Nuclear Potential Barriers: Experiment and Theory

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The heights of nuclear potential barriers are derived from experimental data, corrected for nuclear motion and penetration through the barrier. Barriers differ markedly for different incident particles. It is shown that the heights of barriers to alpha-particles increase with atomic number and that the nuclear radius at the top is proportional to the cube root of the atomic weight. Values for corresponding barrier radii of the neutron and the radioactive elements agree with those given by Dunning and by Gamow. It is suggested that the attraction operative in alpha-particle collisions is only a second order force and that the anomalies in proton barriers are due to a first order force effective much further outside their tops. A method for obtaining information about this force is suggested. The energies of resonance levels are tabulated and an approximate linear increase with atomic number is discussed.

INTRODUCTION

IN order to explain artificial disintegration by charged particles and their anomalous scattering a potential barrier of the type used by Gamow, Gurney and Condon in discussing radioactive disintegration is postulated. In the past few years experiments by many workers have furnished much information as to the nature of these barriers and it is the first aim of this paper to review the evidence, showing how the critical energies of impacts on light nuclei as known at present conform to simple rules. A second part of the paper discusses the bearing of the experimental work on nuclear theory, taking up in particular the possibility of determining the attractive force between a neutron and a proton.

The nuclear potential barrier to an approaching positive particle consists of a smooth Coulomb rise diminished for close approach by a potential term of opposite sign varying much more rapidly with the distance. For the purposes of this paper the important features of such a barrier are:

(1) The energy at the top of the barrier, E_T (barrier height).
(2) The radius at the top, r_T (related to (1)).
(3) The energy values of resonance levels below the top which may permit easy entrance.

The general methods for determining barrier heights have been described in a previous paper.[1] It is here intended to amplify the account there given, applying corrections for nuclear motion and for penetration through the barrier, which alter the conclusion to be drawn from the data.

PENETRATION THROUGH NUCLEAR BARRIERS

It is well known that high energy particles incident on light nuclei are not scattered according to the Rutherford formula. It is also known that many light nuclei disintegrate when bombarded by high energy particles. Both events are explained by the penetration of the barrier by the incident particle; in the first case the particle escapes again while in the second a new nucleus is formed.

In Paper I it was shown that the experimental work on anomalous scattering and on disintegration yields permit the derivation of a series of values for a not very definitely defined minimum penetration level which was there termed the barrier height. The values found in this way prove to be a linear function of atomic number, which, if rigorously true, would mean the barrier height is not closely related to the volume of the particles in the nucleus. In what follows an attempt is made to include a correction for penetration through the barrier below the top assuming that Gamow's penetration formula holds and that the probability of disintegration for each separate element depends mainly on the chance of entry of the alpha-particle. The values so derived for the barrier heights are not expected to be exact, but their variation from element to element should show the same behavior as the exact heights.

In a more complete treatment the incident beam of charged particles must be considered as

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[1] Hereafter called Paper I.
E. Pollard, Phil. Mag. 16, 1131 (1933).

a plane de Broglie wave resolvable into a series of partial waves of angular momentum O or $h/2\pi$ or $2h/2\pi$, etc. In experiments on anomalous scattering it appears that the components of angular momentum higher than zero are of great importance and must be taken into account, while, as will be shown later, disintegration experiments fit a simple theory in which terms of higher order than zero are neglected. This means that for the absorption of a particle it must strike the nucleus approximately head-on. If this limitation is imposed, the probability of penetration is given by

$$P=A\ \exp\left\{-\frac{2(2m)^{\frac{1}{2}}}{\hbar}\int_{r_0}^{r}(V-E)^{\frac{1}{2}}dr\right\},\quad (1)$$

A, a function which is nearly unity and is here taken to be exactly unity;
m, the mass of the impinging particle;
$\hbar$, the quantum of angular momentum;
r_0, r, the inside and outside radii at energy E of the incident particle;
V, the potential energy of the particle at distance r.

For a given barrier we can evaluate P at different values of the energy of the incident particle by graphical integration. We find that P is nearly the same for any potential giving a sharp inward drop. If we choose

$$V=2Ze^2/r-2Ze^2k/r^{12}\qquad (2)$$

to represent the barrier (where k is a different constant for each individual element) and plot curves for P against energy (called excitation curves) we find that the difference between the energy for 10 percent penetration (called $E_{1/10}$) and that for topping the barrier (E_T) increases with the barrier height. For the region covered by experiment it is possible to plot a curve relating $E_{1/10}$ with E_T (whether the change in E_T is due to the variation of Z or k): this is given as Fig. 1. The excitation curves further show that near 10 percent probability P varies rapidly with E so that an error in estimating where P is 1/10 produces only a small error in $E_{1/10}$ and so a small error in deriving E_T; this is made the basis of reducing experimental results to give barrier heights.

