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# On the Longitude Effect of Cosmic Radiation

M. S. VALLARTA, *Massachusetts Institute of Technology*

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It is shown that if the equivalent magnetic dipole describing the earth's magnetic field at large distances is placed at the magnetic center of the earth as calculated by Schmidt, the theory of the latitude and azimuthal effects developed by Lemaitre and Vallarta also accounts for the longitude effect found by Clay, van Alphen and Hooft and, independently, by Millikan and Neher. Further it is shown that if the distribution function of magnetically deflectable particles capable of reaching the earth's tropical belt is substantially an exponentially decreasing function of their energy, the theory quantitatively agrees with experiment.

CLAY, van Alphen and Hooft[1] and, independently, Millikan and Neher[2] have found experimentally that the total intensity of cosmic radiation measured by an ionization chamber at sea level is different for the same geomagnetic latitude and different longitudes. They have suggested that this longitude effect is to be interpreted as due to an asymmetry of the earth's magnetic field about the earth's center. The purpose of this note is to examine this point of view in the light of the theory of the latitude and azimuthal effects developed by Lemaitre and Vallarta,[3] and to find out what conclusions can be drawn from the experimental data concerning the energy distribution function of magnetically deflectable particles capable of reaching the tropical belt.

The magnetic field at the surface of the earth (and elsewhere) can be represented by an expansion of Legendre polynomials about any arbitrary point chosen as origin, in particular about the center of the geoid, but as Schmidt[4] has pointed out the expansion takes a particularly simple form, only one term of the first order and one of the second remaining if the origin is chosen at the magnetic center of the earth, a point about 300 km from the center of the geoid at geographic coordinates lat. 10 N, long. 168 E. If the equivalent magnetic dipole is placed not at the center of the geoid, but at the magnetic center of the earth with its axis pointing towards the geomagnetic poles, the immediate consequence is that the distance from the geomagnetic equator, or from any geomagnetic parallel, to the dipole is a periodic function of longitude. Since changing the distance to the dipole also changes the opening of the allowed cone[5] for any given energy, one must

[1] J. Clay, P. M. van Alphen and C. G.'t Hooft, Physica **1**, 829 (1934).

[2] R. A. Millikan and H. V. Neher, Phys. Rev. **47**, 205 (1935).

[3] A discussion of the present status of the theory is given in a paper by Lemaitre, Vallarta and Bouckaert, Phys. Rev. **47**, 434 (1935), where full references to previous literature are given.

[4] A. Schmidt, Zeits. f. Geophys. **2**, 38 (1926). The introduction of the actual magnetic field of the earth, using Schmidt's results, as done in the present paper, was already suggested by Lemaitre and Vallarta, reference 6, p. 88, footnote (7).

[5] The theory of the determination of the cone is outlined in reference 3 and more fully discussed in Lemaitre's and in Bouckaert's papers there quoted.

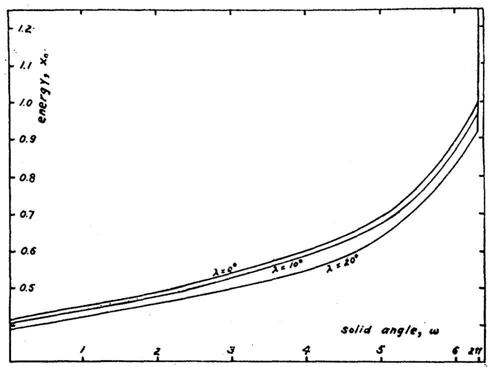


FIG. 1. The solid angle of opening of the cone at three geomagnetic latitudes.

