error (about 100 particles only were counted at each point for the nitrogen curve). We do not consider that this curve alone is sufficient to disprove the presence of deuterons but that taken with the fact that the expected short range group for bombardment by full range alpha-particles is absent, it enables a final conclusion to be drawn in favor of protons.

It may be pointed out that the difference between the proton curve and that obtained by bombarding $Ca(OD)_2$ is direct evidence that the projected particles are deuterons, as concluded from measurements of their range by Rutherford and Kempton.[13]

In conclusion we wish to express our thanks to Professor A. F. Kovarik for his interest and advice, to Dr. C. T. Lane for advice in running the magnet, to Dr. Donald Cooksey for assistance in the counter design, and to Professor H. C. Urey for a gift of heavy water.

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[13] Rutherford and A. E. Kempton, Proc. Roy. Soc. **A143**, 724 (1934).

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# Nuclear Shells: Angular and Magnetic Momenta of Nuclei

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**Part I. Angular momenta.** The experimental data on the number of isotopes per atom show marked regularities, which suggest closed shells in the nucleus. These regularities have been rigorously followed in arranging the first thirty elements into an isotopic system, with proton and neutron shells. In order to correlate both the angular and magnetic momenta of nuclei, it is necessary to choose certain $j$ values of the terms arising from the proton and neutron configurations, as the deepest terms. After this choice has been made, the assumption, that the $j$ values of the lowest terms are added vectorially with the $j$'s oppositely directed, makes it possible to account for all of the observed $i$ values. Apparently the $S$ states are exceptions to the general rule and have their $j$'s in the same direction.

**Part II. Magnetic momenta.** Nuclear magnetic moments are discussed from the viewpoint of proton and neutron shells in the nucleus. The generalized $g$-formula is used to calculate the proton and neutron contributions to the $g$-factor. These two contributions are then combined by $jj$ coupling to give the nuclear $g$ values. A magnetic moment of $+2.7$ nuclear magnetons for the proton, and $+1.75$ nuclear magnetons for the neutron are used in the calculations. The deepest proton and neutron terms fix the values of $l$, $j$ and $s$, so that, once having chosen these deepest terms, no arbitrariness exists in the calculations. A discussion of the correlated data is given.

## PART I

FORTY-EIGHT elements have been found to have a nuclear spin greater than zero. Three ($He^4$, $C^{12}$ and $O^{16}$) are known to have zero spin. Twenty of these elements, about which definite information is known, are among the first thirty elements of the periodic table. For this reason, an attempt to correlate proton and neutron shells with nuclear momenta has been made.

In addition to the known nuclear "$i$" values, the number of known isotopes per atom provide valuable information which can be used as a guide for deciding where nuclear shells are filled. Isotopic regularities are nicely shown by the Chart of Isotopes which has been compiled by Bartlett.[1] Part of those data which are pertinent to the present discussion is given in Table I. The elements are listed in the first column, the number of known isotopes in the second, and the number of expected isotopes in the third. Numbers larger than one mean that that number of isotopes occur with consecutive mass numbers. Where the notation 1, 1 or 1, 1, 1 (Cl, A) is used, each comma denotes a missing mass number between known mass numbers for that element.

Table I suggests[2] that one isotopic regularity

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[1] Bartlett, Rev. Sci. Inst. **6**, 61 (1935).

[2] Bartlett (Nature **130**, 165 (1932)) has suggested shells similar to these.

TABLE I. *Number of isotopes per atom.*

| Element | No. of known isotopes | No. of expected isotopes | Element | No. of known isotopes | No. of expected isotopes |
|---|---|---|---|---|---|
| H | 3 | 3 | Cl | 1, 1 | 1, 1 |
| He | 2 | 3 | A | 1, 1, 1 | 1, 1, 1 |
| | | | K | 1, 1 | 1, 1 |
| Li | 2 | 2 | | | |
| Be | 1 | 2 | Ca | 1, 3 | 5 |
| B | 2 | 2 | Sc | 1 | 1, 1 |
| C | 2 | 2 | Ti | 5 | 5 |
| N | 2 | 2 | V | 1 | 1, 1 |
| | | | Cr | 1, 3 | 5 |
| O | 3 | 3 | Mn | 1 | 1, 1 |
| F | 1 | 1 | Fe | 1, 2 | 5 |
| Ne | 3 | 3 | Co | 1 | 1, 1 |
| Na | 1 | 1 | Ni | 1, 3 | 5 |
| Mg | 3 | 3 | | | |
| Al | 1 | 1 | Cu | 1, 1 | 1, 1, 1 |
| Si | 3 | 3 | Zn | 1, 3, 1 | |
| P | 1 | 1 | Ga | 1, 1 | |
| S | 3 | 3 | | | |

ends with He, which therefore is considered to close the first shell. The next regularity ends with N; the third with S; the fourth with K; and the fifth with Ni. Thus He, N, S, K and Ni may be considered as the elements where successive shells will be approximately closed. There remain to be examined the details of the proton-neutron shell filling process.

