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Frequency and Magnitude of Cosmic-Ray Bursts as a Function of Elevation

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Measurements of the magnitude and frequency of cosmic-ray bursts have been made at elevations of 185, 1620, 3240 and 4300 meters above sea level for periods of time ranging from 189 to 336 hours, using one of the new cosmic-ray intensity meters of the Carnegie Institution. Only bursts releasing more than 10^7 ion pairs are considered. The rather meager data which could be collected in these short times of observation indicate that: (1) Burst-frequency decreases with burst magnitude in a way which can be represented by a series of exponentials. (2) These large bursts contribute only a small fraction of the total ionization. (3) The magnitude of the largest burst observed increases rapidly with altitude, one observed at 4300 meters elevation exceeding 10^9 ion pairs or 3×10^{10} electron volts of energy released in the chamber.

INTRODUCTION

DURING the past summer the first of the new cosmic-ray intensity meters[1] being built at the University of Chicago under the direction of Professor A. H. Compton and under the sponsorship of the Carnegie Institution of Washington, was taken to Colorado for a field test at different altitudes. Observations were made on the University of Denver campus at 1620 meters elevation, at Echo Lake at 3240 meters elevation and at the summit of Mt. Evans at 4300 meters elevation.

The field test of the instrument showed it to be sound in principle and sufficiently rugged to perform perfectly under the most adverse conditions. During the course of the field tests records extending over some eight hundred hours were accumulated, and to these through the cooperation of Drs. E. O. Wollan and R. L. Doan of the University of Chicago, we have been able to add over 250 hours of observations made at Chicago, 185 meters elevation, giving us data at four levels. From these records we have made a study of bursts of ionization (Hoffmann Stösse), investigating their frequency, magnitude and other properties in relation to barometric pressure and mean intensity of ionization.

CRITERIA FOR RECOGNIZING BURSTS

Fig. 1 shows a section of record for the period 10:00 P.M., July 23, to 5:00 A.M., July 24, taken with the instrument operating at the summit of Mt. Evans, elevation 4300 meters. The transverse lines on the record represent intervals of one hour, and the longitudinal lines are millimeters deflection. The smooth white line is a record of barometric pressure, and the jagged black line indicates the position of the electrometer needle and hence the variation of cosmic-ray intensity above and below the mean value to which the compensation mechanism is set. At the end of each hour the electrometer is automatically reset to an arbitrary zero and every four hours its sensitivity is measured, although the sensitivity dots are not easily perceived in this reproduction. Several bursts of ionization are shown in the record of Fig. 1. The one at 10:30 P.M. corresponds to more than 765×10^6 ions and sent the indicator off scale when the sensitivity and zero were set at optimum values for average observing. Those of the cluster between 2:00 A.M. and 5:00 A.M. range in magnitude from 46×10^6 to 102×10^6 ions.

The first step in a study of the recorded bursts is to set up criteria for distinguishing between true bursts and statistical variations which may have the appearance of bursts. The traces of the bursts possess several general characteristics which help to distinguish them, such as direction of deflection of the indicator, initial and average steepness of the deflection, duration and magnitude of the deflection.

After selecting the bursts on this basis, an estimate was made (see Appendix) of the probable number of statistical variations which had been included because they were indistinguishable from bursts. To do this, the time necessary to collect the ions from a burst was calculated and the calculation checked by measurements from the record. Next, the average number of rays traversing the chamber during

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[1] Compton, Wollan and Bennett, Rev. Sci. Inst. 5, 415 (1934).