In experiments using alpha-particles as projectiles disintegration does not occur in general until a closest distance of approach of about

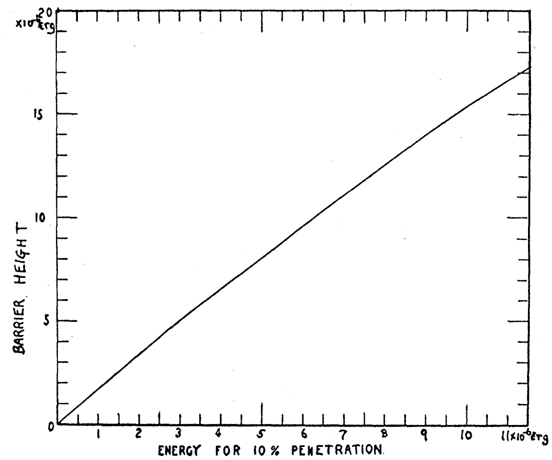

FIG. 1. Relation between barrier height and energy for 10 percent penetration.

5×10^{-13} cm is reached. The number of particles which pass within this distance of some nucleus is approximately 10^{-5} of the total impinging on the target. The yields of disintegration are found to be roughly one in 10^6 indicating that nothing is observed unless penetration is considerable. The same is true of anomalous scattering. In this paper it is assumed that disintegration and anomalous scattering become definitely detectable when the probability of penetration is 1/10 and this value used to derive the energy of the top of the barrier from Fig. 1. It is likely that the region of definite detection deduced from good yield curves lies between $P=1/5$ and 1/20 and if these limits are admitted the error in determining E_T is 10 percent.

It is only necessary for Eq. (1) to hold for the region in which experimental detection occurs. It is found that values given by Chadwick for beryllium and by Haxel for aluminum fit the formula and this is taken as experimental justification for its use.

The probability of entry at lower energy values is governed more by the width at larger radii than by the height so that this latter cannot be deduced unless exact absolute yields are measured. The majority of the experimental evidence available is therefore confined to disintegration and scattering with alpha-particles.

Resonance levels

As first pointed out by Gurney, the probability of entry of a particle is much greater if it has an energy lying within an unoccupied (resonance) level in the nucleus. The experimental evidence for this was first found by Pose. The interest in resonance levels is in their mean energy values and in their energy widths. These widths affect the yield of disintegration products.

Experimental Evidence

In considering energies derived from experimental curves, the motion of the nucleus must always be taken into account. A critical incident particle energy can be reduced to the potential energy at the closest distance of approach by multiplying by the factor: $M_n/(M_i+M_n)$ where M_n is the mass of the nucleus and M_i the mass of the incident particle.

(a) Alpha-particle scattering

Riezler[2] has investigated the variation of the numbers of scattered alpha-particles with scattering angle for beryllium, boron, carbon and aluminum, varying the incident energy in the cases of boron and carbon. For the first three elements he finds that the number scattered increases rapidly as the distance at closest approach (computed for point charges with Coulomb fields) becomes less. He explains this by supposing that at a certain distance several components of angular momentum begin to contribute appreciably to the anomalous scattering. For boron this closest approach for classical scattering is 4.8×10^{-13} cm; for carbon 5.5×10^{-13} cm. The derivation includes correction for the motion of the nucleus. For aluminum he finds the zero order component is sufficient to account for the scattering provided it is disturbed only at distances less than 6.0×10^{-13} cm. The results for beryllium do not yield a definite value for this particular barrier radius, but show clearly that its barrier is lower than that of boron. If we assume the radii given correspond to 10 percent penetration we have, from Fig. 1:

Element	$E_{1/10}$	E_T
Aluminum	9.8×10^{-6} erg	15.0×10^{-6} erg
Carbon	4.98	8.1
Boron	4.74	7.8
Beryllium	still less	

Helium. Scattering experiments have been carried out by Chadwick, and Rutherford and Chadwick.[3] The scattering at 45° to the incident particle path should, on account of the identical nature of the He nucleus and the alpha-particle, be double that expected in classical theory. Chadwick finds this value is exceeded for common alpha-particle ranges, falling to 2.22 at a range of 1.4 cm. Assuming this corresponds to $E_{1/10}$ we find 2.5×10^{-6} erg after correcting for nuclear motion, and using Fig. 1 this gives a barrier height of 3.9×10^{-6} erg.

Hydrogen. Chadwick and Bieler[4] observed the projected H nuclei when hydrogen is struck by alpha-particles, for various angles and various incident particle energies. Here the numbers counted should agree with classical theory except where penetration occurs. They give the following figures:

Range of α-particle	$F(31.3)-F(21.4)$ obs.	*Inverse square* calc.
2.9 cm	3.0	1.0
2.0	1.9	1.8
1.6	1.8	2.3
1.0	4.9	4.3

Then between 2.0 and 2.9 cm range, penetration becomes appreciable. Taking the energy corresponding to 2.0 cm for the value of $E_{1/10}$ we find a value 2.0×10^{-6} erg for the barrier height.