Table II shows in detail the way protons and neutrons are added. First, two $S$ protons ($S_p$) and two $S$ neutrons ($S_n$) are put in. This completes the first shell with $He^4$. Next the addition of six $P$ protons ($P_p$) and six $P$ neutrons ($P_n$) forms $O^{16}$ which closes the second shell (or subshell). Then ten $D$ protons ($D_p$) and ten $D$ neutrons ($D_n$) are added, which gives $A^{36}$ and closes that shell (or sub-shell). With $Ca^{40}$ two more $S_p$ and $S_n$ have been added to complete another S shell. Following this scheme it is necessary to add ten $D_p$ and fourteen $F_n$ to account for the next regularity which ends with Ni. Thus $Zn^{64}$ closes the $D_pF_n$ shell.

Elsasser[3] has shown from theoretical considerations that $S$, $P$, $D$, $S$, $F$ shells are to be expected as the first nuclear shells. The present system confirms that prediction except for the last shell, which is found to be a $D-F$ shell.

In order to correlate the observed nuclear spins with the proposed isotopic system, the following assumptions are made:

1. The proton has a spin of $\frac{1}{2}(h/2\pi)$ units of angular momentum in the nucleus.
2. The neutron has a spin of $\frac{1}{2}(h/2\pi)$ units of angular momentum in the nucleus.
3. $S$, $P$, $D$, $F$ proton and neutrons have orbital angular momenta $l=0$, 1, 2, 3, like electrons outside of the nucleus.
4. That the protons and neutrons outside of a closed shell each give rise to a set of deepest terms (similar to electron deepest terms for a similar configuration).
5. That the deepest proton term and the deepest neutron term determine the deepest nuclear level; and that the resultant angular momentum of the nucleus, $i$, is given by combining vectorially the $j$'s of the deepest proton and neutron terms, *with the provision that these $j$'s are oppositely directed.*

Since the regularities in the isotopic data of Table I have been rigorously followed, there is no arbitrariness in the choice of the number of protons and neutrons in each shell for this model. The only arbitrariness which exists, is in the choice of the deepest term of a proton or neutron multiplet. Two quantities, $i$ the spin and $\mu$ the magnetic moment, limit this choice so that although for some elements (see discussion of F in Part II) there is more than one combination of terms which will satisfy the $i$ values, there is only one which gives best agreement with both the magnetic and spin data. This means that the deepest terms listed in Table II have been chosen so that they are in agreement with the angular momentum of the nucleus and at the same time fit the magnetic moment as well as possible.

At the right side of Table II are listed the observed $i$ values, the deepest proton and neutron terms, and the calculated $i$ values. As an illustration consider the case of $Li^7$ in detail. The deepest proton term is ${}^2P_{1/2\ 3/2}$ from one $P_p$; the deepest neutron term is a ${}^3P_{210}$ from two $P_n$. The ${}^2P$ is normal, the ${}^3P$ inverted. Thus we have $j=2$ opposite to $j=\frac{1}{2}$, and the vector sum is $3/2=i$.

Whenever possible, the deepest terms of elements for which $i$ and $\mu$ are unknown have been chosen so that they agree with the terms arising from similar proton and neutron configurations of nuclei for which $i$ and $\mu$ are known. This is illustrated by the choice of terms for elements from $Li^6$ to $O^{16}$. For example, $N^{14}$ has terms $P_p{}^5\,{}^2P_{1/2}$, $P_n{}^5\,{}^2P_{3/2}$; thus $N^{15}$ must have the same deepest proton term since the proton configurations of $N^{14}$ and $N^{15}$ are identical. For

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[3] W. M. Elsasser, J. de phys. et rad. [7] 4, Part 2, 549 (1933).