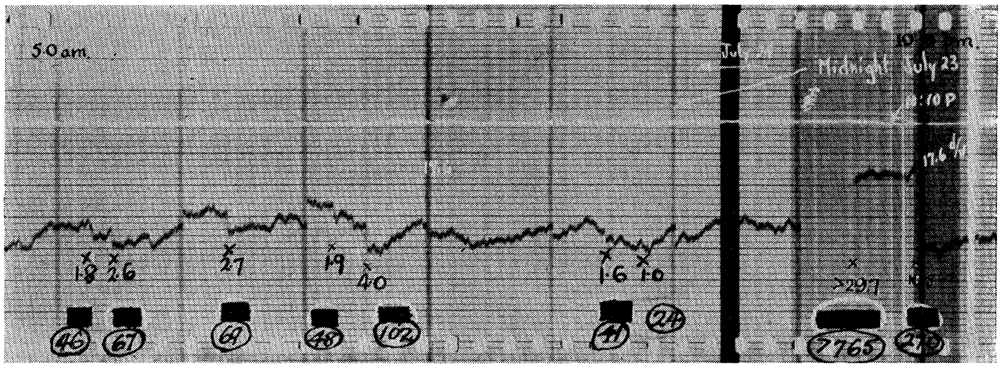

Fig. 1. Section of record for the period 10 p.m. July 23 to 5 a.m. July 24. Instrument operating at summit of Mt. Evans.

the time of collection of the ions was calculated from measurements of average ionization, assuming 135 ions per cm in air at atmospheric pressure (see Appendix). On the assumption that these rays come singly and are randomly distributed in time we have calculated the expected frequency of statistical fluctuations of magnitudes as large as the smallest of those we have included in our tabulations. In all cases the expected number of such fluctuations is negligibly small compared to the number of bursts observed.

However, a fraction of the rays will occur in groups as a result of their originating in bursts either of small numbers of tracks or so placed that only small numbers enter the chamber. Observations with cloud chambers indicate that the fraction of the rays occurring in groups is small near sea level. At other elevations the magnitude of this fraction is unknown. Hence it is not possible at present to calculate the effect of grouping on statistical fluctuations. This effect introduces some uncertainty into our values for numbers of bursts in the smaller magnitude groupings though it seems doubtful that this uncertainty is larger than that due to the small number of observations.

Burst-Frequency as a Function of Magnitude

In Fig. 2 we have grouped and plotted all the bursts observed at each of the four elevations. The minimum magnitude included at the three upper levels was 20×10^6 ions, and at the lowest level, 10×10^6 ions. The group magnitude chosen for the two upper levels was 20×10^6 ions, and for the two lower levels, 10×10^6 ions. Any other reasonable grouping leads to a similar result.

We next resorted to a smoothing process which enabled us to pass smooth curves through the block curves resulting from the grouping. This process is illustrated in detail in Fig. 3, with the data taken at the summit of Mt. Evans. The block curve I is integrated by addition to give the points of curve II indicated by the crosses.

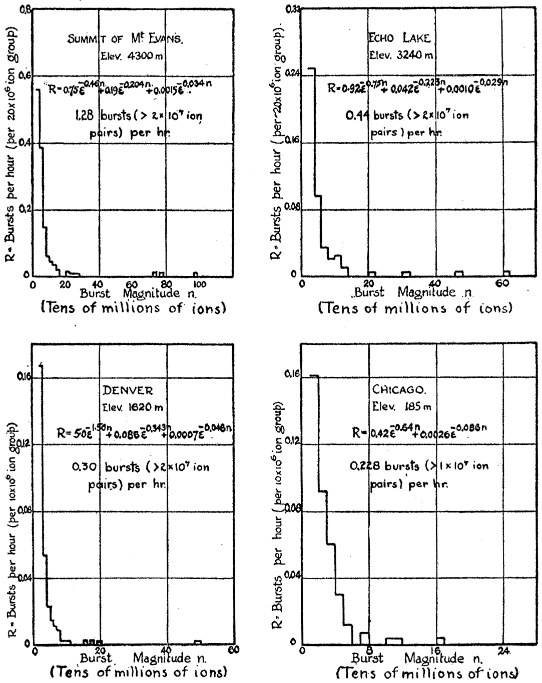

Fig. 2. Burst-frequency as a function of burst magnitude at four elevations.

