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A Spectroscopic Determination of *e/m*

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A determination of *e/m* was made by accurately measuring the difference in wave number between the Hα lines of ordinary and heavy hydrogen. The observations were made with a Fabry and Perot etalon used in conjunction with a spectrograph of sufficient resolving power to separate the interference patterns of the two Hα lines. The effects of the weak fine structure components were eliminated and values of *e/m* were derived from the two main components. The final mean gives $e/m = 1.7579 \pm 0.0003 \times 10^7$. From this determination the ratio of the mass of the hydrogen atom to the mass of the electron is found to be $1835.6 \pm 0.2_5$. There is no indication of systematic errors in the work so that the probable errors may be fairly taken as representing the actual uncertainties in the results.

Introduction

It was pointed out by Birge[1] in 1929 that the mean value of *e/m* determined from the acceleration of free electrons was $(1.769 \pm 0.002) \times 10^7$ whereas spectroscopic methods gave $(1.761 \pm 0.001) \times 10^7$. That the difference between these values might be real was suggested by the small size of the probable errors. This led to the possible interpretation that the value of *e/m* for free electrons was not the same as for electrons within the atom. In view of the importance of this question considerable work has been done recently in determining *e/m* by both methods. The more important results are listed in Table I. Evidently the discrepancy shows a tendency to disappear with later and more refined experimental methods.

Table I.

Date	Observer	*e/m* (spectroscopic)	*e/m* (acceleration)
1930	Perry and Chaffee[2]		1.761 ±0.001
1931	Campbell and Houston[3]	1.7579 ±0.0025	
1932	Kirchner[4]		1.7587 ±0.0009
1933	Dunnington[5]		1.757 ±0.0015
1933	Spedding, Shane and Grace[6]	1.758	
1934	Gibbs and Williams[7]	1.757 ±0.001	
1934	Kinsler and Houston[8]	1.7570 ±0.0010	

Of the various spectroscopic methods the one that is probably most accurate consists in determining the difference in the Rydberg constant corresponding to two different hydrogen-like atoms. Observationally this resolves itself into the measurement of the difference in wave number of corresponding lines in the spectra of two such substances. This method was applied by Houston[9] using ordinary hydrogen and ionized helium. In application the principal defect lies in the different fine structure of the lines to be measured. When through the work of Professor G. N. Lewis, experimental amounts of water containing high concentrations of the heavy hydrogen isotope were available, it became possible to make determinations by the above method relatively free from this difficulty. The fine structure is the same for corresponding lines of H^1 and H^2 and only affects the results to a small extent through the difference in temperature widening. The value given by Spedding, Shane and Grace in the above table is a preliminary result derived by this method. The only other determination in which H^1 and H^2 were used is that of Gibbs and Williams.[7]

The present paper reports the final results of our investigation. The procedure possesses, we believe, three points of advantage over that used by Gibbs and Williams. First, sufficient spectroscopic resolving power was employed to yield separate interference fringe systems for $H^1\alpha$ and $H^2\alpha$ whereas in the experiments of Gibbs and Williams the two systems overlapped. The

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[1] Raymond T. Birge, Rev. Mod. Phys. **1**, 1 (1929).

[2] C. T. Perry and E. L. Chaffee, Phys. Rev. **36**, 904 (1930).

[3] J. S. Campbell and W. V. Houston, Phys. Rev. **38**, 581 (1931).

[4] F. Kirchner, Ann. d. Physik **12**, 503 (1932).

[5] F. G. Dunnington, Phys. Rev. **42**, 739 (1932).

[6] F. H. Spedding, C. D. Shane and Norman S. Grace, Phys. Rev. **44**, 58 (1933).

[7] R. C. Gibbs and R. C. Williams, Phys. Rev. **44**, 1029 (1933).

[8] L. E. Kinsler and W. V. Houston, Phys. Rev. **45**, 104 (1934).

[9] W. V. Houston, Phys. Rev. **30**, 608 (1927).

spectroscopic separation of the fringe systems resulted in greater accuracy of photometric measurement. Second, in view of the resulting simplicity of the fringe systems, a more complete analysis of the effects due to the faint unresolved components of the lines was possible. Finally, through the method of cooling the discharge tube, the lines obtained showed improved sharpness.

The determination of e/m was intimately bound up with a study of the fine structure of Hα and a measurement of the fine structure constant, $1/\alpha$, of which the results are published in the succeeding article.

Experimental Procedure

The source of illumination was a modified Wood's tube operated from a 15,000-volt transformer and completely immersed in liquid air. An image of the source was focused between the plates of an etalon interferometer whose separation of 0.782 cm was chosen so as to give a relative displacement of half a fringe between the two main components of the Hα lines. The interference fringes were focused on the slit of a large grating spectrograph by a lens of 32 inches focal length. The dispersion of this spectrograph in the first order was sufficient to give separate images of $H^1\alpha$ and $H^2\alpha$.[10] A typical spectrogram is illustrated in Fig. 1.

