Hyperfine Structures in La III. Nuclear Magnetic Moment of Lanthanum

M. F. CRAWFORD AND NORMAN S. GRACE, *McLennan Laboratory, University of Toronto*

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Hyperfine structures of the resonance lines of La III have been measured and the hfs separations of $6s\,^2S_{1/2}$, $6p\,^2P_{1/2,\,3/2}$ determined. A consistent value of 2.8 nuclear magnetons is obtained for the magnetic moment of the lanthanum nucleus from these separations. The $g(I)$ factors of several M odd Z odd nuclei with $I=7/2$ are compared.

HYPERFINE structures in La I and II have been studied by Meggers and Burns,[1] White and Anderson[2] and Anderson.[3] The latter investigators have shown from their measurements of La I structures that the nuclear mechanical moment of $_{57}La^{139}$ very probably is $7/2\cdot h/2\pi$. The value of the nuclear magnetic moment also is of interest. As it can be determined most unequivocally from the hyperfine structures of a one-electron spectrum we have measured the structures of the resonance lines of La III and from the observed separations evaluated the nuclear magnetic moment.

La III[4] was excited by an electrodeless discharge[5] in $LaCl_3$ vapor. The lines were photographed in the fourth, fifth and sixth orders of the department's new twenty-one foot concave grating[6] with exposures as short as ten minutes. The resonance lines 3517A and 3172A ($6s\,^2S-6p\,^2P$) were clearly resolved into four and two components, respectively, with the separations and intensities (visual estimates) listed in Table I. The intervals recorded for each multiplet transition are the averages of measurements made in two orders on several sets of plates. The observed intensity is placed above and the theoretical intensity below each component.

TABLE I. *Hyperfine structure separations and intensities in the resonance lines of La III.*

WAVE-LENGTH	CLASSIFICATION	Hfs SEPARATIONS, cm^{-1}, INCREASING $\nu\rightarrow$			
3517A	$6s\,^2S_{1/2}-6p\,^2P^\circ_{1/2}$	10 0.000 10	6 0.240 7	3 1.093 3	9 1.327 10
3172A	$6s\,^2S_{1/2}-6p\,^2P^\circ_{3/2}$	10 0.000 10	8 1.02 8		

The structure of 3517A, which is completely resolved, can be interpreted uniquely when the theoretical intensity relations[7] are taken into consideration. The analysis gives the doublet splitting of $6s\,^2S_{1/2}$ as normal and equal to 1.09 cm^{-1}, and that of $6p\,^2P_{1/2}$ as normal and equal to 0.237 cm^{-1}. The interval factors calculated on the basis of $I=7/2$ from the above separations are $a_{6s}=0.272$ cm^{-1} and $a_{6p}(1/2)=0.059$ cm^{-1}.

The structure of 3172A is not completely resolved. Six components are predicted whereas two are observed with a separation of 1.02 cm^{-1}, slightly smaller than the splitting of $^2S_{1/2}$ determined from 3517A. This decrease is attributable to a small splitting in the $6p\,^2P_{3/2}$ term.

The magnitude of the $^2P_{3/2}$ splitting was determined by a graphical analysis.[8] A graph of the hyperfine pattern as a function of the ratio $a_{6p}(3/2)/a_{6s}$ was constructed using $I=7/2$ and $a_{6s}=0.272$ cm^{-1}. The components were assigned their theoretical intensities and the centroids (of intensity) of the two groups, each with three components, were then drawn. The position on the graph where the separation between the two centroids equalled 1.02 cm^{-1}, the measured interval, corresponded to $a_{6p}(3/2)=0.010$ cm^{-1}. The total hyperfine separation of $^2P_{3/2}$ derived from this value is 0.12 ± 0.02 cm^{-1}.

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Independent calculations of the nuclear g-factor can be made from the three interval factors a_{6s}, $a_{6p}(1/2)$, $a_{6p}(3/2)$.

For the $6s$ electron of La III $g(I)$ is given by the formula[9]

$$g(I)=\frac{3\cdot a_{6s}\cdot n^3_{\text{eff}}\cdot 1838}{8R\alpha^2 Z_i Z_0^2 K(1/2, Z_i)}, \qquad (1)$$

where $Z_i=57$, $Z_0=3$, $n_{\text{eff}}=2.646$, $K(1/2, Z_i)=1.43$, and $a_{6s}=0.272$ cm^{-1}. These values substituted in Eq. (1) give $g(I)=0.814$. According to Fermi and Segrè[10] the right-hand side of Eq. (1) should be multiplied by a correction factor when the ^2S sequence is non-Rydbergian. Although only two members of the ^2S sequence of La III have been identified, extrapolation from the quantum defects of the corresponding sequences of Cs I and Ba II indicates that the correction factor is approximately equal to $1/1.05$.[4] The inclusion of this factor in Eq. (1) reduces the value of $g(I)$ to 0.78.

In the case of the p electron $g(I)$ is given by the formula[9]

$$g(I)=\frac{a_{6p}(j)\cdot Z_i}{\Delta\nu}\cdot\frac{j(j+1)(l+\frac{1}{2})}{l(l+1)}\cdot\frac{\lambda(l, Z_i)}{K(j, Z_i)}\cdot 1838. \qquad (2)$$

For the $6p\ ^2P_{1/2}$ state $j=1/2, l=1, Z_i=Z-4=53$, $\lambda(l, Z_i)=1.06$, $\Delta\nu=3096$ cm^{-1}, $K(j, Z_i)=1.35$, and $a_{6p}(1/2)=0.059$ cm^{-1}. The substitution of these values in Eq. (2) gives $g(I)=0.82$. For the $6p\ ^2P_{3/2}$ state only three of the above factors have different values, namely: $j=3/2$, $K(j, Z_i)=1.06$, and $a_{6p}(3/2)=0.010$ cm^{-1}. These data substituted in Eq. (2) give $g(I)=0.88$.

