The Fine Structure of Hα

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The fine structure of Hα was investigated using Fabry and Perot interference patterns produced in the light of ordinary and heavy hydrogen. The auxiliary spectroscopic dispersion was sufficient to separate the patterns of $H^1\alpha$ and $H^2\alpha$. Accurate photometric methods were used in interpreting the spectrograms and a new method of analyzing the observed patterns was developed. Following are the principal results: (1) Three components of each line are directly observable, while the two remaining components predicted by theory could not be resolved. (2) The observational evidence indicates definitely the same relative intensities of the components for $H^1\alpha$ and $H^2\alpha$. (3) The relative intensities insofar as they are determined depart markedly from the theoretical values. These departures are consistent with the hypothesis of a diminished population of atoms in the 3*D* energy states. (4) The fine structure constant was determined yielding $1/\alpha=137.4\pm0.2$. There is some evidence of systematic dependence of this result on conditions of excitation in the source so that the probable error overestimates the accuracy of the result. The correct value probably lies between 137 and 138.

INTRODUCTION

IN a note published in the Physical Review[1] of July 1, 1933, we reported preliminary results on the fine structure separation of the Hα doublet derived from a study of the lines of ordinary and of heavy hydrogen. After eliminating as far as possible the effects of the fainter components, the separation was found to be 0.324 cm^{-1}, leading to a value for the reciprocal of the fine structure constant $1/\alpha=138$. The spectrograms on which this report was based were secured with rather low pressure in the discharge tube, and it was felt that there might be some dependence of the results on this condition. Accordingly for a more complete study, spectrograms were taken with varying conditions of pressure. An improved optical arrangement was used to give more extended interference fringe systems, and finally the method of analyzing the photometric curves was considerably refined. The present paper reports the final results of this investigation.

Since the publication of our preliminary results, important papers on the subject by Houston and Hsieh[2] and by Williams and Gibbs[3] have appeared. From an analysis of the lines Hβ to Hϵ in ordinary hydrogen Houston and Hsieh find a value of $1/\alpha=139.9$. They conclude that the theory of fine structure requires correction to reconcile their observations with the latest value given by Birge,[4] namely, 137.4. Williams and Gibbs, from a study of $H^1\alpha$ and $H^2\alpha$, find doublet separations corresponding to values of $1/\alpha=141.7$ and 138.8, respectively. The results derived in the present paper are in close accord with Birge's latest value. We feel that the difference between our results and those of the observers mentioned above may be largely explained on the basis of the more detailed method of analyzing the observations which we used.

DESCRIPTION OF APPARATUS

The source of illumination was a modified form of Wood's tube with a length of 150 cm between electrodes and an internal diameter of 1 cm. The entire length of the tube including the electrodes was immersed in liquid air. The light to be photographed was taken longitudinally from the central 50 cm of the tube, and emerged through an extension projecting out of the liquid air. The source of hydrogen consisted of a few drops of water containing approximately equal parts of H^1 and H^2 furnished through the kindness of Professor G. N. Lewis. This water was contained in a small side tube which could be opened into the discharge tube by means of a

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[1] Frank H. Spedding, C. D. Shane and Norman S. Grace, Phys. Rev. **44**, 58 (1933).
[2] W. V. Houston and Y. M. Hsieh, Phys. Rev. **45**, 263 (1934).
[3] R. C. Williams and R. C. Gibbs, Phys. Rev. **45**, 475 (1934).
[4] Raymond T. Birge, Science **79**, 438 (1934).

stopcock. In preparing the discharge tube for operation it was pumped with the discharge running for about a day. Toward the end it was flushed out several times by admitting small amounts of the water vapor. Just before starting a run water vapor was admitted, the tube was immersed in liquid air, and the discharge was started. It would run for about one second and then stop due to absorption of the hydrogen by the cold aluminum electrodes. The tube was then removed and the discharge turned on while the tube warmed up. Another charge of water vapor was admitted and the process was repeated. At each stage the discharge at liquid air temperature would persist several times longer than at the preceding stage. After repeating the process a suitable number of times the discharge would continue for several hours.

