Energy and Angular Momentum in Certain Optical Problems

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An experimental proof of the theory that circularly-polarized light possesses angular momentum is reviewed, and a simplified modification is described. The experiment involved moving interference fringes, and a general condition, necessary but not sufficient, for obtaining such fringes is enunciated. Its application to five other optical problems is discussed. It is found: (a) That moving interference fringes should be obtainable from an arrangement with two diffraction gratings; (b) that if polarized light is reflected obliquely from a metal mirror revolving in its own plane, the absorption by the metal should vary by an amount proportional to the speed of rotation; (c) in two other cases support is found for the view that all photons possess the same amount of angular momentum, the state of polarization of the light depending on their statistical arrangement.

I.

THE question whether circularly-polarized light really possesses the angular momentum $h/2\pi$ per photon has been discussed by Henriot[1] from the point of view of conservation of angular momentum and energy when such light is passed through a half-wave plate so that its direction of rotation is reversed. If there is a corresponding reversal of angular momentum, the light must exert a torque on the plate, and accordingly if the plate rotates in its own plane work will be done. This (positive or negative) amount of work must reappear as an alteration in the energy of the photons, i.e., in the frequency of the light, which will result in moving fringes in any suitable interference experiment. Henriot shows now by the ordinary analytical methods of physical optics that the fringes will in fact move, and at the right speed (two for each turn of the half-wave plate), and the inference seems unescapable that the torque is real.

The analytical treatment of the actual interference problem is not new, although the important inference as to the angular momentum seems to have been overlooked until Henriot's paper. Moreover, the experimental verification has also been carried out. Analysis and experiment are both due to Righi,[2] who developed the subject in connection with the question of beats in optical problems, and the experiment is shortly described in Wood's *Physical Optics*.[3] Righi's arrangement may be somewhat simplified, so that the effect can readily be observed with an ordinary student's optical bench and Fresnel biprism, in the following manner:

A disk of the required thickness (about 0.064 mm) of good quality mica about 1.5 cm in diameter is mounted centrally on the end face of a brass rod 5 mm in diameter, the end having been turned in a lathe. It is placed in front of one half of the biprism; a stationary piece of the same mica is placed in front of the other half; and a strip of black paper about 2 mm wide is placed along the dividing edge since this is straight while the edge of the disk is curved. (A larger disk may be used, but only if it is very flat as well as very parallel; even so, the black strip will usually be indispensable.) It is advisable to remove the eyepiece from its holder and to verify directly that all the light from one image of the slit goes through the one piece of mica, and all the light from the other image goes through the other; it is surprising how disturbing even a small error in this respect is. If the brass rod has a nick turned in it, it may be supported in two V notches cut in the edge of a bent sheet of metal, and it may then be turned slowly by hand, causing the disk to rotate in its own plane. However, if the disk is in fact as small as here suggested, it is better to wrap a thread round the rod to turn it, as it lies very nearly in the line of sight. If now any bright yellow circularly-polarized light is sent through the system, it will be found that the fringes do move when the disk is rotated, that their speed is $2n$ per second if it

[1] É. Henriot, Comptes rendus **198**, 1146 (1934).

[2] A. Righi, Mem. d. accad. d. scienze di Bologna [IV] **4**, 247 (1882). (Contains a number of other instructive and beautiful experiments of a similar nature.) A shortened account appears also in J. de physique théor. et appliqué **2**, 437 (1883).

[3] Wood, *Physical Optics*, 2nd ed., p. 341.

turns n times per second, and that their direction is towards the side of the moving mica when the directions of rotation of the mica and the light vector are the same[4] and *vice versa*. It appears certain, therefore, that circularly-polarized light can in fact transfer the angular momentum $\pm h/2\pi$ per photon to matter, producing a torque which must in principle be measurable.

It has been claimed[5] that an infinite plane wave cannot possess angular momentum even if it is circularly polarized, and the suggestion has been made that diffraction effects at the edge of the disk must be responsible for the torque if there is one. This view now appears open to question, since it is clearly possible in principle to reduce the energy and momentum of the diffracted light to any desired extent by surrounding the disk with a large stationary half-wave plate in the same plane, separated from it everywhere by a gap less than a wavelength in width. The elementary theory of the change in frequency of the light would not be affected by this precaution, and the torque would thus be the same to the first order. Whether the light actually "possesses" the angular momentum need not be discussed here; the interaction[6] of the light and the disk will result in a torque both in this case and also if the light is, for example, absorbed by it. Experiments have been in progress in this laboratory for some time to detect the resulting torque directly.