In recent unpublished experiments Pollard and Margenau have investigated the point at which anomalous scattering begins for hydrogen and deuterium. They find that for head-on collisions with hydrogen the point of departure from classical scattering is between 4.5 and 5.0 microergs alpha-particle energy while with deuterium it is between 3.8 and 4.3 microergs. Correcting for nuclear motion these give 0.95 ±0.05 and 1.35 ± 0.08 microergs, respectively. Taking these as $E_{1/10}$ and using Fig. 1 we get for the barrier heights:

Hydrogen: 1.6×10^{-6} erg,
Deuterium: 2.2×10^{-6} erg.

The values for the barrier heights derived from scattering are tabulated later, marked (S); they are plotted as squares in Fig. 2.

Since the barrier height is always deduced from the point where anomalous scattering is first detected a complete theory is not needed.

[2] W. Riezler, Proc. Roy. Soc. **A134**, 154 (1931).

[3] J. Chadwick, Proc. Roy. Soc. **A128**, 114 (1930); Rutherford and J. Chadwick, Phil. Mag. **4**, 605 (1927).

[4] J. Chadwick and E. S. Bieler, Phil. Mag. **42**, 923 (1921).

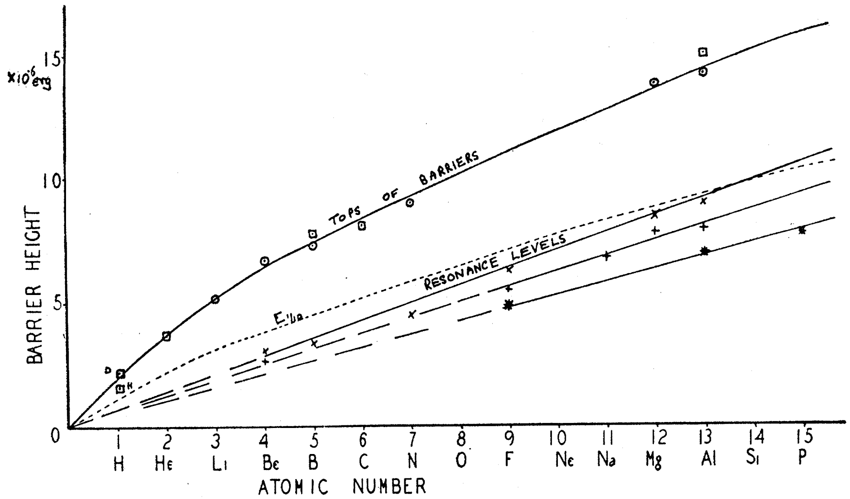

FIG. 2. Barrier height and resonance levels as function of atomic number.

(b) Excitation curves for disintegration by alpha-particles

Under bombardment by sufficiently energetic alpha-particles the majority of light elements emit some form of disintegration product. If care is taken to detect all of one such product and the yield is plotted for varying alpha-particle energy, an excitation curve is obtained. When the results are reduced to the ideal case of bombardment of a thin layer, the excitation curves show maxima at low energies with a smooth upward rise for higher energies. The maxima correspond to resonance levels and the smooth rise to penetration through the barrier. From an estimate of $E_{1/10}$ made on this part of the excitation curve the barrier height can be deduced.

It is assumed that the product of disintegration has, in general, no effect on the potential barrier against the incident particle; there is some evidence that this is true since the boron barrier is roughly the same whether neutron or proton is emitted and the aluminum barrier whether proton or positron. Where the energy conditions are such that a nuclear particle cannot be released until it has received kinetic energy from the alpha-particle in excess of E_T (e.g., neutron from Li^7) the critical value is higher than the barrier height and disintegration with production of some other product (in this case γ-rays) must be used to derive E_T.

It is also known from Blackett's experiments that every alpha-particle which penetrates into a nucleus does not cause disintegration and so penetration is not the only factor governing the yield curve. It is likely, however, that the other factors do not cause so rapid a variation in yield as the change in penetration with energy and so they have been treated as constant.

A résumé of the experimental work is now given and a complete table of known resonance levels and barrier heights for alpha-particle impact summarizes the findings.