In all cases it was possible to regulate the relative intensities of $H^1\alpha$ and $H^2\alpha$ by admitting small amounts of the nearly pure gases[5] from bulbs sealed on the apparatus for the purpose. On the other hand, considerable difficulty was encountered in maintaining the pressure at a value sufficient to give the two strong components of the lines with the theoretical relative intensities. When a fresh charge of gas was admitted to the tube during its operation the strong components would have nearly theoretical relative intensities but this persisted for only a few seconds. Probably this condition could have been maintained if a continuous flow of gas through the tube had been provided, but this was impossible owing to the small amount of heavy hydrogen available. Following Houston's procedure the last three plates were taken with a certain amount of helium in the tube in order to maintain sufficient pressure. This gave more nearly correct relative intensities for the strong components.

Attempts were made to increase the sharpness of the lines by reducing the current and by an intermittent discharge on $\frac{1}{4}$ of a second and off $1\frac{3}{4}$ seconds. This resulted in no improvement. Finally the tube was run while immersed in liquid hydrogen. This was unsatisfactory because it was very difficult to maintain the discharge, and no increased sharpness in the lines resulted.

The tube was operated from a 15,000-volt transformer. At various times the current was measured and found to range between 25 and 30 milliamperes with very little dependence on pressure. The difference in potential between the electrodes was, of course, much smaller than 15,000 volts except at such times as the discharge was on the point of ceasing due to low pressure.

An image of the source was focused between the plates of a Fabry and Perot etalon by a system of lenses arranged to give a fairly uniform illumination over a spectrograph slit length of $1\frac{1}{2}$ inches. The reflecting power of the silvered surfaces of the plates was 0.69. A lens of 32 inches focal length formed an image of the fringe system given by the interferometer on the slit of the spectrograph. This spectrograph is of the Littrow type possessing a plane grating of 6 inches aperture and a lens of 30 feet focal length. The first order was used and the dispersion was such that $H^1\alpha$ and $H^2\alpha$ were separated on the photographs by 1 mm. Thus a slit width of practically this amount could be used without overlapping of the two lines. The photographs with this apparatus showed the two lines $H^1\alpha$ and $H^2\alpha$ as sections taken across the centers of their interference ring systems. The spacing of the interferometer plates, 0.782 cm, was chosen to give for each line the fringes of one of the two main components almost exactly midway between pairs of adjoining fringes of the other main component.

Intensities of the Components

Six plates were selected for measurement and discussion. Of these, Nos. 31, 33 and 34 were taken with no helium in the discharge tube while for Nos. 40, 41 and 42 helium was introduced with a resulting higher total pressure. Determinations of the temperature of the source for each plate were afforded by the quantities b, referred to in the preceding paper, which measure the sharpness of the main components of the Hα line. If it be assumed that all widening is due to temperature, a simple calculation yields a result which may be taken as an upper limit to the temperature. The results ranged between 160°

[5] We are greatly indebted to Professor G. N. Lewis who prepared some highly concentrated H^2 gas for this purpose.