TABLE II. *Proton—neutron shells.*

| ATOMIC NO. | ELEMENT | ISOTOPE | 1 | | 2 | | | | 3 | | | | | | 4 | | | | | | | | OBS. *i* | *LS* COUPLING DEEPEST TERMS | | CALC. *i* | PROTON TERMS | NEUTRON TERMS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $S_p$ | $S_n$ | $S_p$ | $S_n$ | $P_p$ | $P_n$ | $S_p$ | $S_n$ | $P_p$ | $P_n$ | $D_p$ | $D_n$ | $S_p$ | $S_n$ | $P_p$ | $P_n$ | $D_p$ | $D_n$ | $F_p$ | $F_n$ | | PROTON TERMS | NEUTRON TERMS | | | |
| 1 | H | 1 | 1 | | | | | | | | | | | | | | | | | | | | 1/2 | $^2S_{1/2}$ | | 1/2 | | |
| | | 2 | 1 | 1 | | | | | | | | | | | | | | | | | | | 1 | $^2S_{1/2}$ | $^2S_{1/2}$ | 1 | | |
| | | 3 | 1 | 2 | | | | | | | | | | | | | | | | | | | | $^2S_{1/2}$ | $^1S_0$ | 1/2 | | |
| | | | 1 | 1 | | 1 | | | | | | | | | | | | | | | | | | $^2S_{1/2}$ | $^3S_1$ | 3/2 | | |
| 2 | He | 3 | 2 | 1 | | | | | | | | | | | | | | | | | | | | $^1S_0$ | $^2S_{1/2}$ | 1/2 | | |
| | | | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | $^3S_1$ | $^2S_{1/2}$ | 3/2 | | |
| | | 4 | 2 | 2 | | | | | | | | | | | | | | | | | | | 0 | $^1S_0$ | $^1S_0$ | 0 | | |
| 3 | Li | 6 | 2 | 2 | | | 1 | 1 | | | | | | | | | | | | | | | | $^2P_{1/2\,3/2}$ | $^2P_{3/2\,1/2}$ | 1 | N | I |
| | | 7 | 2 | 2 | | | 1 | 2 | | | | | | | | | | | | | | | 3/2 | $^2P_{1/2\,3/2}$ | $^3P_{2\,1\,0}$ | 3/2 | N | I |
| 4 | Be | (8) | 2 | 2 | | | 2 | 2 | | | | | | | | | | | | | | | | $^3P_{2\,1\,0}$ | $^3P_{2\,1\,0}$ | 0 | I | I |
| | | 9 | 2 | 2 | | | 2 | 3 | | | | | | | | | | | | | | | 1/2? | $^3P_{2\,1\,0}$ | $^4S_{3/2}$ | 1/2 | I | |
| 5 | B | 10 | 2 | 2 | | | 3 | 3 | | | | | | | | | | | | | | | | $^4S_{3/2}$ | $^4S_{3/2}$ | 0 | | |
| | | 11 | 2 | 2 | | | 3 | 4 | | | | | | | | | | | | | | | | $^4S_{3/2}$ | $^3P_{2\,1\,0}$ | 1/2 | | I |
| 6 | C | 12 | 2 | 2 | | | 4 | 4 | | | | | | | | | | | | | | | 0 | $^3P_{2\,1\,0}$ | $^3P_{2\,1\,0}$ | 0 | I | I |
| | | 13 | 2 | 2 | | | 4 | 5 | | | | | | | | | | | | | | | | $^3P_{2\,1\,0}$ | $^2P_{3/2\,1/2}$ | 1/2 | I | I |
| 7 | N | 14 | 2 | 2 | | | 5 | 5 | | | | | | | | | | | | | | | 1 | $^2P_{1/2\,3/2}$ | $^2P_{3/2\,1/2}$ | 1 | N | I |
| | | 15 | 2 | 2 | | | 5 | 6 | | | | | | | | | | | | | | | | $^2P_{1/2\,3/2}$ | $^1S_0$ | 1/2 | N | I |
| 8 | O | 16 | 2 | 2 | | | 6 | 6 | | | | | | | | | | | | | | | 0 | $^1S_0$ | $^1S_0$ | 0 | I | I |
| | | 17 | 2 | 2 | | | 6 | 6 | | | | | | 1 | | | | | | | | | | | $^2D_{3/2\,5/2}$ | 3/2 | | N |
| | | 18 | 2 | 2 | | | 6 | 6 | | | | | | 2 | | | | | | | | | | | $^3F_{2\,3\,4}$ | 2 | | N |
| 9 | F | 19 | 2 | 2 | | | 6 | 6 | | | | | 1 | 2 | | | | | | | | | 1/2 | $^2D_{5/2\,3/2}$ | $^3F_{2\,3\,4}$ | 1/2 | I | N |
| 10 | Ne | 20 | 2 | 2 | | | 6 | 6 | | | | | 2 | 2 | | | | | | | | | 0 | $^3F_{2\,3\,4}$ | $^3F_{2\,3\,4}$ | 0 | N | N |
| | | 21 | 2 | 2 | | | 6 | 6 | | | | | 2 | 3 | | | | | | | | | | $^3F_{2\,3\,4}$ | $^4F_{3/2\,5/2\,7/2\,9/2}$ | 1/2 | N | N |
| | | 22 | 2 | 2 | | | 6 | 6 | | | | | 2 | 4 | | | | | | | | | | $^3F_{2\,3\,4}$ | $^5D_{1\,2\,3\,4\,0}$ | | N | PI |
| 11 | Na | 23 | 2 | 2 | | | 6 | 6 | | | | | 3 | 4 | | | | | | | | | 3/2 | $^4F_{5/2\,7/2\,9/2\,3/2}$ | $^5D_{1\,2\,3\,4\,0}$ | 3/2 | PI | PI |
| 12 | Mg | 24 | 2 | 2 | | | 6 | 6 | | | | | 4 | 4 | | | | | | | | | | $^5D_{0\,1\,2\,3\,4}$ | $^5D_{1\,2\,3\,4\,0}$ | | N | PI |
| | | 25 | 2 | 2 | | | 6 | 6 | | | | | 4 | 5 | | | | | | | | | | $^5D_{0\,1\,2\,3\,4}$ | $^6S_{5/2}$ | 5/2 | N | |
| | | 26 | 2 | 2 | | | 6 | 6 | | | | | 4 | 6 | | | | | | | | | | $^5D_{0\,1\,2\,3\,4}$ | $^5D_{3\,2\,1\,0\,4}$ | ? | N | PI |
| 13 | Al | 27 | 2 | 2 | | | 6 | 6 | | | | | 5 | 6 | | | | | | | | | 1/2 | $^6S_{5/2}$ | $^5D_{3\,2\,1\,0\,4}$ | 1/2 | | PI |
| 14 | Si | 28 | 2 | 2 | | | 6 | 6 | | | | | 6 | 6 | | | | | | | | | | $^5D_{4\,3\,2\,1\,0}$ | $^5D_{3\,2\,1\,0\,4}$ | ? | I | PI |
| | | 29 | 2 | 2 | | | 6 | 6 | | | | | 6 | 7 | | | | | | | | | | $^5D_{4\,3\,2\,1\,0}$ | $^4F_{9/2\,7/2\,5/2\,3/2}$ | 1/2 | I | I |
| | | 30 | 2 | 2 | | | 6 | 6 | | | | | 6 | 8 | | | | | | | | | | $^5D_{4\,3\,2\,1\,0}$ | $^3F_{4\,3\,2}$ | 0 | I | I |
| 15 | P | 31 | 2 | 2 | | | 6 | 6 | | | | | 7 | 8 | | | | | | | | | 1/2 | $^4F_{9/2\,7/2\,5/2\,3/2}$ | $^3F_{4\,3\,2}$ | 1/2 | I | I |
| 16 | S | 32 | 2 | 2 | | | 6 | 6 | | | | | 8 | 8 | | | | | | | | | | $^3F_{4\,3\,2}$ | $^3F_{4\,3\,2}$ | 0 | I | I |
| | | 33 | 2 | 2 | | | 6 | 6 | | | | | 8 | 9 | | | | | | | | | | $^3F_{4\,3\,2}$ | $^2D_{5/2\,3/2}$ | 3/2 | I | I |
| | | 34 | 2 | 2 | | | 6 | 6 | | | | | 8 | 10 | | | | | | | | | | $^3F_{4\,3\,2}$ | $^1S_0$ | 4 | I | I |