Because of the difficulty of detection of disintegration products and the need for the plotting of a series of curves to determine the various nuclear levels, there is some disagreement between experimental results obtained by different workers. The major discrepancy today is between the work of Pose and his collaborators using electrometer detection and that of Chadwick and Constable using a linear amplifier. It is not intended to discuss the pro's and con's of a matter which can best be settled by further experiments, but a serious point of difference in the interpretation can be mentioned. Pose and his school find that alpha-particles of the same energy can, (a) cause resonance exciting a group of definite energy or, (b) penetrate without resonance and excite a group of different energy. Chadwick and Constable find that an alpha-particle of definite energy produces definite

TABLE I. *Summary of experimental work.*

Element	Worked on by	Findings and notes	Deductions
Li	Curie and Joliot[5]	γ-ray excitation commences at 1.7 cm. (Neutron excitation higher; explained by need of energy to release neutron.) Agrees with Webster.	Barrier height 5.3×10^{-6} erg.
Be	Curie and Joliot[5] Bothe and Becker[6]	Bombarded thick layer and found for neutron and γ-ray excitation a broad resonance level (1.0–1.8 cm) running into a smooth rise after 1.8 cm.	Resonance level at point where penetration becomes appreciable. Assuming $E_{1/10}$ to be given by 2.0 cm barrier height is 6.3×10^{-6} erg. Resonance levels at 3.1×10^{-6} erg and 2.6×10^{-6} erg.
	Bernadini[7]	Thin layer. Broad resonance level with max. at 1.95 cm and smooth rise after 2.35 cm.	
	Chadwick[8]	Resonance level at 1.15 cm again at 1.5 cm with smooth rise after 2.3 cm.	
B	Curie and Joliot[5]	Resonance level from 1.0 to 1.6 cm followed by rise above 2.3 cm.	Barrier heights and resonance levels approx. the same for two isotopes. Mean barrier height 7.2×10^{-6} erg, mean resonance level 3.2×10^{-6} erg.
	Chadwick[8]	Resonance level from 1.4 to 2.0 cm with rise above 2.4 cm. Both these are for neutron excitation and probably refer to B^{11}.	
	Heidenreich[9]	Proton excitation. Finds yield rising from 2.1–2.3 cm.	
	Pollard[10]	Resonance level from 1.2 to 1.7 cm.	
	Miller, Duncanson, and May[32]	Evidence for resonance level at 1.7 cm with penetration beginning at 2.1 cm.	
	See Paper I.	Indirect derivation gives 4.2×10^{-6} erg for value of $E_{1/10}$. These refer to B^{10}.	
N	Pollard[11]	Broad resonance level at 2.2 cm followed by penetration after 3.0 cm.	Barrier height 9.0×10^{-6} erg. Resonance level 4.5×10^{-6} erg.
F	Chadwick and Constable[12]	Proton excitation from thick layers. Find two resonance levels at 2.2–3, 2.7 cm and a region 3.3–3.9 cm which may be penetration or broad resonance.	No certain evidence for penetration. Probably resonance level energies; 4.8, 5.5, 6.7×10^{-6} erg.
	Pose[13]	2.7 cm level not found. Evidence for 2.2 and 3.3 cm levels.	
	Curie and Joliot[5]	Excitation of neutrons occurs above 2.7 cm range.	
	Bonner and Mott-Smith[14]	Groups of neutrons which can be explained by Chadwick and Constable's levels.	
Na	Atty König[15]	Resonance level at 3.5 cm for excitation of protons.	Resonance level at 6.8×10^{-6} erg.
Mg	Duncanson and Miller[16]	Proton excitation from thin layer with Ra C α-particles. Find resonance levels at 4.25 and 5.0 cm followed by rise after 5.25 cm.	Barrier height 13.5×10^{-6} erg. Resonance levels at 4.2, 6.0, 7.8, 8.5×10^{-6} erg.

[5] I. Curie and F. Joliot, J. de phys. et rad. **6**, 285 (1933).
[6] W. Bothe and F. Becker, Zeits. f. Physik **76**, 421 (1932).
[7] G. Bernadini, Zeits. f. Physik **85**, 555 (1933).
[8] J. Chadwick, Proc. Roy. Soc. **A142**, 5 (1933).
[9] F. Heidenreich, Zeits. f. Physik **85**, 675 (1933).
[10] E. Pollard, Phys. Rev. **45**, 555 (1934).
[11] E. Pollard, Proc. Roy. Soc. **A141**, 384 (1933).
[12] J. Chadwick and J. E. R. Constable, Proc. Roy. Soc. **A135**, 54 (1932).
[13] H. Pose, Zeits. f. Physik **72**, 537 (1931).
[14] T. W. Bonner and L. M. Mott-Smith, Phys. Rev. **46**, 258 (1934).
[15] A. König, Zeits. f. Physik **90**, 197 (1934).
[16] W. E. Duncanson and H. Miller, Proc. Roy. Soc. **A146**, 413 (1934).

