

individual projectiles. Section II, Fig. 1 represents two reactions in which deuterons are involved, section II gives a behavior characteristic of atoms of higher atomic number in which a neutron is added and the nucleus does not immediately disintegrate, but exhibits radioactivity.

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April 12, 1935.

- <sup>1</sup> W. D. Harkins and S. L. Madorsky, Phys. Rev. 19, 136 (1922).  
<sup>2</sup> W. D. Harkins, Phil. Mag. 43, 315 (1921).  
<sup>3</sup> W. D. Harkins and D. M. Gans, Phys. Rev. 46, 397 (1934).

#### The Seasonal Variation of Ionization in Region F<sub>2</sub> of the Ionosphere

In his recent letter<sup>1</sup> in this journal Dr. E. O. Hulburt refers to the measurements of noon maximum ionization in Region F<sub>2</sub> and states that there appears to be a discrepancy between the results obtained in different latitudes. It is my object in this letter to show that by means of a different interpretation of the published data the discrepancy can be satisfactorily removed.

If we reject the contention of Kirby, Berkner and Stuart<sup>2</sup> that critical frequency measurements merely indicate absorption phenomena and accept an interpretation<sup>3</sup> of their electron-limitation significance in terms of a pronounced seasonal variation of molecular temperature, we are obliged to conclude that noon Region F<sub>2</sub> maximum ionization in temperate latitudes does not follow the expected seasonal variation and is actually slightly higher on a winter noon than on a summer noon. Dr. Hulburt correctly points out that if my interpretation on these lines is accepted there is a discrepancy between this result and other evidence. He refers in particular to the ratio (1.5 to 1.8) for summer to winter noon ionization, quoted by me in a general ionospheric discussion in London, and which was, in fact, calculated from Dr. Hulburt's own values of short wave "skipped distances." The discrepancy can therefore be narrowed down to the difference between the interpretation of critical-frequency measurements made with medium wavelengths at short distances and with short wavelengths at large distances. It means, briefly, that my interpretation of the local ionospheric measurements indicates that the maximum Region F<sub>2</sub> noon ionization is slightly less in winter than in summer, whereas Dr. Hulburt's "skipped distance" data indicate, as he himself has shown,<sup>4</sup> that Region F<sub>2</sub> ionization is 1.5 to 1.8 times as great on a summer noon as on a winter noon.

It would be improper for me to question the accuracy of Dr. Hulburt's measurements of "skipped distances" were it not for the fact that other measurements of similar character, made by C. R. Burrows<sup>5</sup> in America, yield entirely different results which are in good agreement with the local ionospheric measurements, in that the measured critical frequency is less on a summer noon than on a winter noon. It is true that Burrows interprets his results in terms of absorption-limitation, but if we regard both local and long-distance measurements as referring to electron-

limitation phenomena (as Dr. Hulburt and I agree that they should be) there is accord between the conclusions drawn from both sets of measurements.

It may not be superfluous to add that this discussion is restricted to noon measurements and not to the maximum value of ionization density which may be experienced throughout the whole of a summer or winter day.

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March 23, 1935.

- <sup>1</sup> Hulburt, Phys. Rev. 47, 422 (1935).  
<sup>2</sup> Kirby, Berkner and Stuart, Bur. Standards J. Research 12, 16 (1934).  
<sup>3</sup> Appleton, Phys. Rev. 47, 89 (1935).  
<sup>4</sup> Hulburt, Phys. Rev. 31, 1018 (1928); 35, 240 (1930); 39, 977 (1932).  
<sup>5</sup> Burrows, Proc. Inst. Rad. Eng. 19, 1634 (1931).

#### Nuclear Magnetic Moment of Na<sup>23</sup>

Through the courtesy of Professor Hartree we were supplied in the summer of 1933 with s.c.f. functions for Na<sup>+</sup> and with series electron functions for 3s, 3p, 4p calculated by J. McDougall in collaboration with him. We computed then by means of these functions the expected ratio between the hyperfine structure interval factor  $a$  in cm<sup>-1</sup> and the nuclear moment  $\mu$  in nuclear magnetons as well as the absolute value of the doublet splitting  $\Delta\nu$  for the 3p and 4p terms.

The theoretical values of the gross doublet splittings disagree with experiment by roughly the same large factor by which the Hartree function result for the h.f.s. disagrees with the Landé-Fermi-Goudsmit formulas. These discrepancies were brought out in the symposium on nuclear moments<sup>1</sup> in June, 1934, and it was emphasized that in view of them one cannot be sure of the theoretical conclusions about the values of nuclear moments for any but the simplest atoms.