| 17 | Cl | 35 | 2 | 2 | | | 6 | 6 | | | | | 9 | 10 | | | | | | | | | 5/2 | $^2D_{5/2\,3/2}$ | $^1S_0$ | 5/2 | I | I |
| 18 | A | 36 | 2 | 2 | | | 6 | 6 | | | | | 10 | 10 | | | | | | | | | | $^1S_0$ | $^1S_0$ | 0 | I | I |
| 17 | Cl | 37 | 2 | 2 | | 2 | 6 | 6 | | | | | 9 | 10 | | | | | | | | | | $^2D_{5/2\,3/2}$ | $^1S_0$ | 5/2 | I | I |
| 18 | A | 38 | 2 | 2 | | 2 | 6 | 6 | | | | | 10 | 10 | | | | | | | | | | | $^1S_0$ | 0 | | |
| 19 | K | 39 | 2 | 2 | 1 | 1 | 6 | 6 | | 1 | | | 10 | 10 | | | | | | | | | 3/2 | $^2S_{1/2}$ | $^3S_1$ | 3/2 | | |
| 18 | A | 40 | 2 | 2 | | 2 | 6 | 6 | | 2 | | | 10 | 10 | | | | | | | | | | | $^1S_0{}^1S_0$ | 0 | | |
| 20 | Ca | 40 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | | | | | 0 | $^1S_0$ | $^1S_0$ | 0 | | |
| 19 | K | 41 | 2 | 2 | 1 | 2 | 6 | 6 | | 1 | | | 10 | 10 | | 1 | | | | | | | 3/2 | $^2S_{1/2}$ | $^3S_1$ | 3/2 | | |
| 20 | Ca | (41) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | | | | 1 | | | | | | |
| | | 42 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | | | | 2 | | | | | | |
| | | 43 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | | | | 3 | | | | | | |
| | | 44 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | | | | 4 | | | | | | |
| 21 | Sc | 45 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 1 | | | 4 | 7/2 | $^2D_{5/2\,3/2}$ | $^5\mathrm{I}_{6\,7\,8\,4\,5}$ | 7/2 | I | PI |
| | | (47) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 1 | | | 6 | | | | | | |
| 22 | Ti | 46 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 2 | | | 4 | | | | | | |
| | | 47 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 2 | | | 5 | | | | | | |
| | | 48 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 2 | | | 6 | | | | | | |
| | | 49 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 2 | | | 7 | | | | | | |
| | | 50 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 2 | | | 8 | | | | | | |
| 23 | V | (49) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 3 | | | 6 | | | | | | |
| | | 51 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 3 | | | 8 | 7/2 | $^4F_{9/2\,7/2\,5/2\,3/2}$ | $^7F_{1\,2\,3\,4\,5\,6\,0}$ | 7/2 | I | PI |
| 24 | Cr | 50 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 4 | | | 6 | | | | | | |
| | | (51) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 4 | | | 7 | | | | | | |
| | | 52 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 4 | | | 8 | | | | | | |
| | | 53 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 4 | | | 9 | | | | | | |
| | | 54 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 4 | | | 10 | | | | | | |
| 25 | Mn | (53) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 5 | | | 8 | | | | | | |
| | | 55 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 5 | | | 10 | 5/2 | $^6S_{5/2}$ | $^5\mathrm{I}_{5\,6\,7\,8\,4}$ | 5/2 | N | PI |
| 26 | Fe | 54 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 6 | | | 8 | | | | | | |
| | | (55) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 6 | | | 9 | | | | | | |
| | | 56 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 6 | | | 10 | | | | | | |
| | | 57 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 6 | | | 11 | | | | | | |
| | | (58) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 6 | | | 12 | | | | | | |
| 27 | Co | (57) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 7 | | | 10 | | | | | | |
| | | 59 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 7 | | | 12 | 7/2 | $^4F_{3/2\,5/2\,7/2\,9/2}$ | $^3H_{5\,6\,4}$ | 7/2 | N | PI |
| 28 | Ni | 58 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 8 | | | 10 | | | | | | |
| | | (59) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 8 | | | 11 | | | | | | |
| | | 60 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 8 | | | 12 | | | | | | |
| | | 61 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 8 | | | 13 | | | | | | |
| | | 62 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 8 | | | 14 | | | | | | |
| 29 | Cu | (61) | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 9 | | | 12 | | | | | | |
| | | 63 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 9 | | | 14 | 3/2 | $^2D_{3/2\,5/2}$ | $^1S_0$ | 3/2 | N | I |
| | | 65 | 2 | 2 | 2 | 2 | 6 | 6 | | 2 | | | 10 | 10 | | | | | 9 | | | 14 | 3/2 | $^2D_{3/2\,5/2}$ | $^1S_0$ | 3/2 | N | I |
| 30 | Zn | 64 | 2 | 2 | 2 | 2 | 6 | 6 | | | | | 10 | 10 | | | | | 10 | | | 14 | | | | | | |