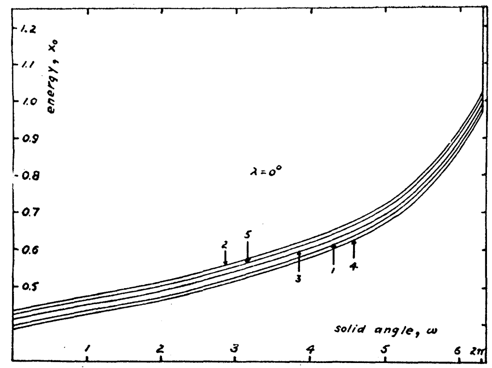


FIG. 2. The solid angle of opening of cone at several points on geomagnetic equator.

expect different intensities at different longitudes and consequently a longitude effect. We now show not only that this asymmetry of the earth's magnetic field is fully competent to account for the observations of Clay, van Alphen and Hooft, and of Millikan and Neher, but also that their intensity measurements along the geomagnetic equator throw valuable light on the energy distribution function of magnetically deflectable particles which give rise to both the latitude and the azimuthal effects.

In Fig. 1 the solid angle cut off from the unit sphere (i.e., the sky) by the allowed cone is plotted as a function of energy, measured in terms of our energy parameter[6] $x_0$, for geomagnetic latitudes 0°, 10° and 20°. The solid angle in each instance has been obtained by graphical integration from the calculations of Bouckaert.[7] In Table I we collect the results of a calculation of the distance from the earth's magnetic center to several points on its surface, together with the fractional change from the standard value of the energy parameter due to the fractional change in distance from the standard earth's radius (6370 km).

TABLE I. *Calculated distances from the earth's magnetic center to several points on its surface.*

| Point No. | Geomag. lat. | Geogr. lat. | Geogr. long. | Dist. (km) | $\Delta x/x_0$ | Approximate location |
|---|---|---|---|---|---|---|
| 1 | 0 | 12 S | 76 W | 6519 | 0.0234 | Lima, Peru |
| 2 | 0 | 0 | 160 W | 6120 | −0.0393 | SS route Honolulu to Sydney or Melbourne |
| 3 | 0 | 10 N | 75 E | 6390 | 0.0031 | SS route Bombay to Colombo |
| 4 | 0 | 6 S | 15 W | 6650 | 0.0440 | SS route Southampton to Cape Town |
| 5 | 0 | 12 N | 110 E | 6220 | −0.0236 | SS route Hong Kong to Singapore |
| 6 | 0 | 10 S | 100 W | 6370 | 0 | 1000 miles SW of Galapagos |
| 7 | −10 | 2 N | 104 E | 6240 | −0.0204 | Singapore |
| 8 | 10 | 2 S | 80 W | 6520 | 0.0236 | Guayaquil |
| 9 | −20 | 6 S | 106 E | 6220 | −0.0236 | Batavia |
| 10 | 20 | 9 N | 79 W | 6460 | 0.0141 | Panama |

From the value of $\Delta x/x_0$ given in this table and from Fig. 1 one easily finds the solid angle of the cone (the full light region in the language of previous papers) for the different points in the table. The solid angle is plotted as a function of energy, expressed in terms of our energy parameter, for these points, in Figs. 2, 4 and 5.

Referring to Fig. 2 it is seen that for any given energy the full light region at the geomagnetic equator is greatest along curve 4 (for the point on the earth where the steamer lane from Southampton to Cape Town intersects the geomagnetic equator, approximately), is less for the same energy along curve 1 (Lima, Peru, approximately), then less along curve 3 (where the steamer lane from Bombay to Colombo intersects the geomagnetic equator), curve 5 (steamer lane from Hong Kong to Singapore) and least along curve 2 (steamer lane from

[6] G. Lemaitre and M. S. Vallarta, Phys. Rev. **43**, 89 (1933), Eq. (12). This amounts to choosing as our energy unit 1/10.60 ergs, or $5.96\times10^{10}$ electron volts, for electrons or positrons.

[7] L. Bouckaert, Ann. de la Soc. Sci. de Bruxelles, **A54**, 174 (1934). Along the equator the angle of opening of the cone in the east-west vertical plane may be calculated from Störmer's formula $\sin\theta = -2\gamma_1/x \cos\lambda + \cos\lambda/x^2$ with $\lambda=0$ and $\gamma_1=1$, because there the vertex of the cone touches Störmer's circular cone as defined by the above expression. The angle of opening of the cone in the east-west vertical plane may *not* be calculated by this formula for any other latitude, since for no other latitude does this condition hold, nor may the solid angle of opening of the cone be calculated from this formula for any latitude including the equator.

