease in the depth of the potential hole. It should be noted that for low velocities a relatively small *reduction* (thirty percent) in the magnitude of the interaction is accompanied by an *increase* of several hundred percent in the collision cross section. The reason for this is plain from (5). As the bottom of the potential well is raised the eigenvalue is squeezed out and vanishes at a definite depth. At this point $\sigma(0)$ is infinite. Immediately beyond this

[4] Dunning, Pegram, Fink and Mitchell, Phys. Rev. **47**, 416 (1935); Bonner, Phys. Rev. **45**, 601 (1934). See also, Dunning, Phys. Rev. **45**, 586 (1934); Chadwick, Proc. Roy. Soc. **A142**, 1 (1933); Meiter and Philipp, Naturwiss. **20**, 929 (1932).

[5] The units are $m_ec^2=510{,}000$ e.v. for energy and $(h^2/4\pi^2m_em_pc^2)^{1/2}=8.97\times10^{-13}$ cm for length.

[6] Wigner, Zeits. f. Physik **83**, 253 (1933).

[7] For the justification of this assumption see Bethe and Peierls, Proc. Roy. Soc. **A149**, 176 (1935).

depth there exists no discrete energy level, but σ is still very large.

Table I exhibits the results of the analysis.

TABLE I. *Cross sections computed from (7) and (8).*

	Rectangular potential well (7)		Gaussian error potential (8)
p*	x	$\sigma\times 10^{24}$ cm^2	$\sigma\times 10^{24}$ cm^2
0.0	0.077	13.6	13.6
0.5	0.109	5.2	
1.0	0.134	2.4	2.4
1.5	0.156	1.3	
2.0	0.173	0.78	0.78

* v (in units of 10^9 cm/sec.) $=1.40\ p$.

These results demonstrate that the inequalities (1) are consistent with the experimental cross sections. But they do more than that. The function $f(W)$ has the form $g(\beta_0+v/c)$; clearly $c\beta_0$ must be interpreted as a velocity. The only velocity other than v which can possibly be associated with the neutron-proton scattering process is the average relative velocity of the particles in the deuteron. A numerical integration yields the value $23m_ec^2$ for the internal kinetic energy of the deuteron (compare Part I. Table III); hence the average relative velocity is 0.22 c, only ten or twenty percent larger than β_0c. Since J_1 is less than J the acceleration experienced by the particles during a collision must be somewhat less than the acceleration to which particles in the deuteron are subjected. This enables us to understand why β_0c is smaller than the average relative velocity of the particles in the deuteron and yet of the same order of magnitude.

Heisenberg[3] has suggested that the interaction of neutrons with protons is associated with or caused by a high frequency exchange process. The frequency is determined by the relation

$$h\nu\sim J(r)m_ec^2\sim(hc/2\pi e^2)m_ec^2. \qquad (9)$$

It is not necessary, and perhaps undesirable, to say much about the physical nature of the exchange process. The idea of exchange forces transcends in its generality the simple nuclear theories (those of Heisenberg and of Majorana) in which it has consciously been used as a guide and hence may be applied without presupposing a definite physical model. What is essential is that the frequency given by (9) is associated with the interaction. The crudest sort of classical reasoning based on the interpretation of β_0c+v as the average velocity of the particles during the collision suggests that the collision time has the order of magnitude $(e^2/m_ec^2)/(\beta_0c+v)$. The number of exchanges which occur during a collision is then

$$n\sim 2\nu(e^2/m_ec^2)/(\beta_0c+v)\sim c/\pi(\beta_0c+v). \qquad (10)$$

There is time for only two or three exchanges even with the very slowest neutrons. It is not surprising then that J_1 should decrease rapidly with increasing W. The relation

$$J_1(r)\sim e^{-g/\pi n}J(r) \qquad (11)$$

obtained by combining (10) with (7) and (8) brings out the point that J_1 is equal to J when n is infinite (the case of binding). It may be concluded that the experimental results on scattering are consistent with and, perhaps, even lend support to the general qualitative picture of exchange forces.