II.

Henriot's paper employs a general principle which it is instructive to apply to other cases also. It may be stated as follows: Moving interference fringes can be obtained only if one of the two beams of light does more work than the other, before they are brought to interference; and if the fringes do move, their velocity can usually be calculated from the work done.

The most familiar example of this is the case of the Michelson interferometer. Let $E=nh\nu$ be the energy arriving per second on the movable mirror, which we may suppose to have unit area. The energy-density in front of it is then $2E/c$, and the radiation pressure p is equal to this, since the incidence is normal. If the mirror moves at a velocity v, the rate of working will be pv, and the photons must be taxed to supply this amount. Thus $2nh\nu v/c=nh\delta\nu$, giving the Doppler effect for a virtual source moving with velocity $2v$.[7] The analogy with the rotating half-wave plate is, as Henriot says, exact in this respect. It is also exact, as Wood says, in that both problems can be treated statically; movement is possible only if all the positions moved through are themselves possible positions.

The analogy is nevertheless not complete. The natural width of the spectral line employed will limit the path difference for *observable* interference, in the case of the interferometer, while with the half-wave plate the initial conditions are exactly reproduced every time the total rotation reaches $m\pi$, where m is any integer. This arrangement thus gives us a *permanent* source of light whose frequency has been modulated in one direction only, while with all such devices as an oscillating Kerr cell or a radio transmitter the modulation is in both directions at once, and with arrangements involving the Doppler effect the source is not permanent. Furthermore, as far as moving interference fringes are concerned, the only limitation on the allowable range of wavelengths is that imposed by the effective useful range of the half-wave plates (provided one does not require to see a large number of fringes at once), so that a good filter is amply monochromatic enough. Righi in fact used white light.

III.

If Righi's experiment be performed with plane polarized light, there is of course no couple on the plate, no work is done, and the fringes move neither way. The change from full speed one way, for left circular light, to full speed in the opposite direction, for right circular light, is moreover continuous, as is the change in the work done; but it does not involve intermediate speeds for elliptical light. Evidently there can be

[4] Fringes must always move away from the source of higher frequency, so that the Doppler effects would keep the two sources in phase for an observer moving with the fringes. Righi states the opposite, but it is evidently an oversight in interpreting his own analysis. (Mem. Acad. Bol. [III] **8**, 645 (1877).)

[5] P. Ehrenfest, J. Russian Phys. Soc. **48**, 17 (1911). (I have not yet seen this paper however.)

[6] P. S. Epstein, Ann. d. Physik **44**, 593 (1914); M. Abraham, Physik. Zeits. **15**, 914 (1914).

[7] See also J. Larmor, Encyclopedia Britannica, 11th ed., Article *Radiation*.

only one condition of the fringes for any one position of the disk, and only the one speed is possible. Here as so often the conception (Fresnel, Airy) of two opposite circular vibrations of different velocities (rotary polarization), or different frequencies (light polarized in a rotating plane), or different amplitudes (elliptical polarization), is the most useful, if not indeed the only useful one. As the ellipticity changes then, the one set of fringes gets continuously fainter while the other gets stronger, and with plane polarized light they are equally strong, so that we see fringes appearing and disappearing, with no motion except that the maxima and minima exchange places every quarter turn.

Incidentally, since the field is uniformly illuminated whenever the axes of the two half-wave plates make an angle of $\pm 45°$ with each other, and since this is evidently true whatever the plane of polarization of the light may have been, it is true for ordinary light also, so that we arrive at the following rather pretty application of the Fresnel-Arago rules:

If *unpolarized* light pass through a slit and biprism, and if two half-wave plates be placed one over each half of the biprism with their axes at 45° to each other, there will be no fringes. (There will also be none if one of the half-wave plates is removed.) Thus even unpolarized light is *observably* affected by passage through a half-wave plate.

IV.

The principle enunciated above may be applied also to the following case, the possibility of which was suggested to me by Mr. G. A. Downsbrough. If light from a slit be allowed to fall normally on two diffraction gratings placed side by side, and if a diffracted beam from one grating be brought to interference with a diffracted beam from the other, moving fringes must result if one grating moves (slowly) in its own plane perpendicularly to the ruled direction. (If the gratings have slightly different spacings a biprism may presumably be dispensed with.) For the photons have received a transverse momentum $h\nu \sin\theta/c = hN/d$ if N is the order of spectrum and d the grating-space of the moving grating. It is then easily verified that, whatever the wavelength, the rate of movement of the fringes must be N times the rate of movement of the grating if this is expressed in lines per second. Technical difficulties, which might be overcome by engraving a grating on a long flexible film, are all that prevent this experiment from being continued indefinitely also.