TABLE I.—*Continued.*

Element	Worked on by	Findings and notes	Deductions
	Klarmann[17]	Possibly resonance levels at 4.2×10^{-6} and 6.9×10^{-6} erg.	
Al	Duncanson and Miller[16]	Proton excitation from thin layer. Resonance levels at 4.25 and 5.25 cm with rise after 5.6 cm.	Barrier height 14.4×10^{-6} erg. Resonance levels at 8.9, 8.0, 7.5, 6.8, 6.3, 5.8, 5.2×10^{-6} erg.
	Haxel[18]	Thicker layer. Rise after 5.6 cm. Unanalyzed resonance between 2.7 and 5.25 cm.	
	Chadwick and Constable[12]	Resonance levels at 3.9, 3.45, 3.1, 2.7 cm.	
	Diebner and Pose[19]	Two such levels at 2.3 and 3.5 cm.	
	Ellis and Henderson[20]	Excitation of induced radioactivity. Evidence of unanalyzed resonance levels above 3.6 cm range, followed by smooth rise after 6 cm range.	
P	Chadwick, Constable and Pollard[21]	Single group of protons excited by particles of maximum range 3.9 cm.	Probably one of the resonance levels is at 7.4×10^{-6} erg.

[17] H. Klarmann, Zeits. f. Physik **87**, 411 (1934).
[18] O. Haxel, Zeits. f. Physik **90**, 376 (1934).
[19] K. Diebner and H. Pose, Zeits. f. Physik **75**, 373 (1932).
[20] C. D. Ellis and Henderson, Proc. Roy. Soc. **A146**, 208 (1934).
[21] J. Chadwick, J. E. R. Constable and E. Pollard, Proc. Roy. Soc. **A130**, 463 (1930).

TABLE II. *Critical values for entry of alpha-particles into light nuclei.*

1	2	3	4	5	6	7		8	9	10	
			$R_{1/10}$	$E_{1/10}$ (10^{-6} erg)		Barrier height		R_{res}	$E_{\text{res.}}$	$E_{\text{res.}}$ (corr.	
Element	Z	M	(cm)	uncorr.	corr.	(10^{-6} erg)	(mev)	(cm)	(10^{-6} erg)	(10^{-6} erg)	(mev)
Hydrogen (S)	1	1	1.60	4.75	0.95	1.6	1.0				
Deuterium (S)	1	2	1.30	4.05	1.35	2.2	1.4				
Helium (S)	2	4	1.4	4.3	2.15	3.9	2.4				
Lithium	3	6, 7	1.7	4.9	3.15	5.3	3.3				
Beryllium	4	9	2.0	5.5	3.8	6.3	4.0				
								1.5	4.5	3.1	1.9
								1.15	3.7	2.6	1.6
Boron	5	10, 11	2.3	6.0	4.4	7.2	4.5				
								1.4	4.3	3.2	2.0
Boron (S)					4.74	7.4	4.9				
Carbon (S)	6	12			5.0	8.1	5.1				
Nitrogen	7	14	3.0	7.2	5.6	9.0	5.6				
								2.2	5.8	4.5	2.8
Fluorine	9	19						3.6	8.2	6.7	4.2
								2.7	6.7	5.5	3.4
								2.2	5.8	4.8	3.0
Sodium	11	23						3.5	8.0	6.8	4.3
Magnesium	12	24, 25	5.25	10.2	8.7	13.5	8.5				
		26						5.0	9.9	7.8	4.9
										6.9	4.3
										4.2	2.6
Aluminum	13	27	5.6	10.7	9.3	14.4	9.0				
								5.25	10.25	8.9	5.6
								4.25	9.0	8.0	5.0
								3.9	8.6	7.5	4.7
								3.45	7.9	6.8	4.3
								3.1	7.3	6.3	4.0
								2.7	6.7	5.8	3.6
								2.3	6.0	5.2	3.3
Aluminum (S)					9.8	15.0	9.4				
Phosphorus	15	31						3.7	8.4	7.4	4.6

groups; that, in fact, a system of levels inside the nucleus, together with the known energy of the impinging alpha-particles, uniquely defines the energies of the groups of products. This is the simpler view and is adopted in this paper, although, clearly, further careful experiment is needed.

Where reasonable agreement has been reached the accepted value has been chosen; otherwise individual results have been tabulated.

The experimental work is summarized as compactly as possible in Table I.

These results are set out in Table II. In columns 4, 5 and 6 the range, uncorrected value of $E_{1/10}$ and $E_{1/10}$ corrected for nuclear motion are given. Ranges are at 15°C and 760 mm and Duncanson's data[22] are used to derive energies from ranges.

A similar review was made in Paper I, with less experimental data. Corrections were not applied for the motion of the nucleus or for the fact that penetration occurs below the top of the barrier. The figures there given correspond to column 5 in the above table.

(c) Experiments with protons and deuterons

From low energy excitation curves it appears that barriers to deuterons are higher than to protons and that the heights increase with atomic number; more detailed deduction cannot be made.

