The Elastic Scattering of Electrons in Argon and Krypton

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Scattering coefficients have been experimentally determined for the elastic scattering of electrons in argon and krypton for the angular range 5° to 150° and incident electron energies 25 to 950 volts. In argon it was found that one maximum and two minima are present up to 150 volts. In the scattering by krypton maxima of two different orders are observed, the first being prominent from 25 to 100 volts, the second appearing at 100 volts for large angles. With increasing energies the first order maximum changes position progressively to smaller angles and merges with the main body of the scattering at about 200 volts in argon and 350 volts in krypton. From 300 to 800 volts one minimum only occurs in argon. There is a single maximum in the range 350 to 950 volts in krypton. The experimental results are compared with theoretical curves. At low voltages (for A) the agreement is only qualitative when Henneberg's phases are used, while (for Kr), using Holtsmark's phases, the agreement is excellent. At higher voltages (500 to 800 volts in A, and 700 to 900 volts in Kr) the scattering is given quite accurately by the Born approximate equation for a limited angular range.

INTRODUCTION

THE elastic scattering of electrons by the rare gases has been investigated experimentally by several physicists. Accurately determined scattering coefficients[1] for these gases are now available for certain angular and primary electron energy ranges. The following Table I summarizes the experimental results obtained up to the present. Only those researches are here mentioned which contain results in tabulated form.

A glance at this table indicates that for the rare gases He and Ne the scattering has been investigated over sufficiently extensive a range of both angles and primary electron energies to enable one to check the curves computed theoretically. As regards argon and krypton there is a definite lack of information at large angles (above 100°) and at small angles (below 20°) throughout the range of primary electron energies 25 to 900 volts. For this reason it was thought advisable to determine the scattering for these two gases over the range of angles 5° to 150° and incident electron energies of 25 to 900 volts.

EXPERIMENTAL PROCEDURE

The experimental method used will not be discussed in detail since the apparatus has been described in a previous report.[5] The argon and krypton were used directly from the manufac-

TABLE I. *Summary of experimental results on the scattering of electrons by rare gases.*

Gas	Angular range	Voltage range	Ref.
Helium	10°–150°	25–700	2
	20°–140°	4–50	3
	15°–167°	1.8–19.2	4
Neon	10°–150°	15–800	5
	10°–120°	29–412	6
	10°– 40°	813	6
	30°–130°	6–30	3
	15°–167°	0.99–15.9	4
	20°–155°	50–150	7
Argon	15°–120°	42–780	6
	20°–130°	6–30	3
	40°–160°	30–150	7
	15°–167°	0.6–12.5	4
Krypton	15°–120°	41–800	6
	15°–167°	0.83–10.6	4
Xenon	15°–120°	42–800	6
	15°–167°	0.61–8.1	4

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[1] The scattering coefficient is defined as the number of electrons scattered in a direction θ to the original beam, through unit solid angle, per unit length of path of the original beam, per single electron in the beam, per single atom in unit volume.

[2] A. L. Hughes, J. H. McMillen and G. M. Webb, Phys. Rev. **41**, 154 (1932).

[3] E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A133**, 637 (1931).

[4] C. Ramsauer and R. Kollath, Ann. d. Physik **9**, 756 (1931); **12**, 529 (1932).

[5] A. L. Hughes and J. H. McMillen, Phys. Rev. **43**, 875 (1933).

[6] F. L. Arnot, Proc. Roy. Soc. **A133**, 615 (1931).

[7] C. B. O. Mohr and F. H. Nicoll, Proc. Roy. Soc. **A138**, 469 (1932).

turer's containers; the labels thereon stated the gases were better than 99 percent pure.

In the experiments with argon the gas was allowed to leak out of the container through a small capillary tube continuously; the rate of pumping was so adjusted that the proper pressure (to insure single electron collisions only) was maintained. In the work with krypton this method could not be used due to its comparative rarity and high cost. The method was to seal the pumps off from the apparatus after it had been baked out for two hours at about 250°C, allow a small quantity of gas to enter the collision chamber and then circulate the gas by means of a fast pump in series with a trap cooled with solid CO_2 to remove impurities. The increase in pressure under these conditions with the electron gun operating was less than three percent over a period of four hours. A single run required, on the average, about an hour.