Photometry and Measurement of Plates

Each plate was standardized by photographing on it light of approximately the wavelength of Hα, passed through an optical wedge of known transmission. Microphotometer tracings were then made of these standards as well as of $H^1\alpha$ and $H^2\alpha$ with the Zeiss microphotometer belonging to the Physics Department.

The tracings were accurately measured in both coordinates. The ordinates were reduced to relative intensities by means of the reduction curves derived from the tracings of the standards. Owing to the slightly non-uniform intensity in the illumination of the slit, it was necessary to use the relative intensities of the successive fringe maxima and minima to derive a curve of correction factors by which all measured intensities were multiplied to reduce them to the case of uniform illumination. The intensities thus reduced were ready for plotting.

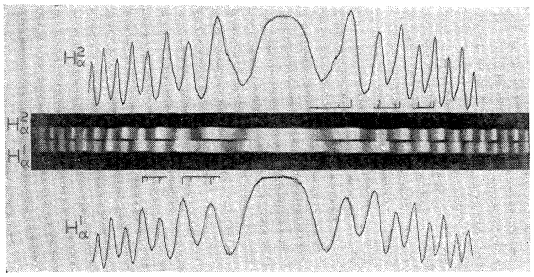

FIG. 1. Spectrograms and microphotometer curves of $H^2\alpha$ and $H^1\alpha$.

It was necessary to alter the scale of abscissae in such a way as to give a uniform scale in terms of orders of interference. Theoretically the linear distance of a point from the center of the fringe system is very nearly proportional to the square root of the difference between the order of interference at the point and the order at the center of the fringe system. Thus if the distances of points from the center be squared and the resulting numbers be taken as abscissae, one should get a scale proportional to the order of interference. When the intensities were plotted against these abscissae, it was found that the fringe maxima corresponding to a given component were nearly but not quite equally spaced. These departures from equal spacing were due to errors in selecting the center of the fringes and to small distortions in the optical system. In no case were the second differences as much as 4 percent of the first differences. The spacings of the maxima were then differenced, and from the first and second differences the scale of abscissae was corrected so that the separations of adjacent maxima due to a given component were all equal to unity. It was found that the first and second differences changed slightly in passing from one side of the fringe center to the other due to distortions, but as we might expect, these quantities remained the same for $H^1\alpha$ and $H^2\alpha$. Therefore mean values were used for these two lines but for each side of the fringe center the appropriate mean values were taken. The arbitrary zero values from which the reduced abscissae were calculated were chosen the

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[10] A more detailed description of the apparatus is given in the following article.

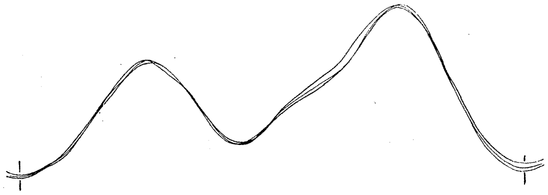

FIG. 2. Typical intensity curves.

same for $H^1\alpha$ and $H^2\alpha$ so that the curves for these two lines would have a relative displacement corresponding to the decimal part of their difference in order of interference. The whole number in this case was 6.

With this final scale of abscissae the intensity curves were plotted over a range of one order, the successive orders being superposed on the same diagram. A typical example is shown in Fig. 2 where the three orders used on one side of the center are plotted in juxtaposition. When the fringes on both sides of the center had been plotted in this manner, the means of the ordinates for equal values of the abscissae were determined and used to plot a single curve known as the master curve for the line. In the case of plates, Nos. 31, 33 and 34, three fringes on each side of the center were used while for the remaining plates three fringes on one side of the center and two on the other were plotted. The omission of one fringe was due to a distortion in the optical system that affected the spacings of the outer fringes on one side of the center in such a way that it was impossible to bring them into accurate juxtaposition with the others by the use of second differences only. The master curves derived in this manner served as a basis for all succeeding measurements and calculations.

REDUCTIONS AND OBSERVATIONS

Theoretically the $H\alpha$ line consists of five components whose calculated intensities are 9.00, 7.08, 1.13, 1.00 and 0.20.[11] These are shown in Fig. 1 of the succeeding article. For convenience of reference these components will be called 1, 2, 3, 4, 5, in order of decreasing theoretical intensity. The purpose of the present work was to determine the difference in wave number between the corresponding main components of $H^1\alpha$ and $H^2\alpha$. To evaluate accurately these differences in wave number, it was necessary to subtract from the master curve the three small components. The observations indicate that the theoretical intensities of these components are not to be closely relied upon. The selection of the intensities which were used for this purpose is discussed in the following paper. As the observations indicated no definite difference in relative intensity for the components of $H^1\alpha$ and $H^2\alpha$, mean values for these two lines were determined and used in the reductions.