*Note:* N means terms are normal; I means terms are inverted; PI means terms are partly inverted.

elements of higher atomic number than oxygen, the data are not complete enough to make the term assignment unambiguous. Obviously the predicted values of $i$ can only be correct if the proper deepest terms have been chosen, and in all cases the experimental values of $i$ and $\mu$ must eventually determine the deepest terms.

If the proposed isotopic system be admitted as correct, the following undiscovered isotopes would be expected to exist: $He^5$ (or $Li^5$), $Be^8$, $Ca^{41}$, $Sc^{47}$, $V^{49}$, $Cr^{51}$, $Mn^{53}$, $Fe^{55,\,58}$, $Co^{57}$, $Ni^{59}$, $Cu^{61}$. Some of these have been predicted by Beck[4] and Bartlett.[5] It seems especially probable that $Sc^{47}$, $V^{49}$, $Mn^{53}$, $Co^{57}$ and $Cu^{61}$ should exist for they are necessary if a smooth neutron filling scheme exists for these elements. In view of the proposed system there is no reason why the others should not exist in such small quantities that they have escaped detection with the mass spectrograph. Their omission would leave bad holes in the filling scheme.

The $i$ values for elements in $S$ shells seem to be exceptions to the general rule. $H^2$ has one proton and one neutron in the nucleus which gives rise to two $^2S_{1/2}$ terms and by subtracting the $j$'s, $i=0$. But $i$ is known to be equal to one. Thus it becomes necessary to *add* $j$'s when $l=0$ in order to get $i$. The case of $K^{39,\,41}$, $i=3/2$ is also interesting because it indicates that one $S_p$ and two $S_n$ have the same spin as one $S_p$, two $S_n$ and two other $S_n$ in another sub-shell. If this interpretation is correct it means that one $S_p$ and two $S_n$ give the terms $^2S_{1/2}$ and some other term whose $j$ value is one, the $j$'s of which then add to give $i=3/2$. This can be explained best by giving $K^{39}$ the configurations (outside of closed shells) $2S_p\,2S_n\,3S_n$ which gives rise to the proton term $^2S_{1/2}$ and the neutron term $^3S_1$. This $^3S_1$ will be the deepest term for a $2S_n\,3S_n$ configuration but will lie higher than the $^1S_0$ term from $2S_n{}^2$. Correspondingly, the configurations for $K^{41}$ will be $2S_p\,2S_n{}^2\,3S_n\,4S_n$, and again the terms will be $^2S_{1/2}$ and $^3S_1$, the $2S_n{}^2$ giving rise to $^1S_0$ which contributes nothing to $i$.