TABLE I. *Master curves.*

Decimal of order of interference	31		33		34		40		41		42	
	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$
0.725		20.7	22.0		30.0			23.5		20.8		
.750	20.4	22.2	20.8	21.8	30.6	17.4	34.1	23.4	31.8	21.2	33.0	23.0
.775	20.0	24.0	20.6	23.3	32.0	18.0	31.6	24.4	32.4	22.6	33.2	23.7
.800	20.2	25.8	21.0	25.1	33.9	19.1	35.8	26.1	33.7	24.5	33.9	25.2
.825	21.6	27.4	22.5	26.6	35.8	20.8	37.1	28.7	35.4	27.3	35.2	27.6
.850	23.3	29.1	24.5	28.4	37.8	23.4	38.9	32.1	37.4	30.7	36.9	31.0
.875	25.8	31.3	27.4	31.3	40.2	26.6	40.9	35.5	39.6	34.9	39.1	35.2
.900	28.8	34.9	31.2	35.1	42.5	30.1	43.3	40.2	41.7	39.7	41.4	39.5
.925	32.1	39.8	35.3	40.3	45.1	33.6	45.3	44.2	44.0	44.3	43.4	43.7
.950	35.2	44.8	39.5	44.9	47.3	36.5	46.8	47.3	46.0	47.9	45.2	47.1
.975	37.6	48.8	42.4	48.2	49.0	38.7	48.1	49.3	47.3	50.4	46.5	49.2
.000	38.9	51.2	44.2	49.9	50.0	39.8	48.9	50.0	48.1	51.1	47.2	50.0
.025	39.6	49.7	44.6	49.0	49.8	39.1	48.8	48.7	47.5	49.8	47.0	49.1
.050	39.0	45.2	43.7	45.4	48.2	37.1	47.6	46.0	46.0	46.8	46.0	46.4
.075	36.8	39.2	41.6	39.8	46.0	34.3	45.7	42.7	43.9	42.5	44.5	43.0
.100	34.0	32.8	38.6	33.1	43.2	31.0	43.4	39.1	40.9	37.8	42.2	39.1
.125	31.1	27.2	34.8	26.8	39.8	27.5	40.9	35.6	37.8	33.5	39.3	35.3
.150	28.7	22.8	31.2	22.0	35.8	24.6	37.7	32.6	34.8	29.8	36.3	32.2
.175	26.8	19.4	28.4	19.0	32.1	23.3	34.7	30.5	31.9	27.5	33.7	29.6
.200	25.6	17.4	26.4	17.1	28.4	23.5	32.1	29.6	29.8	26.3	31.5	28.5
.225	24.8	16.3	25.5	16.0	25.6	25.4	30.2	29.8	28.6	26.3	29.9	28.2
.250	25.1	16.0	25.4	15.8	23.8	27.8	29.1	30.7	28.1	27.2	29.1	28.7
.275	26.2	16.4	25.8	16.2	23.2	30.2	29.4	31.8	28.5	28.4	29.3	29.7
.300	27.5	17.4	26.8	16.8	23.8	32.0	30.7	33.1	29.9	30.1	30.4	31.0
.325	29.3	19.1	28.6	18.4	25.6	33.6	32.7	34.7	32.0	31.8	32.4	32.5
.350	31.5	21.3	31.1	20.5	27.9	35.4	35.0	36.6	34.8	33.8	34.8	34.4
.375	33.9	24.7	34.4	24.0	30.4	37.2	38.0	39.0	37.9	36.2	37.8	36.8
.400	36.8	28.4	37.8	28.9	33.2	39.8	41.3	41.8	41.2	39.0	40.0	39.5
.425	40.3	32.7	41.4	35.1	35.8	43.1	44.3	44.4	44.2	41.1	43.8	42.0
.450	43.3	36.8	45.5	41.0	37.8	46.3	46.5	46.7	46.9	43.8	46.2	44.4
.475	45.8	39.8	48.4	45.5	39.2	48.7	48.1	48.8	48.9	45.9	47.9	46.2
.500	47.5	41.3	50.0	48.1	40.0	49.6	49.2	49.0	49.8	46.2	49.0	46.3
.525	47.6	40.1	50.0	46.4	40.4	48.5	49.4	47.8	49.8	44.8	49.1	45.2
.550	45.9	36.4	48.4	43.4	39.9	45.6	48.8	45.2	48.7	42.3	48.1	43.1
.575	43.0	32.6	45.4	38.0	38.9	41.0	47.3	41.6	46.7	38.7	46.6	40.3
.600	39.0	28.5	41.2	31.9	37.2	35.6	45.3	37.8	44.0	34.7	44.8	37.1
.625	34.5	24.8	36.2	26.3	35.4	30.5	42.9	34.0	41.3	30.5	42.6	33.6
.650	30.8	22.1	31.0	22.5	33.2	25.6	40.6	31.0	38.6	26.8	40.4	30.0
.675	27.2	20.8	27.0	20.6	31.5	21.7	38.7	28.0	36.1	24.1	37.8	27.2
.700	24.4	20.4	24.0	20.3	30.6	19.4	36.9	26.0	34.4	22.5	35.6	25.4
.725	22.5	21.2	22.0	20.5	30.4	18.1	35.8	25.4	33.2	21.7	34.4	24.3
.750	21.2			21.8		17.8	35.1		32.5		33.8	24.1

and 322°, the low pressure plates yielding the lower temperatures. This is what might be expected from the superior conductivity of gas at the lowest pressure used.