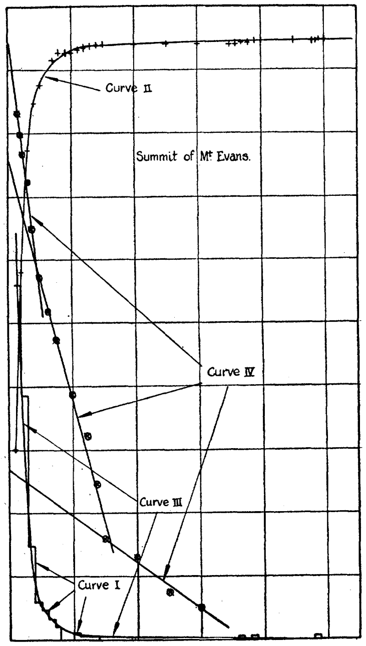

FIG. 3. Method of analysis of burst-frequency curves.

Curve II is then drawn through the crosses, and differentiated graphically to give curve III.

It was desired to obtain a mathematical expression for curve III. To do this we plotted the logarithms of the ordinates of curve III at the corresponding abscissae and obtained the points indicated as crossed circles. Three straight lines appeared to pass through these points very well, indicating that curve III might be reasonably well represented by three exponentials. We also found this to be the case at the two intermediate levels. At the lowest level the data, taken indoors, indicate only two exponentials.

From this plotting process we obtained the equations of the form

$$R=R_1e^{-a_1n}+R_2e^{-a_2n}+R_3e^{-a_3n} \tag{1}$$

shown in Fig. 2 for smooth curves representing the block curves. The values of coefficients and exponents were obtained graphically by determining the coefficient and exponent of the lowest curve directly, subtracting the values obtained from this coefficient and exponent from those of curve III, replotting and similarly obtaining the slope and intercept of the next lowest curve, etc. The equations were checked by calculation of representative points, and as a whole by integration, the integral of R with respect to n leading to the total rate observed when bursts of all sizes between the smallest recorded in the block curve and infinity are included.

TABLE I. *Constants of Eq. (1) calculated from curves.*

ELEVATION	COEFFICIENTS R_1	R_2	R_3	EXPONENTS a_1	a_2	a_3	RATIOS OF EXPONENTS a_1/a_2	a_2/a_3	a_1/a_3
4300 m	0.75	0.19	0.0015	0.46	0.204	0.034	2.25	6.0	13.5
3240 m	0.92	0.042	0.0010	0.75	0.223	0.029	3.35	7.7	25.8
1620 m	5.0	0.086	0.0007	1.5	0.343	0.048	4.37	7.1	31.2
185 m	—	0.42	0.0026	—	0.64	0.086	—	7.4	—

The coefficients and exponents obtained by the above process are given in Table I. These have been reduced in such a way that all apply to a common group magnitude of 10×10^6 ions.

On account of the relatively small number of data at our disposal it is difficult to say whether the trends indicated in the table are significant. The exponents a_1, a_2, a_3 are probably reliable to within approximately 20 percent, but we do not consider the coefficients R_1, R_2, R_3, good to within better than 100 percent. Obviously more extensive data are necessary before laws of variation of burst-frequency with burst-magnitude can be definitely established. The effect of the slate, steel and concrete roof over the apparatus at Chicago seems to have been to eliminate the group of smallest bursts.

In order to investigate the possibility that all cosmic-ray ionization phenomena might occur as a result of bursts, we have calculated the total rate of ionization contributed by bursts of all sizes from those corresponding to but one particle passing through the chamber up to bursts corresponding to an infinite number of particles, on the assumption that all burst phenomena can be represented by the three exponentials obtained above. The result is given in Table II.

The numbers of column 4 indicate that bursts in these classifications account for only an extremely small fraction of the total ionization. However, our data actually deal only with bursts of about 50 or more tracks at the lowest elevation, and about 100 or more tracks at the other elevations. It appears, therefore, that the frequency distribution of the very small bursts (showers) must be quite different. In connection with this point the data on showers given by Evans and Neher[2] were plotted in a manner

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[2] Evans and Neher, Phys. Rev. **45**, 144 (1934).