V.

If plane polarized light falls obliquely on a polished metal surface, the reflected light is circularly polarized if the angle of incidence and azimuth of the plane of polarization are correctly chosen. (The former lies between 70° and 80° from the normal for most metals, and the latter approaches 45° from the plane of incidence for good reflectors.)[8] In general, at other angles, the light is elliptically polarized, and except when either the angle of incidence or the azimuth is either 0 or 90° a certain amount of angular momentum has been produced, whose component normal to the surface is never zero. Thus if the plate rotates in its own plane, work will be done, and if the light be brought to interference with a second beam similarly reflected from a stationary mirror we might at first sight expect moving fringes here also.

They must however unquestionably be stationary in this case, for if the moving reflector be stopped there is only one position possible for them, however far it had previously turned. Thus we must now look elsewhere for the energy. It appears legitimate (as well as unavoidable), to assume that it will be supplied by a change in the absorption, because the whole question of metallic reflection is intimately bound up with absorption. Without going into detail (the question is somewhat intricate), we may predict the result that must be obtained if this view is correct.[9]

[8] Drude, *Lehrb. d. Optik*, p. 338.

[9] *Note added March 4, 1935:* Dr. P. A. M. Dirac suggested to me in conversation recently that the beam of light might be displaced sideways, as well as circularly polarized, by the reflection, and that the amount of displacement might be just enough to compensate the moment of force that would otherwise be acting on the disk. This possibility seems to me to be further supported by the fact that elliptical polarization can be produced by reflection from a contaminated surface of a liquid or isotropic crystal, and that the theory of this effect ignores absorption altogether. (Drude, *Lehrb. d. Optik*, p. 266; however, see also Wood, *Physical Optics*, 2nd ed., pp. 369, 370.)

Let the *reflected* radiation be denoted by $x = a\cos 2\pi\nu t$, $y = b\sin 2\pi\nu t$, so that its intensity is $E = (a^2+b^2)/2 = (n_1+n_2)h\nu$, where n_1 and n_2 are the numbers of left and right-handed photons leaving per second. If we write $a = A+B$, $b = A-B$, the intensity of all the left-handed photons is $A^2 = n_1h\nu$, while that of the right-handed ones is $B^2 = n_2h\nu$. The angular momentum produced per second is thus $(n_1-n_2)h/2\pi = (A^2-B^2)/2\pi\nu = ab/2\pi\nu$. If the angles of incidence and reflection are ψ, and if the plate rotates N times a second, the rate of working will be $Nab\cos\psi/\nu$, and the ratio of work done to incident energy is $RNab\cos\psi/\nu E$ if R is the reflection coefficient for this incidence and azimuth. This ratio is however the change, δR, in the reflection coefficient, so that $\delta R/R = 2Nab\cos\psi/\nu(a^2+b^2)$. If $b/a = \tan\varphi$, so that φ is the azimuth of a Babinet compensator set for extinction, this becomes

$$\delta R/R = (N/\nu)\cos\psi\sin 2\varphi.$$

On account of the extreme smallness of the factor N/ν in any practical case, a derivation of this formula from electronic theory would involve retaining all terms to a high order of small quantities.[10] Experimental verification would also be difficult.

We may note that although the treatment we have given for the Michelson interferometer leads to the correct result, showing that the work done by the radiation pressure appears as a change in the frequency if the reflecting power remains constant, it is not possible to infer conversely that the reflecting power does remain constant when a mirror moves normally to itself. We can apparently draw such conclusions only when both the theorem of the conservation of energy and also that of the conservation of angular momentum are applicable.

VI.

If circularly polarized light is passed through a Nicol[11] its angular momentum is destroyed, and there must be a couple on the Nicol. The resulting light cannot of course interfere completely with the original light, but it can interfere with part of it, producing slightly dim fringes. (The intensity will vary as $3/2+\cos\delta$; in ideal cases it varies as $1+\cos\delta$.) The question whether these fringes will move when the Nicol rotates leads to an interesting little paradox.