High energy experiments have been made by Henderson and Henderson, Livingston and Lawrence[23] on lithium and fluorine bombarded by protons. In these experiments the excitation curves for thick layers of Li and CaF_2 were plotted by counting the emitted alpha-particles. Experiments with a thick layer yield the barrier height with no corrections; for, if the yield is plotted against the *range* of the incident particle, there will be a proportional rise once the probability of entry is unity—the number disintegrated depending only on the thickness traversed. Such a curve is found by Henderson[23] and it can be seen that the lithium barrier has a height between 400,000 and 500,000 electron volts. On the other hand the fluorine barrier is higher than 1,500,000 electron volts since the Gamow penetration formula holds up to this energy. The disintegration of boron has been investigated roughly by White and Lawrence;[24] it would appear that the barrier height is only slightly greater than for lithium, though it cannot be fixed without further work.

It is interesting that no certain evidence of resonance for proton or deuteron disintegration has been reported.*

The available results for barrier heights to a proton are therefore: (corrections are for nuclear motion)

Lithium 0.4–0.5 mev; 0.72×10^{-6} erg; corrected 0.63×10^{-6} erg;

Boron Rather higher;

Fluorine >1.5 mev; 2.4×10^{-6} erg; corrected 2.3×10^{-6} erg.

For *deuteron* bombardment we have the conclusion of Oliphant, Harteck and Rutherford[25] that in the reaction $D+D=H^1+H^3$ the yield is proportional to the penetration after about 100,000 volts. Correcting for nuclear motion this gives 0.085×10^{-6} erg, an extremely low value.

Discussion of Numerical Results

The values for barrier heights and for the higher resonance levels from Table II are plotted against the atomic number in Fig. 2. The barrier heights lie on a smooth curve. The resonance levels lie on straight lines passing through the origin. It can be seen from Eq. (2) that this means resonance occurs at *the same radius* for different nuclei. On the other hand the radii at the top of the barrier increase with increasing atomic number.

Radii of alpha-particle barrier summits

Radii of barriers against alpha-particles cannot be deduced unless the nature of the attractive force is known. But, if this is taken to vary rapidly enough with the distance, an approximation $r_T'=2Ze^2/E_T$ is only slightly too high.

[22] W. E. Duncanson, Proc. Camb. Phil. Soc. **30**, 102 (1934).

[23] M. C. Henderson, M. S. Livingston and E. O. Lawrence, Phys. Rev. **46**, 38 (1934).

[24] M. G. White and E. O. Lawrence, Phys. Rev. **43**, 304 (1933).

* *Note added in proof:* Recent experiments by Hafsted and Tuve indicate that resonance occurs for excitation of gamma-radiation by proton bombardment.

[25] M. L. E. Oliphant, P. Harteck and Rutherford, Proc. Roy. Soc. **A142**, 692 (1934).

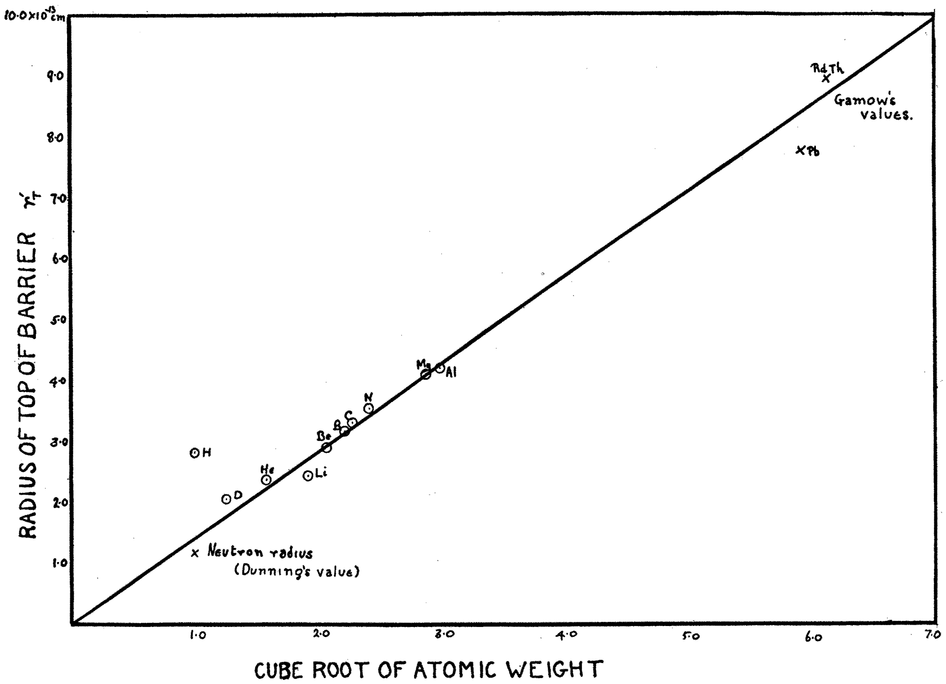

FIG. 3. Showing linear relation between nuclear radius at top of barrier and cube root of atomic weight.