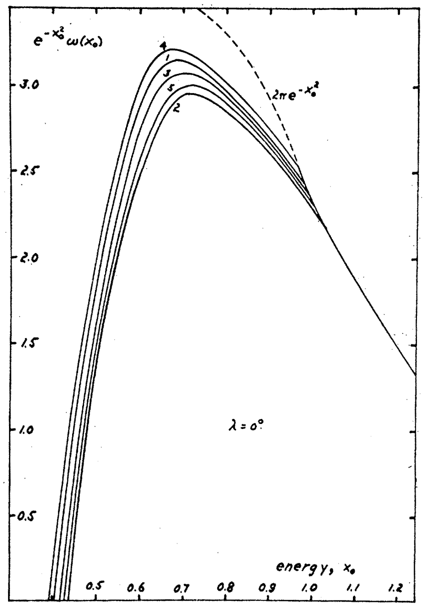


FIG. 3. The function $e^{-x_0^2}\omega(x_0)$ for different points on geomagnetic equator.

Honolulu to Melbourne). Consequently the total intensity must decrease in the same order.

We now calculate the percentage change in total intensity due to the fractional change $\Delta x/x_0$ in the energy parameter $x_0$, and show that the experimental results are consistent with the assumption that the energy distribution function, defined as the number of particles outside the atmosphere having a given energy, is an exponentially decreasing function of the energy. Let $f(x_0)$ be the distribution function and $\omega(x_0)$ the function giving the solid angle of the cone in terms of $x_0$. Then the number of particles coming through the cone corresponding to any particular value of the energy is, by Liouville's theorem,[8] $f(x_0)\omega(x_0)$. We now define the total intensity as the total number of particles which come through all the cones, beginning with those particles which have the least possible energy to reach the top of the atmosphere at the given point. This total intensity is given by

$$I = \int_{\bar{x}_0}^{\infty} f(x_0)\,\omega(x_0)dx_0.$$

[8] A discussion of the application of Liouville's theorem to the present theory, with full literature references, is given in reference 3.

Let us now assume that the distribution function is an exponentially decreasing function of energy. Remembering that for energies of the order of magnitude required to reach the tropical belt the energy is proportional to the square of the parameter $x_0$, we have for the total intensity,

$$I = \int_{\bar{x}_0}^{\infty} e^{-x_0^2}\omega(x_0)dx_0.$$

In Fig. 3 we have plotted as a function of $x_0$ the integrand of the above expression for the points in the geomagnetic equator given in our Table I. From this we have calculated by graphical and analytical integration the intensity $I_0$ at those points on the geomagnetic equator the distance of which from the magnetic center is equal to the earth's radius, i.e., the value of the above integral along curve 6 (or 3). This we have taken as our standard intensity with respect to which the percentage change of intensity due to the longitude effect is calculated. To find the change of intensity at the different points in our Table I we merely have to find by graphical integration the areas of the strips between corresponding curves in Fig. 3. For points 1, 2, 3 or 6, 4 and 5, respectively, we thus obtain $\Delta I_0/I_0 = 1.9$ percent, $-3.7$ percent, 0, 4.2 percent and $-2.5$ percent, respectively. The value of $\Delta I_0/I_0$ is different for the same $|\Delta x/x_0|$ because the areas of the corresponding strips are different.

Similarly for points 7 and 8 at $\lambda = \pm 10°$ the theory demands that the total intensity at the point on the earth described by curve 8 be greater than that described by curve 7 (Fig. 4) and the same calculation outlined above for equatorial points, with the same distribution function gives 5.6 percent as the percentage intensity change $\Delta I_1/I_1$ between these points. Likewise for points 9 and 10 (Fig. 5) at $\lambda = \pm 20$ the theory requires that the intensity at point 9 be less than that at 10, and a similar calculation gives for the percentage intensity difference $\Delta I_1/I_1 = 4.0$ percent.