$H^3$ may be discussed from the above viewpoint. If it is radioactive or slightly unstable the expected configurations would be $1S_p\,1S_n\,2S_n$, which would give rise to terms $^2S_{1/2}$ and $^3S_1$ with $i=3/2$. If it is stable, the configuration $1S_p\,{}^2S_{1/2}$, $1S_n{}^2\,{}^1S_0$ giving $i=1/2$, is most likely.

For $He^3$, $1S_p{}^2\,{}^1S_0$, $1S_n\,{}^2S_{1/2}$ with $i=1/2$ is probably the most likely configuration, though $1S_p\,1S_n\,2S_p\,{}^3S_1\,{}^2S_{1/2}$ with $i=3/2$ is not excluded.

For elements of higher atomic number than $Zn^{64}$ the data on the number of isotopes per element are so incomplete that the position of closed shells cannot be determined from Bartlett's Isotope Chart. Thus the extension of these data to heavier elements is difficult, though enough $i$ values are known to check the system in most places.

## PART II

Landé,[6] Tamm and Altschuler,[7] and Schüler[8] have discussed the idea that one proton or one neutron is mainly responsible for the total spin and the magnetic properties of the nucleus. The writer next wishes to discuss the magnetic moments of nuclei from the standpoint of proton and neutron shells in the nucleus.

### Calculations

Table III gives the elements to be discussed, their deepest proton and neutron terms, and the corresponding values of the nuclear spin. These data are used in calculating the nuclear $G_I$ factors and the nuclear magnetic momenta $\mu$. The contribution of the protons and of the neutrons to $G_I$ are calculated separately by using the generalized $g$-formula

$$g=g_1\frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)}+g_2\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}.$$

The proton contribution $g_p=g$ when $g_1=g_l$, which for protons has a value of one, and when $g_2=g_s$ which has a value of 5.4. This corresponds to a magnetic moment of a proton of 2.7 small magnetons. The values of $j$, $l$ and $s$ are those which correspond to the deepest proton term.

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[4] Beck, Zeits. f. Physik **47**, 407 (1928).

[5] J. H. Bartlett, Phys. Rev. **42**, 145 (1932).

[6] Landé, Phys. Rev. **46**, 477 (1934).

[7] Tamm and Altschuler, Comptes rendus de l'academie des sciences de l'U. R. S. S. **1**, 455 (1934).

[8] Schüler, Zeits. f. Physik **88**, 323 (1934).

TABLE III. *Magnetic moments of nuclei.*
For protons $g_l = 1$, $g_s = 5.4$; For neutrons $g_l' = 0$, $g_s' = 3.5$.

| ATOMIC NO. | ELEMENT | ISOTOPE | PROTON TERM | NEUTRON TERM | $g_p$ | $g_n$ | $G_I$ CALC. | $G_I$ EXP. | $i$ | $\mu$ CALC. | $\mu$ OBS. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | H | 1 | $^2S_{1/2}$ |  | 5.4 |  | 5.4 | 5.0 | 1/2 | 2.7 | 2.5 |
|  | H | 2 | $^2S_{1/2}$ | $^2S_{1/2}$ | 5.4 | 3.5 | 0.95 | 0.7 | 1 | 0.9 | $0.7 \pm 0.2$ |
| 2 | He | 4 | $^1S_0$ | $^1S_0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | Li | 7 | $^2P_{1/2}$ | $^3P_2$ | −0.47 | 1.75 | 2.19 | 2.19 | 3/2 | 3.28 | 3.28 |
| 4 | Be | 9 | $^3P_2$ | $^4S_{3/2}$ | 3.2 | 3.5 | 2.9 | — | 1/2? | 1.45 | — |
| 6 | C | 12 | $^3P_2$ | $^3P_2$ | 3.2 | 1.75 | 0 | 0 | 0 | 0 | 0 |
| 7 | N | 14 | $^2P_{1/2}$ | $^2P_{3/2}$ | −0.47 | 1.17 | 1.58 | $\leqq 0.2$? | 1 | 1.58 | 0.2? |
| 9 | F | 19 | $^2D_{5/2}$ | $^3F_2$ | 1.88 | −1.17 | 5.93 | 6.0 | 1/2 | 2.97 | 3.0 |
| 11 | Na | 23 | $^4F_{5/2}$ | $^5D_1$ | 1.13 | 1.75 | 0.88 | 1.4 | 3/2 | 1.32 | 2.1 |
| 13 | Al | 27 | $^6S_{5/2}$ | $^5D_2$ | 5.4 | 1.75 | 10.27 | 4.2 | 1/2 | 5.13 | 2.1 |
|  |  |  | $^6S_{5/2}$ | $^5D_3$ | 5.4 | 1.75 | −4.33 |  | 1/2 | −2.16 |  |
| 15 | P | 31 | $^4F_{9/2}$ | $^3F_4$ | 2.46 | 0.88 | 6.70 | — | 1/2 | 3.35 | — |
| 17 | Cl | 35 | $^2D_{5/2}$ | $^1S_0$ | 1.88 | 0 | 1.88 | — | 5/2 | 4.70 | — |
| 19 | K | 39 | $^2S_{1/2}$ | $^3S_1$ | 5.4 | 3.5 | 0.53 | .25 | 3/2 | 0.8 | 0.38<br>1.2 |
| 21 | Sc | 45 | $^2D_{5/2}$ | $^5I_6$ | 1.88 | 0.25 | −0.66 | 1.0 | 7/2 | −2.3 | 3.5 |
| 27 | Co | 59 | $^4F_{3/2}$ | $^3H_5$ | −1.63 | 0.12 | 0.7 | 0.8 | 7/2 | 2.45 | 2.8 |
| 29 | Cu | 65 | $^2D_{3/2}$ | $^1S_0$ | 0.12 | 0 | 0.12 | 1.7 | 3/2 | 0.18 | 2.55 |