In Fig. 3 r_T' is plotted against the cube root of the atomic weight; except for hydrogen the points lie near a straight line passing through the origin. Thus we find that:

The cube of the radius at the barrier summit is proportional to the atomic weight—Relation (3).

This relation has been suggested by Gamow[26] and used in a theory of disintegration. The line of Fig. 3 agrees well with evidence from two independent sources: Gamow's theory of radioactive decay and Dunning's deduction of the neutron radius. Gamow's values for the radii of MsTh and Pb barriers are 8.9×10^{-13} and 7.7×10^{-13} cm. The values found from Fig. 4 (by a long extrapolation) are 8.7×10^{-13} and 8.3×10^{-13} cm. Dunning,[27] from a wave-mechanical analysis of his neutron scattering experiments deduces 1.16×10^{-13} cm for the radius of the neutron. From Fig. 3 a particle of mass 1 should have radius 1.4×10^{-13} cm. Neutron scattering should give a smaller radius than that found using charged particles and if this be considered the agreement is very good. Eastman[28] finds that Heisenberg's theory of nuclear stability, postulating Relation 3, gives best agreement with actual nuclei for a proton radius of 1.6×10^{-13} cm. The agreement from this indirect evidence is therefore satisfactory.

It was shown in Paper I that if the nucleus consisted of particles packed within a radius much less than r_T, then for elements in which protons and neutrons are equal in numbers, r_T should be a constant. That this is not so means *the attractive force is only operative very near to the surface of the nuclear structure.* The equivalent of this has already been suggested by Dunning who shows that elastic sphere collisions account best for neutron scattering, indicating that non-Coulomb fields are confined to the near vicinity of the nucleus. Dunning also used Relation 3 in explaining his work, thus indirectly confirming its truth.

Relation 3 may conveniently be re-stated thus: *The volume of a nucleus is proportional to the number of particles in it.*

[26] G. Gamow, Zeits. f. Physik **52**, 510 (1928).
[27] J. R. Dunning, Phys. Rev. **45**, 599 (1934).
[28] E. D. Eastman, Phys. Rev. **46**, 1 (1934).

Radii of proton barriers

Radii of proton barriers around light nuclei cannot be computed by the rules for alpha-particle barriers. In the case of fluorine the proton barrier is nearly four times higher than in the case of lithium. The ratio for alpha-particle barriers is only half as great. If we compute proton barrier radii using the relations previously given, we find the radius for lithium greater than that for fluorine, which is abnormal.

Application of Heisenberg's theory

It is possible that the anomalous behavior of nuclei to proton bombardment can be explained on Heisenberg's theory[29] that nuclei consist entirely of neutrons and protons with non-Coulomb potential functions $J(r)$ between neutron and proton, and $K(r)$ between neutron and neutron giving the major forces apart from Coulombian repulsion. Heisenberg points out that the stability is greatest in light nuclei when neutrons and protons are present in equal numbers and that if two such nuclei approach one another the first order attraction due to $J(r)$ is zero and only a second order attraction exists, analogous to van der Waals forces. The potential term for this force is $|J(r)|^2/\epsilon$. If we suppose $J(r)$ varies rapidly with distance, this second order force will vary much more rapidly and may approximate to a force effective only at the surface of a nucleus. This agrees in character with the attractive force in those alpha-particle collisions wherein the condition for equality of neutron-proton content is fulfilled for nucleus and projectile.

On the other hand, these conditions never hold for proton collisions. Thus, when protons collide with lithium ($Z=3$, mass$=7$) or fluorine ($Z=9$, mass$=19$) not only is the impinging proton unequalized by a neutron, but there is an extra neutron in the bombarded nucleus.

There are therefore first order attractions (a) between the unbalanced proton and the balanced part of the nucleus and (b) between the proton and the odd neutron. In order to show that the barriers of Li and F can be accounted for by the existence of a first order force we can suppose (b) is the force operative at larger distances and neglect (a). Then for simplicity suppose a neutron-proton attraction of the form $V=A/r^p$ (any function varying rapidly and uniformly with r will suffice) and neglect, for a first approximation, all other attractive forces between the nucleus and the proton. Then

$$V=Ze^2/r-A/r^p,$$

from which

$$r_T^{p-1}=Ap/Ze^2.$$

For Li and F, $Z=3$ and 9 and for the ratio of the radii at the summits of the barriers