Experimental Results

(1) Argon

The results obtained for argon are tabulated in Table II in arbitrary units and illustrated graphically in Figs. 1 and 2.[8] The most prominent feature is a maximum occurring at about 90° for the lower voltages. It is very small at 150 volts and at 240 volts is perceptible only in the form of a kink in the scattering curve. A very

TABLE II. *Elastic scattering of electrons in argon.* (Arbitrary units.)

θ \ V	25	50	100	150	240	360	510	780
°								
5							620	602
10		146	118	172	414	1840	262	207
15		98.5	64.8	92.2	144	940	121	86.0
20	45	67.5	40.1	48.0	78.0	520	48.5	37.1
25		46.8	24.8	27.2	39.2	301	27.9	17.0
30	31	33.3	15.8	17.8	24.6	184	16.9	9.91
40	18.0	18.4	6.80	7.80	12.8	89.1	7.8	4.32
50	11.4	8.4	3.38	4.82	8.7	47.2	4.72	2.21
60	6.22	3.0	2.50	3.41	6.2	30.1	2.86	1.24
70	2.95	2.01	2.62	3.02	4.3	19.9	1.85	.763
80	2.57	3.22	3.02	3.17	3.2	14.6	1.32	.565
90	3.09	5.55	3.35	2.69	2.2	10.2	1.20	.455
100	3.91	6.02	2.95	1.50	1.0	9.3	1.09	.400
110	3.69	5.20	2.01	0.83	1.2	10.2	1.26	.378
120	2.69	3.50	1.25	0.93	1.81	12.3	1.36	.400
130	2.19	1.42	1.35	2.42	2.95	15.5	1.70	.431
140	2.12	1.30	2.87	4.68	5.70	20.9	2.26	.525
150	2.30	2.61	5.70	8.61	1.12	28.4	3.02	.660

[8] On these same graphs are plotted theoretical curves which will be discussed later.

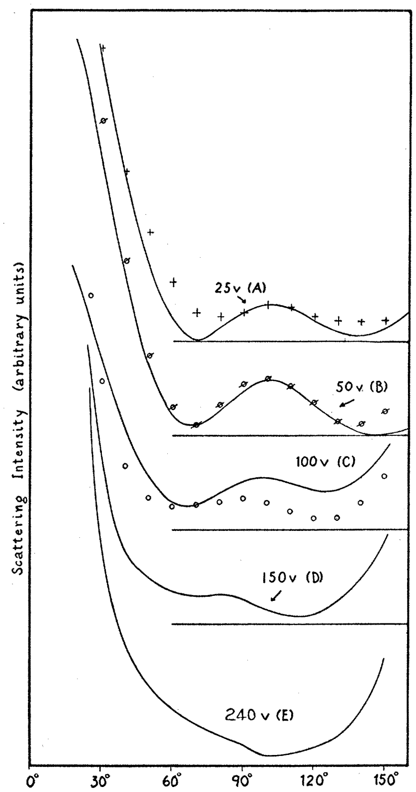

FIG. 1. Elastic scattering of electrons in argon. *A*, *B* and *C*, comparison of experimental points with theoretical curves; *D* and *E*, experimental curves.

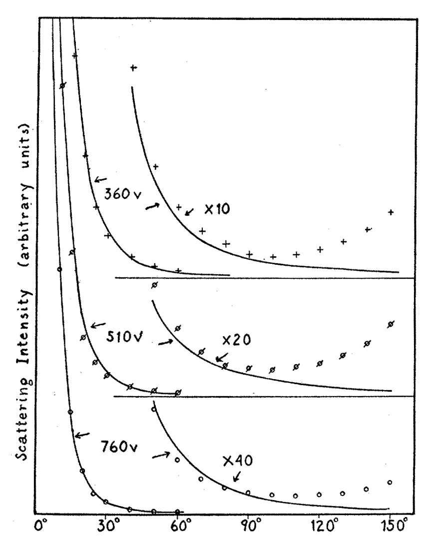

FIG. 2. Elastic scattering of electrons in argon. *Solid lines*, Born's approximate curves (Hartree field); *points*, experimental values.

prominent minimum is also present throughout the whole range of electronic velocities investigated. It shifts its position steadily from 145° at 25 volts to 96° at 760 volts.