The preceding paper describes the photometry and reductions of these plates up to the point of finding the master curves. Table I gives the coordinates of points on the master curves used in this investigation. The method of removing from the master curves the effects of the three faint components for any assigned intensities has also been explained. It remains here to discuss in some detail the assignment of the intensities to be subtracted.

Fig. 1[6] shows a typical master curve for $H^2\alpha$. If the three faint components did not exist, we should expect the fringes of each of the two strong components to possess symmetry about the vertical line passing through their maxima, while the two minima should have equal ordinates. The imposition of these three conditions on the residual curve formed by subtracting the three faint components should provide a means of finding the intensities to be subtracted. In practice it was found that the conditions were insensitive to components four and five in the sense that all three requirements could be

[6] Dr. Harvey E. White kindly furnished this figure.

satisfied within observational error by assigning any reasonable intensities to these components and calculating the intensity of component three by least squares. It was also found that for this purpose the conditions of symmetry for component one had negligible weight compared with the other conditions. These considerations led to the following method of treatment.

Data derived from the master curve were used to calculate a typical fringe as explained in the preceding paper. The scale of ordinates was chosen to give an intensity 1/7.08 times the intensity of the second component. This latter intensity was derived by freeing the master curve by estimation from the effect of the third component which was the only important one for the purpose. Theoretically the five components have intensities 9.00, 7.08, 1.13, 1.00 and 0.20. Thus the typical fringe had an intensity 1.00 based on the theoretical value for the second component. The ordinates of the typical fringe were multiplied by 1.69 thus giving 1.5 times the third component, the abscissae were displaced so as to bring the maximum to the theoretical position of this component and the resulting curve was subtracted from the master curve. This gave the first residual curve. 2.5 times the third component was then subtracted from the master curve, giving the second residual curve. One times the fourth component subtracted from the first residual curve gave the third residual curve, and two times the fifth component subtracted from the third residual curve gave the fourth residual curve.

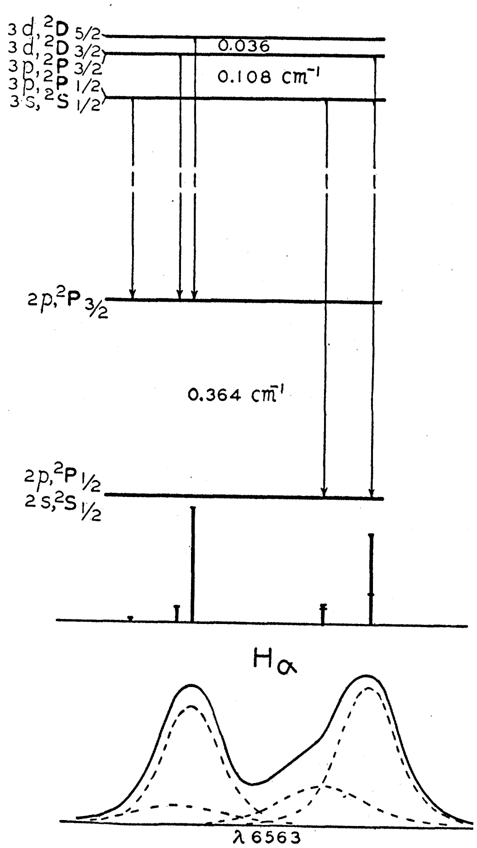

FIG. 1. Fine structure of Hα.