The $g(I)$ values 0.78 and 0.82 calculated from the splittings of $^2S_{1/2}$ and $^2P_{1/2}$, respectively, are in good agreement. The former is the more reliable since the experimental error in the $^2S_{1/2}$ separation is considerably smaller than that in the $^2P_{1/2}$ separation. According to our judgment the former merits four times as much weight as the latter. This weighting gives $g(I)=0.79$.

The $g(I)$ derived from the $^2P_{3/2}$ is not sufficiently accurate to merit consideration in estimating the most probable value of $g(I)$. It, however, agrees with the latter well within the limits imposed by the experimental error in $a_{6p}(3/2)$. Thus the hyperfine splittings of the three terms $6s\ ^2S_{1/2}$, $6p\ ^2P_{1/2, 3/2}$, give a consistent value of 2.8 nuclear magnetons for the magnetic moment of the lanthanum nucleus.

The $g(I)$ of La has also been computed by Anderson[3] from the hyperfine structures of $5d^26s\ ^4F$ of La I. His value, 0.72, is in fair agreement with ours, but as his treatment of the three-electron configuration involves a number of approximations in addition to those made in the one-electron problem our value appears to be the more reliable.

It is interesting to compare the $g(I)$ factor of $_{57}La^{139}$ with those of other M odd Z odd nuclei that have the same spin. $_{55}Cs^{133}$ is one of these. As Cs I and La III are members of the same isoelectronic sequence the $g(I)$ values derived from the splittings of their $6s\ ^2S_{1/2}$ states can be compared with some assurance. According to Jackson[11] the splitting of the $6s\ ^2S_{1/2}$ state of Cs I is 0.305 cm^{-1}. The $g(I)$ obtained from this separation by a computation similar to that used for $6s\ ^2S$ of La III is 0.74. This is in good agreement with the values derived from other terms of Cs I.[11, 12, 13] Thus the $g(I)$ factors of Cs and La are nearly equal. Whether or not the small difference is significant is difficult to decide since the formulae used in the calculation of $g(I)$ are only approximate.

$_{51}Sb^{123}$ also is of the M odd Z odd type with $I=7/2$[14] and $g(I)=0.87$. This $g(I)$ value,[6] computed from the interaction constant of a $5s$ electron in a two-electron configuration of Sb IV, is not as accurate as those of Cs and La. Although a comparison cannot be made with great certainty, it appears that the $g(I)$ of $_{51}Sb^{123}$ is not very different from those of La and Cs; and for the present limit of accuracy the possibility of these three nuclei having the same magnetic moment certainly is not precluded.

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Two other M odd Z odd nuclei $_{27}Co^{59}$ [15] and $_{21}Sc^{45}$ [16] have $I=7/2$ and $g(I)$-factors 0.77 and 1.0, respectively. These $g(I)$ values which the respective writers give only as estimates are of the same order of magnitude as those of $_{55}Cs^{133}$, $_{57}La^{139}$ and $_{51}Sb^{123}$.

The apparent regularity in mechanical and magnetic moments pointed out above suggests that the nuclei Cs^{133}, La^{139}, Sb^{123} and possibly Co^{59} and Sc^{45} have some structural feature in common.

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Symmetry Properties and the Identity of Similar Particles

E. E. WITMER AND J. P. VINTI,* *Randal Morgan Laboratory of Physics, University of Pennsylvania*

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It is demonstrated that in a system of N similar particles, the permissible wave functions are either symmetrical or antisymmetrical with respect to interchanges of the particles, if the following principles are assumed to be valid: (1) The interchange of two like particles produces no change in any measurable property of the system. In particular, if ψ is to be a permissible wave function, $\psi\bar{\psi}$ must be unaltered by such an interchange. (2) All the wave functions obtained from a given permissible wave function by permuting the similar particles are also permissible wave functions for the same eigenvalue.

IT has often been remarked that the principle of antisymmetry of the wave function for electrons appears in quantum mechanics as something simply superimposed on the theory to take into account the Pauli exclusion principle. Also in the treatment of photons a corresponding principle of symmetry must be superimposed to obtain the Bose-Einstein statistics, which one must assume photons to obey in order to obtain the Planck radiation law. These and other facts have led to the empirical principle that for any given type of particle only symmetrical states occur or else only antisymmetrical states. It is the purpose of this paper[1] to show that this empirical principle follows logically from other more fundamental principles, *viz.*:

(1) *The interchange of two like particles in a dynamical system will produce no change in any measurable property of the system.*

(2) *All the mathematical quantities obtained from a given permissible mathematical quantity appearing in the theory, by permuting the similar particles, are also permissible.* We have in mind especially the quantity ψ.

These two principles together we shall call the principle of the identity of similar particles. They appear to be necessary from the physical point of view; in other words they are physical axioms. It has often been assumed that wave functions neither symmetrical nor antisymmetrical would distinguish between like particles but explicit proofs are lacking. We shall show that the symmetry or the antisymmetry follows from the principles stated above.

Let there be N particles of the type considered, besides other particles. The coordinates $(x_i, y_i, z_i, \sigma_i)$ including spin σ_i, if present, of the ith particle we shall indicate by x_i. The wave function of the system may be indicated as follows:

$$\psi=\psi(x_1, x_2, \cdots x_N; b). \quad (1)$$

* Now at the Massachusetts Institute of Technology.

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