TABLE II. *Calculations which show that bursts in these classifications contribute but a small fraction of the total ionization.*

ELEVATION	NO. OF IONS PER SEC. CONTRIBUTED BY BURSTS OF ALL MAGNITUDES	OBSERVED TOTAL NUMBER OF IONS PER SECOND	PERCENT OF TOTAL IONS CONTRIBUTED BY BURSTS
4300 m	0.0261×10^6	5.18×10^6	0.504
3240 m	0.0102×10^6	3.62×10^6	0.282
1620 m	0.0088×10^6	2.50×10^6	0.352
185 m	0.0036×10^6	1.55×10^6	0.245

similar to our burst data, as shown in Fig. 4. A mathematical expression representing this curve is $F=2.2e^{-2.16\tau}+0.063e^{-0.63\tau}$. In this case we have used the number of tracks τ as abscissae and the fraction of the total observed number of showers F as ordinate, the scale at the left applying to F and that at the right to $\log_{10} F$. There result again two exponentials, though these are not directly comparable with our derived exponentials because of the different scales used. The two sets of data can, however, be compared by calculating from our data the expected relative numbers of single-track, double-track, triple-track, etc., showers, using the data at the 185-meter elevation as being most nearly comparable to that obtained in cloud chambers. The ratios are given in Table III.

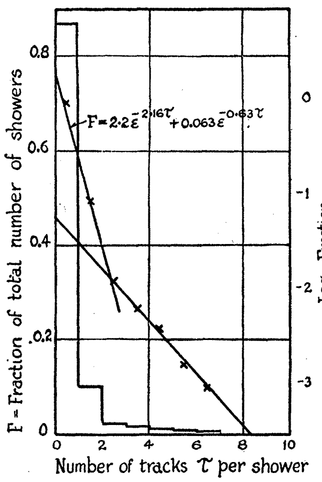

FIG. 4. Plot of data on showers given by Evans and Neher.[2]

TABLE III.

RATIOS	CLOUD-CHAMBER DATA	EXTRAPOLATED LARGE BURST DATA
Double to single	0.113	0.99
Triple to single	0.015	0.97
Quadruple to single	0.008	0.96

The table indicates that the variation with magnitude for very small bursts (showers) of a few tracks must be very much different from that of the large bursts we have observed. These analyses seem to indicate that the relation between burst-frequency and burst magnitude can be represented by a series of many exponentials.

RELATIVE VALUES OF VARIOUS QUANTITIES

In Fig. 5 we have plotted a number of quantities as functions of barometric pressure. In order to see more easily the relative rates of increase of these quantities we have plotted the ratio of the magnitude at each elevation to the magnitude found at Denver. The values at Denver were taken as basic rather than those at Chicago, because at Chicago the instrument was operated under a roof, whereas at each of the other locations it was in a tent. The resulting curves indicate the following:

(1) The values of total ionization taken with the new instrument at the three upper levels plus a fourth taken especially for us outdoors at Chicago by Dr. R. L. Doan, give an absorption curve of the usual shape. The values found for ionization (per cm^3 per second reduced[1] to air at 1 atmosphere) fall below those found for this latitude with the 1932 apparatus by about the amount to be expected with the increased shielding on the new machine.

(2) The total number of bursts increases at about the same rate as the total ionization up to an elevation of about 3240 meters, after which it rises much more rapidly.*

(3) The number of very large bursts, specifically bursts releasing more than 100×10^6 ion pairs within the chamber, increases much more rapidly with altitude than does the total ionization. Further, the rate of increase of this quantity also increases rapidly with elevation.

(4) The magnitude of the largest burst observed at each elevation increases at least as

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* Considering only bursts greater than 20×10^6 ions.

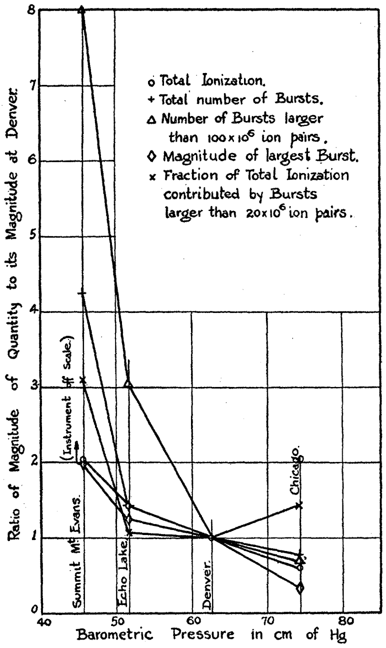

FIG. 5. Plot of various quantities associated with cosmic-ray bursts as a function of barometric pressure.

fast as the total ionization. (At the summit the very large bursts threw the indicator off scale.)