For 3s, by taking into account only the energy of the series electron in the central field, the effective quantum number  $n^* = 1.75$  as compared with 1.63 experimentally. Correcting for interaction with inner shells as has been done by McDougall,<sup>2</sup> we obtain a theoretical  $n^* = 1.68$ . The difference between this and 1.63 is small and the difference between the Landé-Fermi-Goudsmit result and the theoretical one is therefore significant. For 3p the theoretical  $n^* = 2.26$  and the experimental = 2.12. It is vital to orthogonalize the series electron functions to the core functions in computing  $\mu$  and  $\Delta\nu$  because  $1/r^3$  and  $(1/r)(dV/dr)$  are sensitive to the amplitude of the functions at small  $r$ . The factor due to this correction is approximately 14 for 3p.

The effects discussed should similarly be sensitive to perturbations by configurations involving excitation of core electrons. A calculation of the first-order effect of a perturbation of (2s)<sup>2</sup>(3p) by (2s)(3s)(3p) changes the theoretical h.f.s. value by about 8 percent and is not sufficient to matter materially. Similarly the effect of (2p)<sup>5</sup>(3p)<sup>2</sup> on (2p)<sup>6</sup>(3p) is appreciable but not sufficient to bring about agreement. Effects on h.f.s. due to the first power of

coefficients of the perturbing configuration disappear by using Fock's field.<sup>3</sup>

The usual computation of nuclear  $g$  factors is not necessarily wrong, but it is fair to say that it is uncertain on account of the sensitiveness of the theoretical values to perturbations.<sup>4</sup> We intended to postpone publication until sufficiently detailed calculations were made to ascertain the  $g$  factors. Professor J. H. Bartlett kindly informed us that he and his co-workers<sup>5</sup> have made related calculations for  $K$  using the Hartree field and for Na using Fock's field, and it seemed of interest to report the present state of our calculation.

The results of Shoupp, Bartlett and Dunn for  $3s$ ,  $3p$  are easily extended to  $4s$ ,  $4p$ . We summarize in Table I their values, our Hartree field values, and our Fock field values for  $4p$ . The ratio  $\psi_{3s^2}(0)/\psi_{4s^2}(0)=4.75$  by using Fock's functions, and 4.28 by using  $Z_i Z_0^2/n^{*3}$ . For Hartree', Fock' it was supposed that the ratio of the h.f.s. and the multiplet

TABLE I.

$a$	Experiment <sup>7</sup>	$3s$		$3p_{3/2}$		$4p_{3/2}$	
		0.029	$8.3 \times 10^{-4}$	$(7.5 \times 10^{-4})$	$1.87 \times 10^{-4}$		
$\mu$	Hartree	5.85	22	(20)	10.4		
	Fock	2.5*	5.1*	(4.6)	3.1		
	Goudsmit-Fermi <sup>7</sup>	2.0	2.8	(2.6)	2.0		
	Hartree'		3.3	(3.0)	2.4		
Fock'		3.1*	(2.9)	2.3			
$\Delta\nu$	Hartree		2.6	$\text{cm}^{-1}$	1.2	$\text{cm}^{-1}$	
	Fock		10.7*		4.0		
	Experiment		17.2		5.49		

frequency separations is given correctly by the respective functions of Hartree and Fock. The ratio of the two frequency differences as obtained from experiment was then equated to its theoretical value which involves  $\mu$  and hence  $\mu$  was determined. This amounts to defining an effective  $Z_i$  by  $Z_i = -(dV/dr)/(1/r^3)$  where  $V$  is the potential of the central field, and by using this  $Z_i$  in Goudsmit's<sup>6</sup> Eq. (6). The Hartree and Fock values of  $Z_i$  are 8.91 and 8.54. The first of these was communicated by us to Ellett and Heydenburg.<sup>7</sup> The  $Z_i$  and  $\mu$  thus determined are insensitive to progressive errors in the computation of the wave function and depend essentially on its shape for small  $r$ . They may be expected to be nearly the same for any central field calculation. The values of  $\mu$  obtained by means of this  $Z_i$  will be correct if there exists a central field which suffices for the discussion of the hyperfine and the ordinary fine structures; they may be wrong if there are perturbations affecting the multiplet and h.f.s. structures unequally.

The values marked\* were kindly supplied to us by Shoupp, Bartlett and Dunn.

Values of  $\mu$  obtained<sup>8, 9</sup> from  $6s$ ,  $6p$ ,  $7p$ ,  $8p$  of Cs check each other satisfactorily. Measurements of Wood and Fortrat on the principal series of Na disagree with the Landé gross doublet formula while for K, Cs the formula holds well. There is thus some evidence that Na is a poor element for testing the theory. The difference between  $\mu$  ( $3p$ ) and  $\mu$  ( $4p$ ) may be due, however, partly to experi-

mental error as is seen by comparing values in each row of the  $3p$  column.

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April 14, 1935.