Data were now taken from each residual curve as follows. The vertical distance between one of the two maxima and the *highest* minimum was divided into twelve equal parts. At each point of division the mean of the two abscissae on that fringe was calculated, thus giving eleven points each representing the center of the fringe for a particular ordinate. These points were plotted on a new chart and the resulting curve was known as a line of centers. The process was repeated for the other fringe of the master curve. If the three faint components had been properly removed the two lines of centers should be vertical and the reciprocals of their slopes should be zero. In each case the average slope reciprocals, hereafter called "slopes" for the sake of brevity, were determined omitting the upper one and lower two points which were of small weight. From these data for the four residual curves it was then possible to derive coefficients giving the change in slope for a standard change of intensity for each of the faint components. Using these coefficients and the slope of the line of centers for any one of the residual curves, a linear equation could be formed involving corrections to the three intensities used in the residual curve necessary to make the line of centers vertical. The fringe due to the first component was very insensitive to the faint components as regards slope of the line of centers and was therefore not used. The remaining condition, that the two minima should be equal, was imposed in much the same way, namely by the determination of coefficients and the derivation of an equation. Thus there were available two equations involving corrections to the assigned intensities of the three faint components. After much experimenting it was found that components four and five were indeterminate within the errors of observation, so that solutions were made only for component three based on hypotheses concerning the other two. This was done by least squares from the two equations, on the basis of different hypotheses concerning the fourth and fifth components. In this work the approximation of linear relationships between the intensities of the faint components and the various observed quantities was made. This while not strictly true, was found by tests to be correct in the applications within the observational errors. The solutions obtained in this manner gave the most probable values for the third component on the basis of the adopted assumptions regarding the fourth and fifth. All intensities were calculated in units of the theoretical intensity using the second component as a standard. The intensities of the third component are recorded in the first line of each section in Table II. An inspection of the values given in the first two sections reveals no differences between $H^1\alpha$ and $H^2\alpha$ exceeding the possible errors of determination. It was therefore decided to form the mean intensity for $H^1\alpha$ and $H^2\alpha$ in each case and to use these values in calculating the results listed in the remaining portions of the table.

The intensities of the first component on the different plates were calculated on the assumption of theoretical intensity for the fourth and fifth and the corresponding measured intensities for the third. This was accomplished by subtracting from the master curve the faint components with these assigned intensities and measuring the heights of the maxima of the residual curve. Account was taken of the fact that the minimum of the fringe for each component added something to the maximum of the other component and must, therefore, be subtracted. The resulting intensities of the first component measured in terms of its theoretical value are included in the fourth section of Table II. Again the differences between the results for $H^1\alpha$ and $H^2\alpha$ do not exceed the observational errors. We conclude, therefore, that the components have the same relative intensities in $H^1\alpha$ and $H^2\alpha$.

It is clear that all plates agree in giving intensities too low for the first component and too high for the third. This can be explained qualitatively if we assume a population of the $3D$ energy levels less than would occur in thermodynamic equilibrium. The upper level for the first component is a $3D$ state while the second originates partly in a D state and partly in a P state (see Fig. 1). Thus, in accordance with the hypothesis, both components should be weakened but the first should be affected more than the second. This would give in the adopted scale of intensities a relatively faint first component. On the other hand the third component originates from P and S states for its upper level and should, therefore, not be weakened but, relative to the second, it should be strengthened. This is in accord with the observations. The fourth originates from a D state only, since the corresponding line from the P state is forbidden by the selection rules, and it should be weakened like the first. The fifth comes from the S state and should be strengthened relatively to the second. Briefly, the hypothesis of a diminished D population explains the weakened first and the strengthened third components and predicts weakened fourth and strengthened fifth components.

When we examine the changes in intensity from plate to plate it is apparent that the weakening of the first component is accentuated on the low pressure spectrograms whereas no systematic effect is noted for the third component. The errors of observation may be large enough to mask the effect in the latter case, or there may be departures from the equilibrium populations of the P and S states capable of explaining this observation if we assume it to be correct.

If the hypothesis of a diminished population in the $3D$ energy levels is correct, the fourth component should be affected in the same ratio as the first. We therefore assigned intensities to the fourth component on each plate bearing the same ratio to their theoretical values as were found for the first. These were used as a basis for the results which we regard as most probable.

TABLE II.