The agreement of these results with those previously obtained by other investigators is good. Arnot's 240 volt curve is in excellent agreement with the present corresponding curves. Mohr and Nicoll's results at 150 volts are checked except for a very slight maximum which is not present in their curve. The general angular shift of maxima and minima is in agreement with Arnot's observations.

(2) Krypton

The experimental results obtained for krypton are given in Table III in arbitrary units; Figs. 3 and 4 illustrate the results graphically.[8] The main feature of this set of curves is the occurrence of a single maximum at low energies which merges with the main body of the scattering above 250 volts. Above 55 volts a second maximum appears which, as the energy of the colliding electrons is increased, retreats into smaller angles. This second maximum is present up the the highest voltage used (950 volts).

TABLE III. *Elastic scattering of electrons in krypton.* (Arbitrary units.)

θ \ V	25	55	100	250	350	510	780	950
°								
5		3210	7560	(835)	342			790
10		1950	4220	341	286	262	230	192
15		1160	2410	142	960	121	92.3	85.3
20	92.5	657	1310	65.0	45	64.5	48.1	40.1
25	62.8	455	613	31.8	21.5	37.1	26.7	18.6
30	43.3	242	233	13.8	14.5	23.3	16.8	12.4
40	20.6	53.4	20.1	10.1	8.23	9.35	7.32	5.90
50	10.2	15.8	22.5	11.1	4.55	3.87	3.77	2.47
60	4.02	29.2	55.2	6.50	2.55	2.08	2.07	1.10
70	1.92	40.8	53.0	2.1	1.31	1.42	1.26	1.03
80	2.65	32.9	29.2	1.1	1.35	1.60	.917	1.55
90	4.48	18.6	12.8	2.8	2.42	2.74	1.53	1.54
100	5.62	2.43	8.31	4.9	3.35	2.93	1.54	1.15
110	4.37	2.41	8.92	6.5	2.33	1.85	.693	.49
120	2.85	1.45	21.8	4.7	.91	.58	.713	.58
130	1.69	2.88	27.3	2.0	.58	.88	1.56	1.50
140	.79	5.20	16.2	1.4	2.36	2.90	3.05	2.31
150	.43	10.7	2.95	5.3	5.52	7.35	5.22	2.85

The general nature of these curves is the same as found by Arnot. His 54 and 820 volt curves, for example, agree very well with the present 55 and 780 volt curves, respectively, at large angles.

In view of the good agreement of the present results with those obtained by other investigators, it is believed that they are free from serious error due to peculiarities of the apparatus used and accurately represent the true scattering.

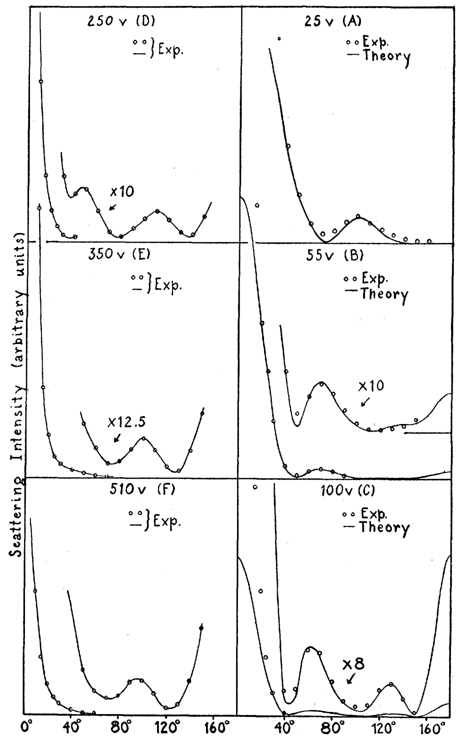

FIG. 3. Elastic scattering of electrons in krypton. *A*, *B* and *C*, comparison of experimental points with theoretical curves; *D*, *E* and *F*, experimental curves.

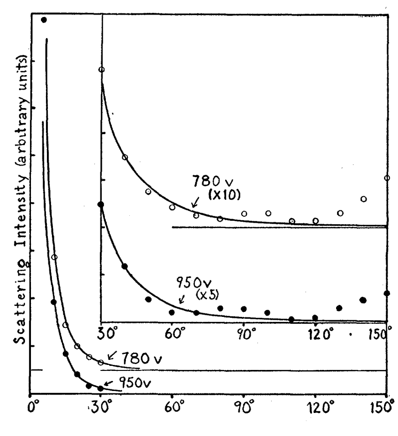

FIG. 4. Elastic scattering of electrons in krypton. *Solid lines*, Born's approximate equation (Thomas-Fermi field); *points*, experimental values.