We now proceed to compare our theoretical results with experiment. It is very fortunate that the steamer lane from Bombay to Colombo intersects the geomagnetic equator at a point, the distance of which from the magnetic center

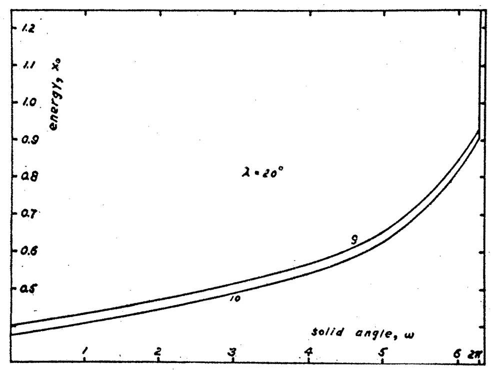


FIG. 4. The solid angle of opening of cone for two points at $\lambda=10°$.

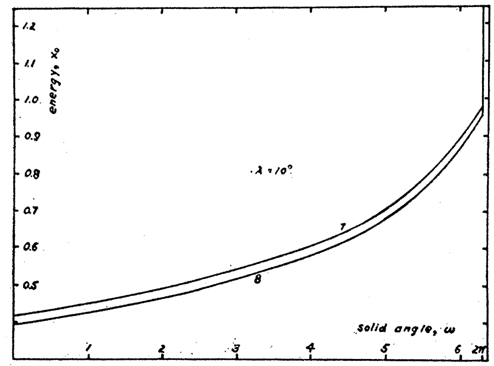


FIG. 5. The solid angle of opening of cone for two points at $\lambda=20°$.

is very nearly the standard earth's radius. Consequently we have taken 1.73 as our standard ionization from which to reckon the intensity changes due to the longitude effect along the geomagnetic equator in the case of Clay, van Alphen and Hooft's measurements. Conditions are not so favorable in the case of Millikan and Neher's measurements because unfortunately they have not reported separately their measurements on the run from Hong Kong to Singapore and from Colombo to Bombay. However, the other point (point 6 in our Table I) which enjoys the same property of being at a distance from the magnetic center equal to the earth's radius is between Lima and the point where the steamer lane from Honolulu to Sydney intersects the geomagnetic equator. By interpolation we have taken as standard ionization in the case of Millikan and Neher's measurements the value 2.60 and have referred the percentage change of intensity due to the longitude effect to this value.

To compare the experimental value of the intensities at Guayaquil and Singapore we multiply the intensities given as ordinates in Millikan and Neher's[2] Fig. 2 by the factor 0.1191 so as to bring their high latitude values in their Fig. 2 into agreement with their high latitude values in their Fig. 5. This gives at Singapore the ionization 2.51 which is 5.2 percent less than the ionization at Guayaquil (2.64).

If one now desires to compare the intensity at Batavia as measured by Clay, van Alphen and Hooft with the intensity at Panama as measured by Millikan and Neher one meets with a grave difficulty. Clearly the ionization measured by the former is not directly comparable with that measured by the latter, presumably because of differences of construction in their respective apparatuses. If one attempts to reduce the former's values to the latter's by multiplying by the factor (1.374) which is required to make the high latitude value of the former agree with that of the latter one obtains the value 2.45 at Batavia which gives about 8 percent less intensity there than at Panama, where according to the latter the ionization is 2.64. If one attempts the reduction by multiplying by the factor (1.395) which is needed to make the former's measurements at Singapore agree with the latter's at the same point one obtains 2.47 as the ionization at Batavia, which is about 7 percent less than at Panama. It would appear that the conversion factor, besides depending on the apparatus, also depends on latitude. Further clarification of this point from the experimental side is earnestly to be hoped for.