The neutron contribution $g_n = g$ when $g_1 = g_l'$ which equals zero for neutrons, and when $g_2 = g_s' = +3.50$. This value of $g_s'$, so chosen because it fits the experimental data best, means that the magnetic moment of the neutron is +1.75 small magnetons. Correspondingly, $j$, $l$ and $s$ assume the values which fit the deepest neutron term. Then $G_I$ is obtained by coupling $g_p$ and $g_n$ according to the equation,

$$G_I = g_p \frac{i(i+1) + j_p(j_p+1) - j_n(j_n+1)}{2i(i+1)} + g_n \frac{i(i+1) + j_n(j_n+1) - j_p(j_p+1)}{2i(i+1)},$$

where $j_p = j$ value of the deepest proton term and $j_n = j$ value of the deepest neutron term.

To illustrate the calculation with a special case, consider $Li^7$. The deepest proton term is $^2P_{1/2}$, the deepest neutron term $^3P_2$. Accordingly, to calculate $g_p$, $j = 1/2$, $l = 1$ and $s = 1/2$. This gives $g_p = -0.47$. For $g_n$, $j = 2$, $l = 1$, and $s = 1$ so that $g_n = 1.75$. In calculating $G_I$, $i = 3/2$, $j_p = 1/2$, and $j_n = 2$. Inserting these values in the above equation makes $G_I$ for Li equal 2.19.

Table III gives the calculated values of $g_p$, $g_n$ and $G_I$, in the 6th, 7th and 8th columns, respectively. The calculated values of $G_I$ are compared with the experimental values in the next column. The last two columns of the table compare the calculated magnetic moments with the experimental values.

In discussing the correlation of proton-neutron shells (Part I) and angular momenta of nuclei it was found that elements in $S$-shells were exceptions to the general rule, and that for $S$-terms, the $j$'s must be added instead of subtracted to get $i$. A similar situation occurs for the magnetic momenta. In calculating $G_I$ the two contributions from $g_p$ and $g_n$ are subtracted for elements ($H^2$ and K) in $S$-shells. The writer can see no obvious explanation for these two anomalies.

## Discussion

$H^1$: The magnetic moment for $H^1$, $\mu = 2.7$, agrees well with the molecular beam value of Stern, Estermann and Frisch,[9] who report $\mu = 2.5$.

$H^2$: The value $\mu = 0.75 \pm 0.2$ from the atomic beam method[10] is supported by the ratio $\mu H^1/\mu H^2 = 4$ from the work of Kalckar and Teller.[11] The calculated value is in good agreement with these values.

N: Nitrogen is the first element of Table III to offer serious disagreement with the reported[12] experimental evidence. The present discussion suggests that the experimental value may be too low and that it would be well to check the value with another experiment.

F: Fluorine offers a good example of the case, mentioned in Part I, where there is more than one combination of terms which will satisfy the $i$ values, but only one which best satisfies both $i$ and $\mu$. There are three possible combinations which will give $i = 1/2$. They are:

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[9] Stern, Estermann and Frisch, Zeits. f. Physik **85** (4), 17 (1933).

[10] Rabi, Kellogg and Zacharias, Phys. Rev. **46**, 163 (1934).

[11] Kalckar and Teller, Nature **134**, 180 (1934).

[12] Bacher, Phys. Rev. **43**, 1044 (1933).

