units). This datum may be used to evaluate the mass of N^{15} as follows:

$$\begin{aligned} N^{15} &= N^{14}+D^2-H^1-(\text{transmutation energy})\\ &=14.0069+2.0136-1.0078-0.0086\\ &=15.0041. \end{aligned}$$

This is to be compared with the value derived from band spectra data by Birge and Menzel[14] of 15.0032.

The 24 cm proton group corresponds to a kinetic energy release in the reaction of 3.3 mv, and the obvious interpretation is that when these protons are emitted, N^{15} is left in a 4.7 mv excitation level.

The experimental observations indicate that about a thirtieth as many protons as alpha-particles are emitted from nitrogen. In other words, the effective collision radius for the formation of N^{15} is about 4×10^{-15} cm.

The fact that the alpha-particle emission is considerably greater than the proton emission may be interpreted to indicate that the nuclear barrier (of C^{12}) for alpha-particles is less than the energy with which they are released (8 mv). This is quite in accord with the estimate of 3.6 mv for the nuclear barrier of carbon deduced from scattering experiments, as given by Pollard.[15]

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[14] R. T. Birge and D. H. Menzel, Phys. Rev. **37**, 1669 (1931).

[15] E. W. Pollard, Phil. Mag. **16**, 1131 (1933).

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Note on the Scattering of X-Rays from Fluids Containing Polyatomic Molecules

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The intensity of the (unmodified) scattered radiation from a fluid containing one kind of molecules is given by:

$$S=(1/\Sigma_n Z_n)\Big[F_1^2+F_2^2\int P_{(R)}dR(\sin kR)/kR\Big],$$

where $F_1^2=\Sigma_n\Sigma_m f_n f_m(\sin kr_{nm})/kr_{nm}$, while F_2^2 depends both upon the molecular structure and upon the relative orientation of the molecules. It is shown that the radial distribution function P_R may be obtained directly from the observed scattering curve if the molecular structure is known:

$$P_{(R)}=4\pi R^2\rho_0+\frac{2R}{\pi}\int_0^\infty k\left[\frac{S\Sigma_n Z_n}{F_1^2}-1\right]\frac{F_1^2}{F_2^2}\sin(kR)dk,$$

where ρ_0 is the density of the fluid in molecules per unit volume.

WHEN x-rays are scattered from a fluid containing only one kind of atoms (strictly speaking we should require the atoms to be equivalent as well), the intensity of the (unmodified) scattered radiation is given by[1]

$$S=(f^2/Z)\Big[1+\int 4\pi r^2\rho_0 w_{(r)}dr(\sin kr)/kr\Big], \quad (1)$$

where S is the intensity expressed in terms of the Thomson scattering. Z is the atomic number, f the scattering power of the atom and $k=4\pi\sin\theta/\lambda$. $4\pi r^2\rho_0 w_{(r)}dr$ (where ρ_0 is the density of the fluid in atoms per unit volume) represents the probability of finding an atom at a distance between r and $r+dr$ from any given one. As shown by Zernike and Prins[2] and by Debye and Menke[1], the radial distribution function may be obtained directly from the observed scattering curve, the result being:

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[1] P. Debye and H. Menke, Ergebn. d. Techn. Röntgenkunde **2**, 1 (1931); compare also reference 2.

[2] Zernike and Prins, Zeits. f. Physik **41**, 184 (1927).

$w_{(r)}=1+g(r)/4\pi r^2\rho_0$ where

$$g(r)=\frac{2r}{\pi}\int_0^\infty k\left[\frac{ZS}{f^2}-1\right]\sin(kr)dk. \quad (2)$$

Eq. (2) has been used with success in several instances.[1, 3, 4, 5]

The object of this note is to point out that the radial distribution function may be obtained directly also in the case of a fluid containing polyatomic molecules provided that the molecular structure is known.

We will assume that the fluid contains only one kind of molecules and that all orientations of any one molecule are equally probable. We find readily:

$$S=\frac{1}{\sum_n Z_n}\left[F_1{}^2+F_2{}^2\int 4\pi R^2\rho_0 W_{(R)}dR\frac{\sin kR}{kR}\right], \quad (3)$$

where Z_n is the number of electrons in the nth atom of the molecule, while $4\pi R^2\rho_0 W_{(R)}dR$ (ρ_0 now being density in molecules per unit volume) represents the probability of finding a molecule at a distance between R and $R+dR$ from any given one. The quantity $F_1{}^2$ is the familiar one

$$F_1{}^2=\sum_n\sum_m f_n f_m(\sin kr_{nm})/kr_{nm}, \quad (4)$$

while $F_2{}^2$ in general is different from $F_1{}^2$. The expression for $F_2{}^2$ depends both upon the molecular structure and upon the relative orientation of the molecules. If we assume that all molecules are parallel at any instant, $F_2{}^2$ becomes identical with $F_1{}^2$. In the other extreme case when the relative orientation is purely random, we find:

$$F_2{}^2=[\sum_n f_n(\sin kr_n)/kr_n]^2, \quad (5)$$

where f_n is the scattering power of the nth atom and r_n its distance from the center of the molecule. This latter expression for $F_2{}^2$ has been used for the special case of a diatomic molecule by Debye[6] and by Menke[3] for the special case of carbon tetrachloride. If the relative orientation is a restricted random one (as for example when the axes of the various molecules remain parallel), the expression for $F_2{}^2$ becomes more complicated. Quite generally, however, $F_2{}^2$ will decrease more rapidly with increasing k than $F_1{}^2$, so that the intermolecular interference (given by the integral term in Eq. (3)) at large values of k will become of less importance than otherwise might have been expected.

If the molecular structure is known, we may calculate $F_1{}^2$ and $F_2{}^2$, and Eq. (3) may be solved for the distribution function $W_{(R)}$ by means of the Fourier reciprocity relation.[7] We find readily:

$W_{(R)}=1+G_{(R)}/4\pi R^2\rho_0$ where

$$G_{(R)}=\frac{2R}{\pi}\int_0^\infty k\left[\frac{S\sum_n Z_n}{F_1{}^2}-1\right]\frac{F_1{}^2}{F_2{}^2}\sin(kR)dk. \quad (6)$$

If the symmetry of the molecule is fairly high (and also if the temperature is high), it is to be expected that the relative orientation of the molecules will be practically random, so that Eq. (5) may be used for $F_2{}^2$.

It has been customary hitherto to account for the diffraction pattern of fluids by assuming a radial distribution function and then calculate the intensity according to Eq. (3). The calculated and observed scattering curves were then compared. (This is for example the procedure followed by Menke in his paper on liquid CCl_4.)[3] By Eq. (6) a more direct method of attack is made possible.

It is worth while noticing that the integrand in Eq. (6) approaches zero with increasing k (the molecules scatter independently), so that the observations need not extend to extremely high values of k in order to give a reliable radial distribution function. This is, of course, in agreement with the well-known fact that the liquid scattering curve approaches the pure gas scattering curve as the scattering angle increases.

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[3] H. Menke, Phys. Zeits. **33**, 597 (1932).

[4] B. E. Warren and N. S. Gingrich, Phys. Rev. **46**, 368 (1934).

[5] B. E. Warren, J. Chem. Phys. **2**, 551 (1934).

[6] P. Debye, Phys. Zeits. **28**, 140 (1927).

[7] Menke's paper (reference 3) contains the statement that one cannot solve for $W_{(R)}$ because $F_1{}^2$ and $F_2{}^2$ are different. This statement must be due to an oversight.