The calculated and experimental results are compared in Table II, and the calculated longi-

TABLE II. *Calculated and observed values of* $\Delta I_1/I_1$ *and* $\Delta I_1/I_1$.

| POINT NO. | $\Delta I_0/I_0$ CALC. | $\Delta I_0/I_0$ OBS. | $\Delta I_1/I_1$ CALC. | $\Delta I_1/I_1$ OBS. | OBSERVERS |
|---|---|---|---|---|---|
| 1 | 1.9 | 1.2 | — | — | Millikan and Neher |
| 2 | −3.7 | −0.8 | — | — | " " " |
| 3 | 0 | 0 | — | — | Clay, van Alphen and Hooft |
| 4 | 4.2 | 4.2 | — | — | " " " " " |
| 5 | −2.5 | −3.8(?) | — | — | Millikan and Neher |
| 7 | — | — | −5.4 | −5.2 | " " " |
| 8 | — | — | | | " " " |
| 9 | — | — | −4.0 | −7.0(?) | Clay, van Alphen and Hooft |
| 10 | — | — | | | Millikan and Neher |

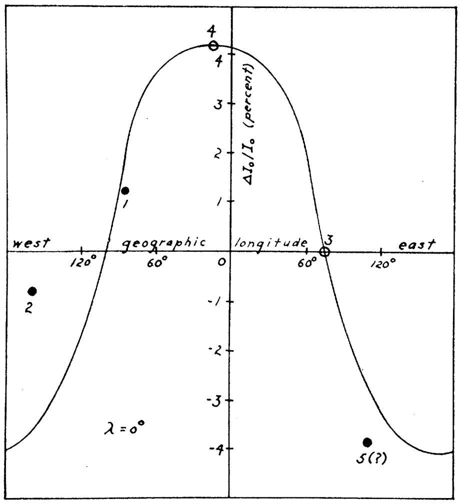


FIG. 6. The longitude effect along the geomagnetic equator.

tude effect at the geomagnetic equator is plotted in Fig. 6 together with the experimental points. The curve is symmetrical about the longitude 12 W approximately, but asymmetrical with respect to the axis of the abscissae.

With the exception of point 2 (SS route Honolulu to Melbourne at geomagnetic equator) it is seen that the agreement is very satisfactory.

The influence of atmospheric absorption will be considered in a future paper, but even without taking this factor into account it is seen from the above discussion that the theory is sufficiently in agreement with experiment. An additional factor which may affect the longitude effect is the bulging of the earth's atmosphere along the geographic equator due to the earth's rotation. Estimates of the magnitude of this effect are difficult to give at the present time, but in general it is clear that absorption would be greatest along the geographic equator and least at the poles. It is possible that the intensity at Lima may be affected by one percent due to this factor, as compared with the intensity at the west coast of Africa (SS lane Southampton to Cape Town at geomagnetic equator, point 4) but there seems to be no hope of accounting in this way for the high intensity measured by Millikan and Neher on the run from Honolulu to Melbourne, for there magnetic and absorption effects cooperate to give the least intensity.

We conclude that the theory of the latitude and azimuthal effects developed by Lemaitre and Vallarta is competent to account quantitatively for the longitude effect, and further that if the energy distribution function of the cosmic radiation which is responsible for these effects is substantially an exponentially decreasing function of the energy, the theory agrees with the measurements of the longitude effect.

My thanks are due to Dr. L. Bouckaert and Mr. E. J. Schremp who have helped me with the graphical integrations and with the discussion of the distribution function suggested in this paper.

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# Difference Bands in the Spectra of the Major Planets

ARTHUR ADEL AND V. M. SLIPHER, *University of Michigan and Lowell Observatory*

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This brief paper contains the first identifications of difference bands in planetary spectra. These absorption bands lie at 816$\mu\mu$, 802$\mu\mu$, 683$\mu\mu$, 673$\mu\mu$ and 584$\mu\mu$. They are due to methane gas and are found in the spectra of Jupiter, Saturn, Uranus and Neptune.

IT is now well established that the atmospheres of the giant planets (Jupiter, Saturn, Uranus, Neptune) contain vast amounts of methane ($CH_4$) in a large excess of hydrogen, and that in the two former ones there is also present a relatively small amount of ammonia ($NH_3$).[1]

[1] A. Adel and V. M. Slipher, Phys. Rev. **46**, 902 (1934); H. N. Russell, Science **81**, 1 (1935).