		Interferometer separation = 0.78190 cm						Interferometer separation = 0.7817? cm							
		31		33		34		40		41		42		Mean	
	Plate number	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$	$H^1\alpha$	$H^2\alpha$
	Intensity of 3rd	1.31	1.25	1.46	1.56	(2.39	2.27)	1.72	1.23	1.11	1.77	1.39	1.10		
4th = 0	Center of 1st	0.017	0.498	0.017	0.504	(0.511	−0.001)	0.504	−0.009	0.504	−0.009	0.508	−0.006		
5th = 0	Center of 2nd	0.527	1.010	0.527	1.012	(1.042	0.515)	1.033	0.506	1.016	0.512	1.028	0.509		
	Separation cm^{-1}	0.326	0.327	0.326	0.325	(0.339	0.330)	0.338	0.329	0.327	0.333	0.333	0.329	0.3300 ± 18	0.3286 ± 9
	Intensity of 3rd	1.54	1.35	1.76	1.74	(2.78	2.36)	2.29	1.70	1.90	2.18	1.91	1.64		
4th = 1	Center of 1st	0.027	0.509	0.025	0.512	(0.523	0.010)	0.508	−0.001	0.517	−0.004	0.513	0.001		
5th = 1	Center of 2nd	0.529	1.010	0.530	1.013	(1.046	0.517)	1.037	0.510	1.022	0.515	1.033	0.513		
	Separation cm^{-1}	0.321	0.320	0.323	0.320	(0.334	0.324)	0.338	0.327	0.323	0.332	0.333	0.327	0.3276 ± 22	0.3252 ± 14
	Intensity of 3rd	1.28		1.51		(2.33)		1.48		1.44		1.24			
4th = 0	Center of 1st	0.017	0.498	0.017	0.504	(0.512	−0.001)	0.506	−0.009	0.502	−0.009	0.509	−0.006		
5th = 0	Center of 2nd	0.526	1.010	0.528	1.012	(1.041	0.516)	1.028	0.509	1.021	0.508	1.026	0.511		
	Separation cm^{-1}	0.326	0.328	0.327	0.325	(0.338	0.330)	0.334	0.331	0.332	0.331	0.331	0.331	0.3300 ± 10	0.3296 ± 9
	Intensity of 3rd	1.44		1.75		(2.57)		2.00		2.04		1.78			
	Intensity of 1st	0.59	0.57	0.67	0.71	(0.61	0.59)	0.81	0.82	0.86	0.91	0.83	0.85		
4th = 1	Center of 1st	0.028	0.509	0.026	0.512	(0.523	0.010)	0.510	−0.002	0.508	−0.002	0.515	0.001		
5th = 1	Center of 2nd	0.527	1.011	0.530	1.013	(1.042	0.517)	1.031	0.513	1.025	0.513	1.030	0.515		
	Separation cm^{-1}	0.319	0.321	0.322	0.320	(0.332	0.324)	0.333	0.329	0.331	0.329	0.329	0.329	0.3268 ± 18	0.3262 ± 13
	Intensity of 3rd	1.42		1.72		(2.56)		1.92		2.00		1.73			
	Intensity of 4th	0.58		0.69		(0.60)		0.82		0.88		0.84			
5th = 1	Center of 1st	0.024	0.505	0.024	0.510	(0.519	0.006)	0.510	−0.003	0.507	−0.003	0.514	0.000		
	Center of 2nd	0.527	1.011	0.529	1.013	(1.042	0.517)	1.031	0.513	1.024	0.513	1.028	0.515		
	Separation cm^{-1}	0.322	0.324	0.323	0.322	(0.334	0.327)	0.333	0.330	0.331	0.330	0.329	0.329	0.3276 ± 15	0.3270 ± 11

POSITIONS OF MAIN COMPONENTS

With given values of the intensities for the faint components it was possible to find the positions of components one and two by using the lines of centers. The means of the abscissae in each of these lines were formed omitting the upper one and the lower three points. This gave the center of the corresponding fringe. By comparing results from the different residual curves, coefficients were found relating changes in these centers to changes in the intensities of the faint components. A center found from a given residual curve combined with these coefficients furnished a linear equation from which the center could be calculated for any intensities of the first component. In this manner the centers of components one and two were determined on the basis of the adopted intensities.

The lines headed "center of 1st" and "center of 2nd" in each section of Table II give the positions of the fringe centers determined in this way on the basis of the various hypotheses concerning the faint components. The differences between fringe centers for the first and second components when divided by twice the interferometer spacing yields the separation in wave numbers for the two strong components. These values are given in the last line of each section. For the reasons given in the preceding article the data derived from plate 34 have not been included in the means.