(5) The fraction of the total ionization contributed by bursts greater than 20×10^6 ion pairs is not constant. This fraction is less at Denver than at Chicago, the higher value at Chicago perhaps being due to bursts originating in the roof over the instrument. This fraction increases very rapidly again above 3240 meters.

Three bursts at the 4300-meter level were of sufficient magnitude to exceed the range of the scale of the instrument at the sensitivity used. The records indicate that these bursts released 10^9 or more ion pairs. Using the value of 135 ion pairs per cm in air, some 5000 tracks would be required to produce so many ions. With 30 electron volts expended for each ion pair the energy becomes of the order of 3×10^{10} electron volts. Since the absorbing power of the argon in our chamber was equivalent to only about 2 mm of lead, whereas the shielding amounted to some 120 mm, it seems reasonable to expect that the total energy released approached 10^{12} electron volts, in agreement with the findings of others.

CONCLUSIONS

For the type of apparatus used and for bursts of magnitude greater than 20×10^6 ion pairs:

(1) The burst-frequency decreases with magnitude of burst in a way which can be represented by a series of exponentials.

(2) The total burst-frequency increases with elevation, at approximately the same rate as the total ionization up to 3000 meters, but considerably more rapidly for higher elevations.

(3) The frequency of very large bursts increases much more rapidly with elevation than the total ionization.

(4) The magnitude of the largest burst likely to be observed increases with elevation more rapidly than the total ionization.

(5) The observed bursts contribute a very small fraction of the total ionization. However, the possibility that all the ionization comes from bursts is not excluded, since the contribution of the smaller but much more frequent bursts is not included.

(6) The number of ions released in a burst is a function of the size of the chamber, those observed in the 1932 apparatus of 1/2-liter capacity amounting to a few tens of millions of ion pairs; those reported by Millikan in a chamber of 1.6 liters capacity amounting to the order of one hundred million ion pairs; those in the new instrument of volume 19.3 liters amounting to about one thousand million ion pairs. This would indicate that the physical dimensions of the instruments have not yet exceeded the physical dimensions of the bursts.

(7) The very large bursts may result either (a) from larger numbers of tracks than the number of constituent entities yet observed for any single atom, indicating the cooperation of a number of atoms; (b) from particles of greater ionizing power than those most commonly recognized in a cloud-chamber work; (c) possibly from materialization of the energy of photons into particles.

ACKNOWLEDGMENTS

It would not have been possible to carry out this work without the cooperation of many people. The Carnegie Institution of Washington supplied the instrument and its transportation. To Professor A. H. Compton we are indebted for advice and cooperation both in the experimental work and in the analysis of the results. Professor J. C. Stearns, of the University of Denver, extended to us the freedom of his laboratory and gave us most valuable advice and help. Mr. Walter Ailinger and the Mountain Parks Division of the City of Denver supplied us with men and trucks for the transportation of the heavy instrument to the mountain summit. Drs. Doan and Wollan, of the University of Chicago, have supplied us with instrument data and constants and have made a series of records for us at Chicago.

APPENDIX

ON DISTINGUISHING BETWEEN BURSTS AND STATISTICAL VARIATIONS

Supposedly the bursts of ionization indicated on the records are produced by a large number of ionizing rays traversing the chamber in a time which is small compared with any natural period inherent in the instrument. Such natural periods in this instrument result from the period of the electrometer itself and from the time necessary for the ions to migrate to the electrodes under the influence of the collecting field. The electrometer period at the sensitivities used is of the order of one second, and negligibly small compared with the time necessary to collect the ions, as is demonstrated below. Hence, where the background of single rays (or multiples involving small numbers of rays) is large, as in this case, a sudden statistical variation in the rate of production of ions may resemble a burst when such a variation takes place in a time which is comparable with the time necessary to collect ions. This time may be calculated approximately, as follows.