In the first place, the work done cannot be accounted for by any change in the absorption here, for the Nicol may be replaced by a thick plane-parallel slab of calcite, and the extraordinary ray (or even the ordinary one) may be absorbed after emergence, without affecting the problem in principle. And the doubly refracting character of a crystal like calcite is so little connected with its (extremely small) absorption in the visible that the usual analytical treatment ignores absorption altogether. Further, even if it could be supposed that the absorption should rise from zero when the crystal rotates in the direction of the circular polarization, it certainly cannot fall below zero, so as to create new photons, when the crystal rotates the other way. These arguments apply equally to both ordinary and extraordinary rays. Thus the observations must be predicted to the first order without introducing any question of absorption, and we should naturally assume that the fringes must move.

They cannot move however. The plane polarized light that emerges from the Nicol will not interfere at all times with the same component of the circular light, since its plane is rotating. It will, however, always interfere with exactly that (plane) component that would have been transmitted by a second Nicol, placed in the second beam and rotating in phase with the first one; and a Nicol is always regarded as letting one component through unaltered. If such a Nicol were present, however, the fringes would be stationary by symmetry. Thus they will be stationary in its absence also. It may be added that if this Nicol were present, work would be done on it also, and yet we cannot ascribe more than half of this to the photons that are stopped by it, for there is not really anything to choose in this respect between the ordinary and extraordinary rays in the case of the calcite plate; it is possible in principle to absorb the ordinary

[10] The "reflection coefficient" is ordinarily defined for normal incidence. The actual reflecting power involved here will depend on ψ and the azimuth of polarization of the incident light.

[11] The nature of the light produced in this case is also discussed by Righi (reference 2), following Airy and Verdet, but he does not consider the behavior of any interference fringes.

ray and then to bring the extraordinary one back onto the axis of rotation without changing its state of polarization.

We thus reach the paradoxical (but correct) conclusion that although the work cannot be accounted for by any change in the absorption by the doubly refracting substance, still the fringes will not move. Further, some or all of the component that is "freely transmitted" does in fact act on, and is therefore affected by, the Nicol. The paradox is bound up with the notion of "plane components," and disappears as soon as we resort to circular ones.

Let the initial circular vibration be $x=\cos \omega t$, $y=\sin \omega t$. The components of these in a direction making an angle θ with the x-axis are $\cos \omega t \cos \theta$ and $\sin \omega t \sin \theta$, so that the Nicol transmits $\cos (\omega t-\theta)$ if its principal section is in this direction. The x and y components of this are:

$$x=\cos (\omega t-\theta) \cos \theta=\tfrac{1}{2}[\cos \omega t+\cos (\omega t-2\theta)],$$
$$y=\cos (\omega t-\theta) \sin \theta=\tfrac{1}{2}[\sin \omega t-\sin (\omega t-2\theta)],$$

so that the emergent light consists of two parts. One is circularly polarized the same way as the original light and has the same frequency; this will interfere with half the amplitude of the original light to produce stationary fringes. The second component is that which does all the work when the Nicol rotates, and in that case ($\theta=\alpha t$, say) it has its frequency altered to correspond; but it cannot interfere at all since its direction of polarization has been reversed. The interaction of these two components is what we call light plane-polarized in a rotating plane, but the notion is a confusing one. It is apparently not true, even for circularly polarized light, that all the photons that get through a Nicol get through unaffected; on the contrary, half get through unaffected, and half interact with it. The interaction turns them over, and if the Nicol also rotates they do work on it.

Exactly the same holds for those photons that are stopped by the Nicol; half have their frequency and state of polarization unchanged, and half are turned over in every case, and do work as well if it rotates. Of course, if these are afterwards absorbed, the heating effect will be altered by rotation.

The emphasis that has usually been placed hitherto on plane, as opposed to circular, components is evidently to some extent of experimental origin. It so happens that differences in velocity and direction of the rays in a crystal, due to "anisotropic restoring forces" that respond to plane components, are more common (among simple substances) and certainly much larger than differences due to a rotational asymmetry that responds to circular components. In addition, however, when we speak of photons as being "turned over" in a crystal, we are speaking a language that is probably not adapted to actual interference phenomena, and it is accordingly difficult to explain why there should be a special phase relation, resulting in a definite plane of polarization, between those that are and those that are not turned over. The difficulty appears to be closely analogous to the difficulty that used to be felt in reconciling the photoelectric effect with interference phenomena, and it appears likely that in the future the language of circular components will be more generally used.[12] However, it sometimes involves abandoning the idea of conservation of angular momentum in individual atomic processes, for example in the emission of π-components in the Zeeman effect.

[12] Raman (Ind. J. Phys. **6**, 353 (1931)) draws support for this view from the intensities of lines in certain cases of the Raman effect.