$$[r_T(\mathrm{Li})/r_T'(\mathrm{F})]^{p-1}=Z(\mathrm{F})/Z(\mathrm{Li})=3.$$

The experimental values for r_T are (roughly) 1.1×10^{-6} for Li and 0.9×10^{-6} for F, giving $p=6.3$ which means an attractive potential $V\sim A/r^6$ which satisfies the general demands of a theory of nuclear barriers. The second order attraction operative for alpha-particles would be expected to be roughly of the form B/r^{12}. To explain Relation 3 it must be shown that the radius at the summit is nearly equal to a radius known to be within the nuclear structure. Now the mass defect curve shows that the addition of an alpha-particle to a light element increases the binding energy by 10^{-5} erg and from (2) we can calculate the corresponding internal radius. The added alpha-particle is, by hypothesis, within the structure. With a potential function as suggested the internal radius at this level is about 5/6 that at the top; hence using r_T to describe the radius of the nuclear structure is a justifiable approximation.

It is intended only to emphasize that a study of proton barriers yields information about the mutual potential energy of a neutron and a proton. The inverse sixth power here derived is only one of the analytical expressions that might serve for a first approximation; a potential field such as $Ae^{-\mu r}$ fits in more with current nuclear theory. Treatment similar to the above yields the value 2.0×10^{12} for μ.

It follows from the above reasoning that barriers against deuterons should be higher than those against protons, since, as for alpha-particles, only second order attraction should take place between deuterons and other even nuclei.

[29] W. Heisenberg, Zeits. f. Physik **77**, 1 (1932); **78**, 156 (1932); **80**, 587 (1933).

The wild point for hydrogen in Fig. 3 is now to be explained by a first order force between its unbalanced proton and the alpha-particle of the type (a) discussed above. This varies more slowly with distance than the second order force and hence extends further beyond the barrier, so that our uncorrected formulae give too large a value for r_T. The deuteron, with one neutron and one proton, should give a normal barrier radius of 1.8×10^{-13} cm. Experiments show that it is much more nearly normal than hydrogen (having a radius of 2.06×10^{-13} cm) but that it is probable that the field of force is a little abnormal compared to heavier nuclei.

In the preceding discussion of proton barriers the first order force between proton and nucleus was neglected in comparison with that between proton and odd neutron; it can be shown that if the former force increases with Z, as would be expected, it does not greatly alter the computed value of p.

Resonance levels for alpha-particles

The linear increase of the energy of a particular resonance level with atomic number has already been pointed out, although the experimental data are not adequate to justify dogmatic assertion. In Fig. 3 it appears that the first line of resonance levels cuts the line for $E_{1/10}$ at about $Z=15$. This means unusual excitation curves should be found for elements of higher atomic number than phosphorus.

An explanation of the linear progression has been given by Margenau and Pollard,[30] in terms of the energy change due to the addition of a charged particle in going from one element to the next higher. The fact that the non-Coulombian forces are greater for lower levels accounts for the smaller energy increase per unit change of Z, since these forces diminish the potential energy due to the added charge.

The widths of the resonance levels are not easily explained. These should obey the relation

$$\Delta E_1/E_1/\Delta E_2/E_2=P_1/P_2$$

where the suffixes refer to any two levels in one element. In aluminum levels are observed at 8.9×10^{-6} and 5.8×10^{-6} erg, having widths in a ratio of at most 2 : 1. This means P_1/P_2 should be not greater than 1.3. Actually the ratio is 130. So large a discrepancy cannot be explained by supposing the levels to differ as to angular momentum and hence the precise nature of nuclear resonance is still obscure. As Massey[31] suggests, alpha-particle exchange may be an important factor.

Disintegration by terms of higher angular momentum than zero

Haxel has suggested that the steady increase in the yield of protons from aluminum between 5.6 and 8.6 cm incident range of the alpha-particle may mean the terms of successively higher angular momentum begin to cause disintegration. His reasoning is based on the value for the barrier height given in Paper I. If the corrected values given here are used, his results are seen to fit on the penetration curve and hence do not need explaining in terms of non-zero order terms (P, D, etc.). Thus there is no definite evidence for disintegration by particles having angular momentum greater than zero.

Experiments of Miller, Duncanson and May

These workers find a rapid fall in the yield curve for disintegration of B^{10} above an energy of 5.9×10^{-6} erg, or 4.2×10^{-6} after correcting for nuclear motion. It is difficult to explain the occurrence of this drop, which, as Paton's[32] work shows, is not followed by a second rise. If the values found for barrier heights in this paper are significant, the fall is not associated with the top of the barrier, but occurs well below.

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[30] H. Margenau and E. Pollard, Phys. Rev. **46**, 228 (1934).

[31] H. S. W. Massey, Proc. Roy. Soc. **A137**, 455 (1932).

[32] R. F. Paton, Zeits. f. Physik **90**, 586 (1934).