If we assume that the intensities to be subtracted are known, there remain to be found the positions and forms of the curves corresponding to these components. Since the small components through blending affect the positions of the main components by at most about two percent of an order of interference, small errors in placing them are not important. They were, therefore, inserted in their theoretical positions as calculated from an assumed fine structure constant $1/\alpha=138$, referred to components 1 and 2 after these had been freed by estimation from blends with the small components.

The calculation of the forms of the curves to be subtracted was more complicated. The procedure was as follows. Assuming a given component to be monochromatic, the intensity curve for its fringe system is

$$J'(\theta)=j'(1+2r\cos\theta+\cdots+2r^n\cos n\theta+\cdots),$$

where $\theta=2\pi$ times the order of interference; r=geometrical mean reflecting power of the interferometer plates; j'=a constant depending on the intensity of the source; $J'(\theta)$=intensity at order $\theta/2\pi$. The components were not monochromatic, but were widened due to a number of causes of which the Doppler widening due to temperature was predominant. We are, therefore, justified in assuming to a sufficient approximation that the intensity distribution in a component is given by the expression $e^{-u^2/a^2}$where u is measured in the same units as θ.

The resulting intensity distribution $J(\theta)$ is given by

$$J(\theta)=\int_{-\infty}^{+\infty}e^{-u^2/a^2}J'(\theta-u)du$$

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[11] W. V. Houston and Y. M. Hsieh, Phys. Rev. **45**, 263 (1934).

or

$$J(\theta)=j(1+2rb\cos\theta+\cdots+2r^nb^{n^2}\cos n\theta+\cdots),$$

where $j=a(\pi)^{\frac{1}{2}}j'$ and $b=e^{-a^2/4}$.

For the spectrograms under consideration r was found to be 0.69 while b was determined separately for $H^1\alpha$ and $H^2\alpha$ on each spectrogram. The method of determining b consisted in finding from the master curve the ratio of the maximum intensity of each of the two components to the intensity at a point one-tenth of an order from the maximum, care being taken to work on the side of the maximum least affected by the fainter components. Then by successive trials a value of b was found for which the ratios of the computed and observed values of $J(\theta)$ agreed. With this value of b, a table was calculated giving a typical fringe corresponding to the reflecting power of the interferometer and the line widening of the spectrogram.

With the typical fringe determined, it was possible for any given intensities of the faint components (1) to multiply the ordinates by a suitable factor, (2) to shift the abscissae by amounts corresponding to the theoretical positions of these components and (3) to subtract the resultant curves successively from the master curve. The result yielded a curve showing the two main components unaffected by the lesser ones to the extent that the assigned intensities were correct.

With the two main components freed from the effect of the fainter ones, the next step was to locate the positions of their centers in terms of fractional orders of interference measured from the assigned zero positions common to both $H^1\alpha$ and $H^2\alpha$. A single determination consisted in selecting an ordinate and taking the mean of the two corresponding abscissae. Seven such determinations were made for each component choosing equally spaced ordinates falling on the steep part of the curve. The mean of these seven values was taken as the center of the component. Then the separation between $H^1\alpha$ and $H^2\alpha$ in orders of interference could be found from either of the two strong components, the whole number being supplied from previous knowledge that with the interferometer spacing used it should be approximately 6.5. For the first three plates the interferometer separation was 0.78190 cm while for the last three it was 0.78179 cm. These values were used in calculating the difference in wave number between $H^1\alpha$ and $H^2\alpha$. For the calculation of e/m the following modification of Birge's formula was used:

$$e/m=\frac{FR_{H^1}(H^2-H^1)}{(R_{H^2}-R_{H^1})H^2(H^1-m)}.$$

The following values of the constants were used:

$$H^1=1.007775,^{12}\qquad R_{H^1}=109{,}677.759,^{13}$$
$$H^2=2.01363,^{12}\qquad F=9651.1\pm0.8.$$

The value of F given by Birge, 9648.9 ± 0.7, was increased by 2.2 ± 0.4 to effect the reduction to the Aston scale of atomic masses.[14]

RESULTS

The collected results are contained in Table II. The values derived from plate 34 have been rejected from the means. This plate was obtained with the gas in the tube at unusually low pressure, in fact, the discharge ceased immediately after the exposure had been made. The low pressure in the tube may account for the abnormally high intensity of the third component. With this high intensity there still remained distortions in the residual curve for the main components of $H^2\alpha$ that could be removed only by increasing the third as well as the fourth and fifth components to such an extent that the residual curve would have had little value. In $H^1\alpha$ this difficulty did not appear but owing to the greater temperature broadening in this case it might well be concealed. These factors together with the strongly discrepant value of the separation derived from the second component were considered sufficient reason for excluding this plate from the final averages.