The longest time involved will result when ions are produced near the inner surface of the outer sphere and have to be drawn all the way to the inner collector. It is true that as soon as the ions begin to separate, their influence begins to be felt by electrostatic induction on the inner collector, but the full change in voltage is not felt until they actually arrive at the collector. Hence this calculation gives the maximum possible time to realize the full effect of the charges. For the sake of simplicity the chamber and collector may be assumed to be concentric spheres of radii r_1 and r_2, respectively, without seriously impairing the accuracy of the result.

The field strength at any radius r is

$$E_r = Vr_1r_2/(r_2-r_1)r^2,$$

where V = total voltage between spheres; r_1 = radius of collector sphere; r_2 = radius of external sphere. Let μ = mobility of ions in cm/sec./volt/cm at 1 atmos. pressure; v_r = velocity in cm per sec. of ions at radius r; p = pressure in atmospheres.

Then

$$v_r = \mu E_r/p = dr/dt, \qquad dt = dr/v_r = p\,dr/\mu E_r;$$

or

$$t = \int_{r=r_1}^{r=r_2} \frac{p\,dr}{\mu E_r}, \qquad t = \frac{p(r_2-r_1)}{V\mu r_1 r_2}\int_{r_1}^{r_2} r^2 dr,$$

$$t = \frac{p(r_2-r_1)}{3\,Vr_1r_2\mu}\,(r_2{}^3 - r_1{}^3) \text{ seconds.}$$

In this case $\mu = 1.37$ (*Smithsonian Tables*); $V = 250$ v; $p = 50$ atmos.; $r_2 = 17.8$ cm; $r_1 = 5.7$ cm (mean of max. and min. diameters of collector). Thus $t = 32$ seconds.

This sets an upper limit for the time necessary to collect the ions from any burst, provided of course the instrument has been in operation long enough to clear away the very large and slow ions found initially in such a chamber. In no case would it be possible to realize as long a period as this, since it would not be possible to produce a large amount of ionization right at the outside wall by means of many particles traveling in straight lines.

Records were taken for two days at Echo Lake at four times normal speed. It was possible to get a fairly accurate (5 percent) measure of the time constant of the apparatus from the large bursts found on these records, and a less accurate measure from large bursts on other records made at normal speed. These experimental values range between 14 and 23 seconds, which is well within the calculated upper limit of 32 seconds.

By knowing this period it is possible to calculate the expected frequency of statistical variations resembling bursts which might be mistaken for them in reading the records. This may be done as follows:

Let n = number of ion pairs per cm path in air at 1 atmos.; f = factor between argon at 50 atmos. and air at 1 atmos. ($f = 67.0$)[1]; nf = number of ion pairs per cm in argon at 50 atmos.; s = mean length of path of particle traversing the chamber, assuming that every particle passes completely across the chamber.

$$s = 4r_2/3 \text{ or } 23.7 \text{ cm.}$$

In calculating the average number of ions per particle, there are a number of possible choices for n ranging from 31 to 135.[2] To get the largest expected frequency of statistical bursts, one should choose the largest of these values, namely, 135. Then $nfs = 214{,}200$ ions per particle.

Let N = total number of ions produced per second on the average. Then N/nfs = average number of particles per second. In time Δt we get $x = N\Delta t/nfs$ particles and $N\Delta t$ total ions.

The probability P that in the interval of time Δt there will be collected not $N\Delta t$ ions but $(N\Delta t + B)$ ions (where B is the number of ions in an apparent burst) or that there will be not $x = N\Delta t/nfs$ particles but $(x+y) = (N\Delta t + B)/nfs$ particles is by Poisson's law[2]

$$P = x^{x+y}e^{-x}/(x+y)!.$$

By Sterling's approximation:

$$(x+y)! = [2\pi(x+y)]^{\frac{1}{2}}(x+y)^{(x+y)}e^{-(x+y)}.$$

Hence

$$P = \frac{x^{x+y}e^{-x}}{[2\pi(x+y)]^{\frac{1}{2}}(x+y)^{x+y}e^{-(x+y)}} = \frac{1}{[2\pi(x+y)]^{\frac{1}{2}}}\left(\frac{x}{x+y}\right)^{x+y} e^{y},$$

and

$$\log P = y \log e + (x+y)[\log x - \log (x+y)] - \tfrac{1}{2}\log 2\pi - \tfrac{1}{2}\log (x+y).$$