<sup>1</sup> G. Breit, Phys. Rev. **46**, 340 (1934).

<sup>2</sup> J. McDougall, Proc. Roy. Soc. **A138**, 550 (1932).

<sup>3</sup> Chr. Møller and M. S. Plesset, Phys. Rev. **46**, 618 (1934).

<sup>4</sup> G. Breit and I. I. Rabi, Phys. Rev. **46**, 231 (1934).

<sup>5</sup> W. E. Shoupp, J. H. Bartlett, Jr. and C. G. Dunn, Phys. Rev. this issue.

<sup>6</sup> S. Goudsmit, Phys. Rev. **43**, 636 (1933).

<sup>7</sup> A. Ellett and N. P. Heydenburg, Phys. Rev. **46**, 583 (1934).

<sup>8</sup> L. P. Granath and R. K. Stranathan, Phys. Rev. **46**, 316 (1934); Bull. Am. Phys. Soc. April 25-27 (1935); Paper 80.

<sup>9</sup> N. P. Heydenburg, Phys. Rev. **46**, 802 (1934).

### The Magnetic Moment of the Na<sup>23</sup> Nucleus

From the most recent experiments with sodium<sup>1-5</sup> it has been concluded that the nuclear spin  $I=3/2$  and that the total h.f.s. splitting<sup>1</sup> for the  $3^2S_{1/2}$  state is  $0.0583 \text{ cm}^{-1}$ , for the  $3^2P_{3/2}$  state  $0.0050 \text{ cm}^{-1}$ , and for the  $3^2P_{1/2}$  state  $0.0083 \text{ cm}^{-1}$ . Ellett and Heydenburg, using formulae developed by Goudsmit<sup>6</sup> and Fermi and Segré,<sup>7</sup> have calculated values of the nuclear magnetic moment. From the  $3^2S_{1/2}$  state, they find  $\mu=2.02$  nuclear magnetons; while from the  $3^2P$  states, they obtain  $\mu=2.3-2.6$  n.m. Hartree wave functions were used by Wills and Breit,<sup>8</sup> who obtained values  $\mu=5.8$  and  $\mu=22.6$  from the  $S$  and  $P$  states, respectively. Since, however, their calculations show the multiplet splitting for the  $2P$  state to be  $2.60 \text{ cm}^{-1}$ , as compared with the observed value of  $17.6 \text{ cm}^{-1}$ , it is fairly obvious that better wave functions than those of Hartree are needed for accurate calculations. It is the purpose of this note to investigate whether or not the functions recently published by Fock and Petrashen<sup>9</sup> will serve the purpose.

The h.f.s. separation of a  $2S$  state is  $\Delta(s) = (8\pi/3)\{(2I+1)/I\}\mu\mu_0\psi^2(0)$ ; that of a  $2P_{3/2}$  state is  $\Delta^2 P_{3/2} = (8/3)\{(2I+1)/I\}\mu\mu_0(1/r^3)$ . The gross structure separation for the  $2P$  state is  $\delta = (3\mu_0^2/hca_0^3)(1/r)(dU/dr)$ , where  $U(r) = -11/r + V(r)$ , and  $V(r)$ , is tabulated by Fock and Petrashen.

We find the following results:  $\lim_{r \rightarrow 0} (f_{3s}/r) = 2.86$ ;  $(1/r^3)_{3p} = 0.144$ ;  $(1/r)(dU/dr)_{3p} = 1.23$ ; from  $2S$ ,  $\mu = 2.5$  n.m.; from  $2P$ ,  $\mu = 5.1$  n.m.; and  $\delta = 10.7 \text{ cm}^{-1}$ . The values of  $(1/r)(dV/dr)$  are given in Table I.

TABLE I.

$r$	$-\frac{1}{r} \frac{dV}{dr}$	$r$	$-\frac{1}{r} \frac{dV}{dr}$	$r$	$-\frac{1}{r} \frac{dV}{dr}$	$r$	$-\frac{1}{r} \frac{dV}{dr}$
0.00		0.24	147	0.60	23.1	1.4	3.43
.02	2400	.26	123	.65	19.9	1.5	2.84
.04	1700	.28	105	.70	17.2	1.6	2.37
.06	1300	.30	90.0	.75	15.0	1.7	1.99
.08	987	.32	78.1	.80	13.2	1.8	1.69
.10	747	.34	68.5	.85	11.6	1.9	1.44
.12	571	.36	60.7	.90	10.2	2.0	1.24
.14	441	.38	56.2	.95	9.12	2.2	↓
.16	345	.40	49.2	1.0	8.04	2.4	↓
.18	273	.45	39.2	1.1	6.37	↓	10
.20	220	.50	32.4	1.2	5.17	↓	↓
.22	179	.55	27.1	1.3	4.20	↓	$\frac{1}{r^3}$