Discussion

A. Low incident electron energies

Faxen and Holtsmark have shown that the elastic scattering of electrons by a spherically symmetrical field is given by [9]

$$I(\theta)=|F(\theta)|^2$$
$$=\frac{1}{4E}\left|\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)(e^{2i\delta_l}-1)\right|^2, \quad (1)$$

where $I(\theta)$ is the scattered intensity at an angle θ, E is the energy of the incident electrons in atomic units ($E=1$ corresponds to 13.54 volts), l is the orbital angular momentum quantum number, and P_l is the Legendre polynomial of the lth degree. δ_l is the phase difference between the asymptotic solution of the equation

$$[d^2/dr^2+E+2V-l(l+1)/r^2]\psi=0, \quad (2)$$

when V is zero, and when V is the potential field of the atom.

Before the actual evaluation of Eq. (1) can be carried out, it is necessary to determine the phase differences. Holtsmark[10] has computed exact phase shifts for argon and krypton for values of E from 0 to 4 using the self-consistent field given by Hartree's method. His calculations also include a correction for polarization. Arnot and Baines[11] have very recently computed phase shifts for krypton ($E=0, 1, 4, 9$) using Jeffry's approximation and Holtsmark's atomic field values.

To compute theoretical curves for krypton in the present work, Holtsmark's exact phase shifts (for $l=0, \cdots, 4$) for 25 and 55 volts were used. For the 100 volt curve, the phase shifts (for $l=0, \cdots, 6$) were obtained by interpolating between the values given by Arnot and Baines for 55 and 121 volt electrons. These three theoretical curves are shown in Fig. 3, and the agreement with the experimental points is seen to be exceptionally good.

Since the phase-shifts have not yet been computed for argon using the Hartree field at moderate voltages, a method developed by Henneberg,[12] using the W-K-B approximation and the Thomas-Fermi field was used. The phase shifts are determined by the equation

$$\delta_l=\int_R^\infty\left[E+2V-\frac{l(l+1)}{r^2}\right]^{1/2}dr$$
$$-\int_{R_0}^\infty\left[E-\frac{l(l+1)}{r^2}\right]^{1/2}dr, \quad (3)$$

where R and R_0 are the roots of the respective integrands, E is the incident electron energy and V is the potential field of the atom under consideration, given by

$$V=(Z/r)\varphi(r/\mu);\quad \mu=0.885/Z^{1/3}. \quad (4)$$

Upon substituting equation (4) in (3) we note that the resulting formula for $\delta_l Z^{-1/3}$ remains invariant if we place

$$EZ^{-4/3}=E_1Z_1^{-4/3};\quad l(l+1)Z^{-1/3}$$
$$=l_1(l_1+1)Z_1^{-1/3}, \quad (5)$$

where E, Z and E_1, Z_1 are corresponding values of the incident electronic energy and atomic number of the scattering atom. If we define a quantity, q, such that

$$q=(Z_1/Z)^{1/3},$$

it is seen that

$$\delta_Z(E)=q^{-1}\delta_{Z_1}(q^4E),$$

and the value of l is changed according to the relation

$$l_Z=q^{-1}(l_{Z_1}+\tfrac{1}{2})-\tfrac{1}{2}.$$

Henneberg has computed the phase shifts for mercury ($Z_1=80$) by graphical integration of Eq. (3) for $l=0, \cdots, 10$ and $E=0$ to 50. Hence we can immediately compute from his results the phase shifts for any atom of atomic number Z sufficiently large to make the use of the Thomas-Fermi field valid. Thus the phase shifts for argon have been determined by this method for primary electron energies from 25 to 100 volts.

Henneberg points out that for low velocity electrons (but not so low that exchange effects are appreciable) one can get an accurate picture of the shape of the angular distribution curves by using in the evaluation of Eq. (1) only those

[9] H. Faxen and J. Holtsmark, Zeits. f. Physik **45**, 307 (1927).