The case where a statistical variation is most likely to be mistaken for a burst occurs when reading from the records the smallest bursts where the background due to single particles and small showers is largest. The background is largest at the summit of Mt. Evans, and the smallest burst which can be conveniently read from the records is one of about ten million ions. The existence of a statistical variation of a given magnitude becomes more probable the longer the instrumental period chosen. Hence we shall take the longest period found experimentally, namely, 23 seconds. A burst of 10^7 ions is equivalent to

46.6 particles on the above assumptions, and the background of 5.18×10^6 ions per second at the summit is equivalent to 24.1 particles per second or 555 particles in a 23-second period. Then

$$x=555, \quad y=46.6, \quad x+y=601.6,$$

and

$$P=1/430.$$

Hence we should get one statistical burst of 10^7 ions in 430 periods of 23 seconds each, or about one every 2.75 hours, which is a rate comparable to that observed. However, if we consider the prevalence of statistical twenty million ion bursts we find we should expect one about every 730 hours, which would amount to about 0.25 percent of the total observed. On this basis we felt safe in including all bursts greater than 2×10^7 ions at the three higher elevations. At Chicago the expected frequency of a statistical 10^7 ion burst is found to be one every 42.2 hours and for a 1.5×10^7 ion burst, one every 18,000 hours. Actually we observed 6.8 bursts between 10^7 and 2×10^7 ions per 42.2-hour period at this elevation, so only bursts less than 10^7 ions were omitted at this elevation.

There is, of course, the possibility that these large fluctuations may result from instrumental accidents. The possibility of incipient arcs, as suggested by Millikan and Neher,[3] is definitely ruled out by the construction of the instrument, as has been pointed out.[4] Furthermore, bursts occur when the total voltage on the chamber is reduced to less than the ionizing potential of argon. Battery fluctuations have no effect on the recording system because of the electrostatic balance arrangement.[1] We are certain that the recording system is amply shielded against electrostatic disturbances, since frequent nearby lightning strokes, as well as corona streaming from tent poles, nearby rocks, etc., at noted times, produced no disturbance of the recording mechanism. Hence we feel justified in retaining the full numbers of bursts as read from the records and plotted in our curves.

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[3] Millikan, Anderson and Neher, Phys. Rev. **45**, 141 (1934).

[4] R. D. Bennett, Phys. Rev. **45**, 491 (1934).

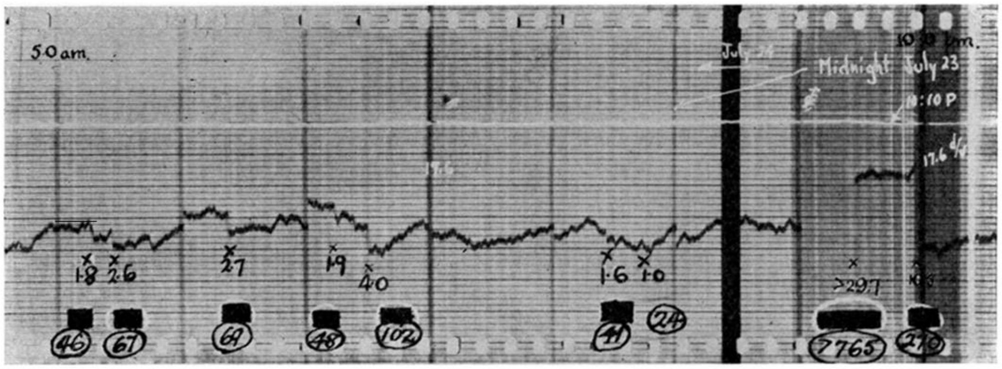

FIG. 1. Section of record for the period 10 P.M. July 23 to 5 A.M. July 24. Instrument operating at summit of Mt. Evans.