DISCUSSION OF RESULTS

It is clear from the results quoted in the table that there is no appreciable difference in doublet separation for $H^1\alpha$ and $H^2\alpha$. This is in disagreement with the results of Williams and Gibbs who find 0.308 and 0.321 for $H^1\alpha$ and $H^2\alpha$, respectively. We feel, however, that our treatment possesses certain advantages over theirs, particularly with reference to our spectrographic separation of $H^1\alpha$ and $H^2\alpha$ and to our more detailed analysis of the intensity curves of the fringe systems.

We regard as the most probable value of the separation the weighted mean of the values for $H^1\alpha$ and $H^2\alpha$ taken from the last section of Table II, the weights being 1 and 2, respectively. From the formula[7]

$$\alpha^2 = (1296/73)(\Delta\nu/R)$$

we find $1/\alpha = 137.4 \pm 0.2$. The probable errors quoted are calculated values based solely on the accordance of the observations and understate the uncertainty of the results. There appears to

[7] See Ruark and Urey, *Atoms, Molecules and Quanta*, p. 135 for the term values used in deriving this equation.

be a distinct systematic difference between the separations depending on the pressure used in the source. It is conceivable that this is real. On the other hand it may be due to wide variation in intensities of the unresolved components. If we assume that the fourth and fifth components have zero intensities on the low pressure plates and theoretical intensities on the high pressure plates, the systematic difference is greatly reduced. The only conclusion to be drawn at present from this difference is that the uncertainty of the separation must be much greater than is indicated by the calculated probable errors. Our best estimate of this uncertainty may be expressed by saying that $1/\alpha$ probably lies between 137 and 138.

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Are the Formulae for the Absorption of High Energy Radiations Valid?

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In this paper we consider the discrepancies between theoretical prediction and experiment for the absorption of cosmic-ray electrons and gamma-rays. By applying a strict criterion for the validity of classical electron theory, it is possible to derive new formulae for impact and radiative energy losses of high energy electrons, which may be regarded as theoretical lower limits for these quantities, and which are in far better agreement with experiment than the formulae given by an uncritical application of quantum mechanics to these problems. These limitations on classical electron theory are consistent with those given by possible unitary classical field theories, but are more incisive than those given by the unitary theory of Born.

1. THE THEORETICAL FORMULAE

THE question of the validity of the theoretical formulae for the absorption of high energy radiations has been brought to a new prominence by recent experimental and theoretical researches. On the one hand the observation of the cloud chamber tracks of cosmic rays has made it possible to extend our knowledge of the specific ionization and energy loss of electrons from particles of a few million volts on up to a few billion.[1] On the other hand two mechanisms of absorption, increasingly important at high energies, have been carefully investigated theoretically:[2] the pair production by gamma-rays, and the radiative energy losses of electrons. The question of whether the formulae derived for the probability of these processes, and the more familiar formulae for the ionization and impact energy losses of fast electrons, should hold for the very high cosmic-ray energies, has often been discussed, and has been explicitly studied by v. Weizsaecker[3] and by Williams.[4] The conclusion to which these researches have led is that the formulae should remain valid. The experiments, however, do not speak for this. We want here to reconsider the question in the light of this discrepancy.

The predictions of the theory are these: (1) The specific primary ionization of an electron (or positron) should pass through a minimum as the energy of the electron increases, and should increase slowly with the energy throughout the entire range of cosmic-ray energies. If the velocity of the electron be $v=\beta c$, then the specific ionization should vary[5] with v according to

$$(1/\beta^2)[\ln \epsilon\beta+\ln (k/\alpha)-\tfrac{1}{2}\beta^2]$$

with $$\epsilon=(1-\beta^2)^{-\frac{1}{2}};\quad \alpha=e^2/\hbar c. \qquad (1)$$

Here k is a constant of the order of 10, depending on the f-values of the atomic electrons of the matter through which the ray is passing. According to this formula one has to expect an

[1] C. D. Anderson and S. H. Neddermeyer, International Conference on Physics, London, 1934.

[2] H. Bethe and W. Heitler, Proc. Roy. Soc. **A146**, 83 (1934).

[3] v. Weizsaecker, Zeits. f. Physik **88**, 612 (1934).

[4] E. J. Williams, Phys. Rev. **45**, 729 (1934).

[5] e.g., H. Bethe, *Handbuch der Physik*, XXIV, 1, 2nd edition, 1932.