[10] J. Holtsmark, Zeits. f. Physik **55**, 437 (1929); **66**, 49 (1930).

[11] F. L. Arnot and G. O. Baines, Proc. Roy. Soc. **A146**, 651 (1934).

[12] W. Henneberg, Zeits. f. Physik **83**, 555 (1933).

terms for which $\delta_l \geq \pi/2$. The reason for this is that the products $(2l+1)$ sin $2\delta_l$ and $(2l+1)$ $(1-\cos 2\delta_l)$, being the coefficients of the lth Legendre polynomial P_l, determine the relative contributions of the various P's. Obviously, since δ_l decreases monotonically with increasing l (for large l, roughly as l^{-3}) that value of δ_l nearest to $\pi/2$ will have most effect in determining the shape of the scattering curve.

To test this out for argon, the theoretical curves for 25, 50 and 100 volts are plotted in Fig. 1 on the same graphs as the experimental points. For each of these curves the first three values of δ_l (for $l=0$, 1, 2) were used. The agreement is best for 50 volts, good for 25 volts and only fair for 100 volts. In this last curve, the shape (i.e., the positions of maxima and minima) is predicted quite accurately, but the absolute magnitude of the scattered intensity is considerably in error. Thus we see that as the incident electron energy increases we must include in our evaluation of Eq. (1) a correspondingly greater number of terms.

This lack of complete agreement between theory and experiment at 100 volts may also be partially explained by the following factors: (1) Use of the Thomas-Fermi rather than the Hartree field; (2) neglect of polarization which, according to Massey and Mohr,[13] is of considerable importance, and (3) neglect of exchange. The excellent agreement of the low voltage krypton experimental results with theory substantiates these conclusions when it is remembered that there the Hartree field was used and polarization was taken into account. In an attempt to get better agreement with experiment, phases for argon are being calculated using the Hartree field as given by Holtsmark.

B. High incident electron energies

When the incident electron energy is sufficiently high, it is of interest to compare the experimental results with Born's approximate equation. Mott[14] has put this equation in a convenient form by making use of the F-factors calculated from the self-consistent field:[15]

$$I(\theta)=(e^4/4m^2v^4)(Z-F)^2 \operatorname{cosec}^4 \theta/2. \quad (6)$$

When the Thomas-Fermi field is used, an integral equation connecting the scattered intensity and scattering angle is obtained which can only be solved by numerical integration. Mitchell[16] and Bullard and Massey,[17] carrying out this integration, have tabulated $I(\theta)Z^{-2/3}$ as a function of $Z^{-1/3}v \sin \theta/2$.

For heavy atoms the Born approximate equation obtained with either of the above atomic fields should be valid; but as the atomic number decreases the Hartree field can be expected to give better results than the Thomas-Fermi field.

In Fig. 2 Eq. (6) has been plotted for 760, 510 and 360 volt electrons in argon. The agreement at the highest voltages is good to angles up to about 100 degrees, while at 360 volts the experimental curve rises above the theory curve below 10° and at large angles.

In Fig. 4 the Born approximate equation has been plotted using the Thomas-Fermi field for krypton at 950 and 780 volts. It is seen that for both curves the experimental results fit the theory up to about 40°; beyond this point the experimental curve oscillates about it.

In order to obtain better agreement between theory and experiment at these high energies, it appears that only one additional factor need be taken into account: distortion. The general effect of including distortion is to introduce maxima and minima in the angular distribution curves. Since the Born approximate equation is valid at small angles we should expect excellent agreement between theory and experiment throughout the complete angular range if distortion were included. The effect of exchange and polarization are undoubtedly small at these energies.

Conclusion

It is a pleasure for the writer to acknowledge his indebtedness to Professor A. L. Hughes, who suggested and directed this research, for his continued interest throughout the course of the work.

[13] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. **A146**, 880 (1934).
[14] N. F. Mott, Proc. Roy. Soc. **A127**, 658 (1930).
[15] R. W. James and G. W. Brindley, Phil. Mag. **12**, 81 (1931).
[16] A. C. G. Mitchell, Proc. Nat. Acad. Sci. **15**, 520 (1929).
[17] E. C. Bullard and H. S. W. Massey, Proc. Camb. Phil. Soc. **26**, 556 (1930).