Radiative Capture of Protons by Carbon

In a previous note[1] we reported that the "potential well" picture of the nucleus and the ordinary theory of dipole radiation give a sufficiently high probability to the $C^{12}+H^1\rightarrow N^{13}+\gamma$ to make it possible to explain the formation of radioactive nitrogen under proton bombardment as due to such radiative capture. In the calculations the captured proton state has an orbital angular momentum $L=0$ and the part of the incident wave responsible for the capture has $L=\hbar$. Some numerical errors in substitutions into Eq. (2) of I gave erroneous information as to the position of the first resonance level for $L=\hbar$. The calculations have since been rechecked and it was found that for the model used in preparing the last table a resonance level lies within the experimental range of voltages.

We were kindly supplied with some unpublished data by Crane[2] and Lauritsen which showed a decidedly steeper rise of the yield with voltage than would be expected from the above-mentioned note. Their results indicate resonance at about 600 kv which fits in with the corrected resonance level. The use of alternating current masks the resonance phenomenon somewhat in these experiments. A very clear and indisputable experimental result has been obtained meanwhile independently by Hafstad and Tuve[3] who find by experiments on thick and on thin targets that there are two resonance peaks for the formation of radioactive nitrogen, at 400 kv and at 480 kv, respectively. These data are in approximate agreement with the recently published curve of Cockroft, Gilbert and Walton,[4] who, however, do not definitely speak of the existence of resonance. Because of resonance the theoretically expected yields are much greater with the type of model used than was previously expected. We made calculations using Eq. (2) of I for three values of the nuclear radius R. In each case the depth of the potential well U was adjusted so as to give resonance at about 560 kv. The binding energy of the proton in the captured state and the yield in thick targets were calculated. For $R\times10^{12}=0.33$, 0.41, 0.52 cm the binding energies are 9.7, 6.0, 3.7 mev and the yields are 4×10^6, 1.6×10^6, 0.6×10^6 captures per microampere per second while $U=21.4$, 13.4, 8.3 mev. The recoil motion of the C^{12} nucleus was taken into account in these calculations. The computations of the yields involved some numerical integration and are not very accurate.

Experiment gives yields which are about 10^{-3} or 10^{-4} of those calculated and it is significant that changes in the nuclear radius in the limits given above change the yield only by a factor of 7.6 while the combined requirement of correct resonance and binding energies fixes the radius between 0.35×10^{-12} and 0.45×10^{-12} cm. We have thus a definite *contradiction between a model with a constant depth and experiment.*[5]

The calculated yield is not sensitive to exact adjustment of the resonance level to the experimental value. Thus for thick targets on the assumption of the 3/2 power law for variation of range with energy one can express the yield per second of bombardment as

$$\left(\frac{M_1Z_2-M_2Z_1}{M_1+M_2}\right)^2\frac{64\pi^5}{137.2}\,\frac{c}{v}\,\frac{N_CN_Pl_0}{(k\lambda)^3k^2}\left|\int\rho\Psi\overline{G}d\rho\right|^2\Big/\left\{\int\left|\overline{G}\right|^2d\rho\cdot\int\Psi^2d\rho\right\},$$

where $\overline{G}/\rho$ is the solution of the radial wave equation for $L=\hbar$ at resonance, N_C is the number of carbon atoms per cm^3, N_P is the number of incident protons per second, l_0 is the range of the protons in carbon at the resonance voltage, Ψ/ρ is the solution of the radial wave equation for $L=0$ and otherwise the notation is as in I. The exponential Gamow factors cancel in this formula because $\overline{G}$ appears to the same power in the numerator and denominator. This expression is not exact theoretically and applies only in cases of sharp resonance for which the upper integration limit of $\int|\overline{G}|^2d\rho$ can be defined without ambiguity as being several times kR. We verified that this expression and Eq. (2) of I give close agreement with results. The penetration through the barrier does not matter much for the final yield, because changes in the width of resonance approximately compensate for changes in $\overline{G}$ at resonance.

The type of change necessary to decrease the theoretically expected yields is that of concentrating the bound proton state inside the N^{13} nucleus so as to decrease its overlapping with $\overline{G}$. This could be accomplished by using a wide and shallow well with an additional deep and narrow sink at very small r. It would seem more likely, however, that the present disagreement between theory and experiment is due to the exclusion principle. A consistent application of the Heisenberg-Majorana or of the Wigner theory of proton neutron interaction will not lead to a simple central field picture.

The half-value breadth is approximately 10 to 20 kv by calculation. The general rise of the Cambridge and Pasadena curves speaks for a larger width while the observations in Washington indicate two peaks each of more nearly the calculated magnitude. The apparent disagreement in the voltage scale of Washington with that of Cambridge and Pasadena is not very significant for the theoretical interpretation of the yield because it does not depend critically on the resonance voltage. There is no real difficulty in finding an explanation for the doublet structure because the vibrations of the α particles with respect to each other and the coupling of their potential valleys can give rise to multiplet structure. Such effects would also give a fine structure of the normal level of N^{13} which may be the cause of the existence of two groups of positrons ejected by N^{13} according to Cockroft's London conference report. More detailed calculations are necessary in this connection.