$^2D_{3/2}\,^3F_2 : g_p = +0.12 : g_n = -1.17 : G_I = -2.46,$
$^2D_{5/2}\,^3F_2 : g_p = +1.88 : g_n = -1.17 : G_I = +5.93,$
$^2D_{5/2}\,^3F_3 : g_p = +1.88 : g_n = +0.29 : G_I = -2.35.$

The only value of $G_I$ which agrees with the value of Brown and Bartlett[13] is $+5.93$.

Na: The terms $^4F_{5/2}\,^5D_1$ have been chosen for Na. They give $\mu = +1.32$ which is not in serious disagreement with the experimental value of $+2.10$. If $^4F_{3/2}\,^5D_0$ were chosen, $\mu = -2.45$ a value which is about right in magnitude but of the wrong sign. Other choices of deepest terms and their respective $\mu$ values are:

$$^4F_{7/2}\,^5D_2 \quad \mu = 3.3 \qquad ^4F_{3/2}\,^5D_3 \quad \mu = 5.7$$
$$^4F_{9/2}\,^5D_3 \quad \mu = 5.0 \qquad ^4F_{5/2}\,^5D_4 \quad \mu = 3.6.$$

It may be that $\mu$ is as big as 3.3. In that case the deepest terms $^4F_{7/2}\,^5D_2$ must be chosen and the deepest terms of Table II correspondingly rearranged.

Al: In Al the deepest proton term is $^6S_{5/2}$, but there are two possible choices of the deepest neutron term, i.e., $^5D_2$ or $^5D_3$. $^5D_2$ gives $\mu = 5.13$ while $^5D_3$ gives $\mu = -2.16$. The latter value agrees well in magnitude with the experimental value 2.1 but not in sign. If the present model is correct, it must be concluded that the Al data can be explained equally well with a negative magnetic moment or that the formulae of Goudsmit[14] and Fermi[15] are particularly inadequate for the case of Al.[16]

K: Millman, Fox and Rabi[17] have found $\mu = 0.38$ from the atomic beam method. Gibbons and Bartlett[18] have obtained $\mu = 1.2$ from a theoretical calculation. There is no serious disagreement between these values since this discussion gives a value midway between the other two.

Sc: Scandium agrees well with the experimental value except for sign and since Kopfermann and Rasmussen[19] assumed the sign to be positive this is not a serious disagreement.

Co: Cobalt agrees well with More's[20] data.

Cu: The deepest proton term as given in the tables is $D_p^9\,^2D_{3/2}$ and the corresponding $G_I = 0.12$. This value presents the most serious disagreement in Table III. If $D_p^8\,S_p$ were the deepest proton configuration a $^4F_{3/2}$ deepest term would result. This would give $G_I = -1.64$ which is about right in magnitude but of the wrong sign.

The magnetic moment of thirteen of the sixteen elements listed in Table III are known from experiment, but the values are not as accurately known as is desirable. A variation of ±10 percent is usually considered very probable and it is not unlikely that some values are inaccurate to ±30 or 40 percent. More accurate values of the atomic wave functions must be known before improved values of the magnetic moment can be obtained. It is possible that most of them will have to be changed. With this in mind one may consider that the calculated and observed $\mu$ of ten ($H^1$, $H^2$, He, Li, C, F, Na, K, Sc, and Co) of the thirteen elements agree well. One (Al) agrees fairly well and only two (N and Cu) are in serious disagreement.

Clearly this agreement between observed and calculated magnetic moments is not as good as one would like, and the only conclusion to be drawn at the present time is that the agreement is neither so bad as to invalidate the scheme entirely nor good enough to substantiate it completely. The point of finding only one combination of the deepest proton and neutron term which best fits both magnetic and spin data seems to be important and when more accurate values of the magnetic moment are known it will be possible to decide if the proposed scheme is valid. Perhaps some modification can be made which will take care of the special cases where there is no agreement at present. Also it may be that the vector model can be used to describe the angular momenta, but that it is not adequate to express the magnetic momenta. However, from the present discussion, it seems that there is a possibility of accounting for the angular momenta of nuclei in terms of proton and neutron shells and that such a tentative model may be helpful in the study of nuclear problems.

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[13] Brown and Bartlett, Phys. Rev. **45**, 527 (1934).

[14] Goudsmit, Phys. Rev. **43**, 636 (1933).

[15] Fermi and Segrè, Zeits. f. Physik **82**, 729 (1933).

[16] Brown and Cook (Phys. Rev. **45**, 731L (1934)) calculated $\mu = 4.8$ for Al. This is a higher value than the Goudsmit, Fermi and Segrè formulas give. It is possible that a still higher value will be obtained, when the atomic wave functions are more accurately known.

[17] Millman, Fox and Rabi, Phys. Rev. **46**, 320 (1934).

[18] Gibbons and Bartlett, Phys. Rev. May 1 (1935) in print.

[19] Kopfermann and Rasmussen, Zeits. f. Physik **92**, 82 (1934).

[20] More, Phys. Rev. **46**, 470 (1934); P, Cl: Tolansky, Zeits. f. Physik **74**, 336 (1932).