The results which we accept as most probable are contained in the last section of Table II. The intensities of the faint components recorded in this section are based on the discussion of intensities given in the succeeding paper.

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[12] Kenneth T. Bainbridge, Phys. Rev. **44**, 57 (1933).

[13] W. V. Houston, Phys. Rev. **30**, 608 (1927).

[14] I am indebted to Professor Birge for suggesting this correction to the value of the Faraday.

TABLE II. *Spectroscopic data and values of e/m.*

	Plate number	31	33	34	40	41	42	Mean
4th=0 5th=0	Intensity of 3rd	1.28	1.51	2.33	1.48	1.44	1.24	
	Separation, from 1st	6.481	6.487	6.487	6.485	6.487	6.485	
	e/m	1.7593	1.7576	1.7576	1.7580	1.7574	1.7580	1.7581±2
	Separation, from 2nd	6.484	6.484	6.475	6.481	6.487	6.485	
	e/m	1.7585	1.7585	1.7609	1.7590	1.7574	1.7580	1.7583±2
4th=1 5th=1	Intensity of 3rd	1.44	1.75	2.57	2.00	2.04	1.78	
	Separation, from 1st	6.481	6.486	6.487	6.488	6.490	6.486	
	e/m	1.7593	1.7579	1.7576	1.7571	1.7566	1.7577	1.7577±3
	Separation, from 2nd	6.484	6.483	6.475	6.482	6.488	6.485	
	e/m	1.7585	1.7588	1.7609	1.7589	1.7571	1.7580	1.7582±2
5th=1	Intensity of 3rd	1.42	1.72	2.56	1.92	2.00	1.73	
	Intensity of 4th	0.58	0.69	0.60	0.82	0.88	0.84	
	Separation, from 1st	6.481	6.486	6.487	6.487	6.490	6.486	
	e/m	1.7593	1.7579	1.7576	1.7574	1.7566	1.7577	1.7578±3
	Separation, from 2nd	6.484	6.484	6.475	6.482	6.489	6.487	
	e/m	1.7585	1.7585	1.7609	1.7588	1.7569	1.7574	1.7580±3

In calculating the probable error of the definitive result, we must take account of the uncertainty in the intensities of components 4 and 5. Lacking definite information we have assumed rather arbitrarily that the probable errors of these were in each case one-half the intensities themselves. A comparison of the means from the first and second sections of the table reveals a difference of 3 in the last place, corresponding to a difference in the strengths of components 4 and 5 equal to their theoretical values. In treating the fourth and fifth components together in this manner it might be suspected that there is some compensation between the effects of the two taken separately. Since their actual intensities need not differ from the theoretical values in the same ratio, this combined treatment might lead to a too optimistic view of the probable error. To test this, all the calculations were made omitting the fifth entirely and including the fourth. The result showed that the fifth alone did not change the separation by as much as one in the last place. Thus there is no appreciable compensating effect of the type mentioned. We may, therefore, take 1.5 in the last place to be the probable error arising from the assumption of theoretical intensity. This combined with the accidental probable error and the probable error of the Faraday, which is one part in 12,000, gives the resultant probable error of 2.7 in the last place.

Therefore, the definitive value is

$$e/m = 1.7579 \pm 0.0003 \times 10^7.$$

Dividing $e/M_{\mathrm{H}}(9576.7 \pm 0.8)$ we obtain

$$M_{\mathrm{H}}/m = 1835.6 \pm 0.2_5.$$

It should be noted in this last result that as the Faraday has been removed, it is no longer necessary to include its probable error.

An inspection of the table shows that the results derived separately from the first and second components agree within the limits of accidental error. This gives some encouragement to the belief that the values derived are fairly free from systematic error. When in addition it is considered that the plates were taken under a variety of conditions of temperature and pressure with resulting strong variations in the intensities and sharpness of the various components and that no appreciable dependence of the results on these conditions is evident, we feel justified in thinking that the calculated probable errors describe closely the actual uncertainties in the results.

The assistance of Dr. Norman S. Grace who collaborated during the early stages of the work is gratefully acknowledged.

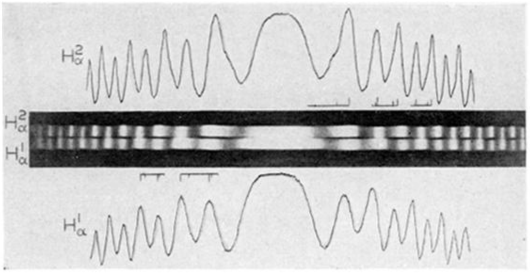

FIG. 1. Spectrograms and microphotometer curves of $H^2\alpha$ and $H^1\alpha$.