If a potential well with variable depth is even a roughly correct picture one should expect similar reactions for heavier elements provided the resonance levels fall in attainable voltage regions.

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[1] G. Breit and F. L. Yost, Phys. Rev. **46**, 1110 (L) (1934). Referred to as I in the text.

[2] We are very much indebted to Professor Lauritsen for supplying us with these data.

[3] Hafstad and Tuve. See letters in this issue. We are grateful for being informed of the results of these experiments before publication.

[4] J. D. Cockroft, C. W. Gilbert and E. T. S. Walton, Proc. Roy. Soc. **A148**, 225 (1935).

[5] Cf. Henry Margenau, Phys. Rev. **46**, 613 (1934) for evidence of similar breakdown of such a model in heavy nuclei.

Hyperfine Structure of Y II Lines

The yttrium spectrum has been excited in a water-cooled hollow cathode lamp of the vertical type by using argon as the exciting gas. A strong Y II spectrum results. The lines have been examined for hyperfine structure with a Fabry-Perot etalon inserted between the collimating lens and the prisms of a three-prism Steinheil spectrograph. A separation of 3, 4, 5, 8, 10, 12 and 20 mm has been used between the etalon plates. The three lines listed in Table I have shown components.

TABLE I. *Hyperfine structure in lines of Y II.*

Classification	Int.	λ air A	ν cm^{-1}	Δν cm^{-}
$4d^2\,{}^3F_3-4d5p\,{}^3D_3$	7	4786.755	20885.15	−0.77
	10	4786.580	20885.92	
$4d^2\,{}^3F_3-4d5p\,{}^3D_2$	10	4900.130	20401.93	+0.15
	7	4900.092	20402.08	
$4d^2\,{}^3F_2-4d5p\,{}^3D_1$	5	4854.089	20591.76	−0.38
	10	4854.870	20592.14	

Since no more than two components have been found for any one line a nuclear moment of $i=\frac{1}{2}$ is indicated A more complete study of the structure is in progress, and the detailed results will be published later.

The writers wish to express their thanks to Professor L. F. Audrieth of the University of Illinois, Chemistry Department, for the supply of Y–Hg alloy which enabled this investigation, and to Professor R. C. Gibbs of Cornell University who has kindly loaned us a pair of Hilger etalon plates.

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Ionosphere Studies

The virtual height of the F_2 region of the ionosphere appears to have a definite seasonal variation as shown in the accompanying graph. Over a period of sixteen months, August, 1933 to November, 1934, inclusive, the virtual height, taken near 1200 EST, varies from approximately 250 km in the winter, to 375 km in the summer. The solid curve represents this seasonal effect. The values shown for the individual months are monthly averages of the virtual heights taken over a frequency range from 4500 kilocycles per second up to the particular frequency where the virtual height increases because of the proximity to the critical frequency.

Two possible explanations of such seasonal variation may be offered. (1) In the summer there exists a greater diffusion of the electrons in the F_2 region. If such was the case the wave, having penetrated into the F_2 region, might travel an appreciable distance in the layer before being returned to earth. Because of the greater path traversed at low group velocity, a greater virtual height would be observed. (2) The F layer is known to separate into two regions, F_1 and F_2, during a summer day. The wave, although returned from the F_2 layer, may have its velocity retarded in traversing the F_1 or any ionized region below the F_2 layer before contacting the latter and again upon leaving it. This would at once indicate a higher virtual height.

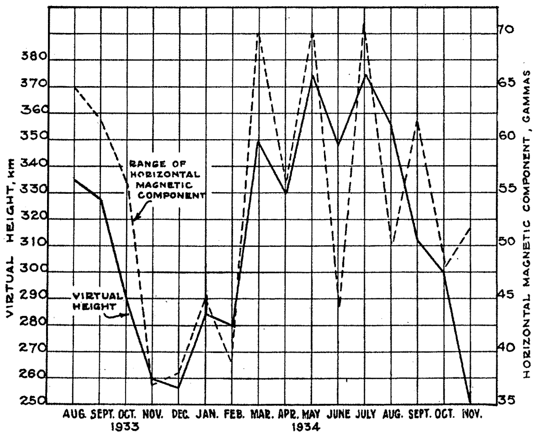

FIG. 1.

It is interesting to note a correlation between the virtual height curve and one for the range of the intensity of the horizontal component of the earth's magnetic field at Cheltenham, for the corresponding period of time. The dashed curve represents an average of the horizontal range for the same days that observations were made on the ionosphere heights.

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