where $-\Delta KE$ is positive and $-\Delta E_m$, a constant characteristic of each reaction, is either positive or negative.

On this basis Fig. 2 indicates

(4) that the total γ-ray energy emitted in the process rises rapidly as the kinetic energy of the bombarding neutron increases.

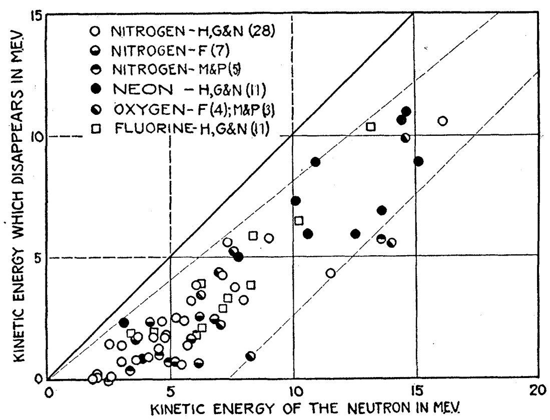

FIG. 2. H, G & N: Harkins, Gans and Newson. F: Feather. M & P: Meitner and Philipp.

A remarkable feature of Fig. 2 is

(5) the absence of points in the lower right hand part of the figure. Thus the γ-ray energy is always high if the neutron energy is high.

If the γ-rays represent quantized levels, then the values of the energy for any atomic species must lie, not on a single smooth curve, but on or near a series of horizontal straight lines, which would give a band as is found in Fig. 2. There may be, in addition, a continuous spectrum. As was pointed out earlier the present degree of experimental accuracy is not sufficient to resolve the definite energy values with any certainty.

The atomic nucleus is found to be a remarkably efficient machine for the conversion of large amounts of kinetic into γ-ray energy.

WILLIAM D. HARKINS
DAVID M. GANS

University of Chicago,
October 10, 1934.

[1] Fermi, Amaldi, D'Agostino, Rasetti and Segrè, Proc. Roy. Soc. **A146**, 483 (1934).
[2] Harkins, Gans and Newson, Phys. Rev. **44**, 945 (1933).

On the Modified Ritz Variation Method

Weinstein's results[1] on bounds for eigenvalues W of

$$(H-W)\psi=0 \qquad (1)$$

may be derived and extended by variational arguments which avoid explicit reference to expansion theorems with their continuous spectrum difficulties. Using Weinstein's notation we assume that self-adjointness of H and functional behavior near the singularities and boundaries[2] allow partial integration to give

$$\int(\alpha H\beta)d\tau=\int(\beta H\alpha)d\tau \qquad (2)$$

for functional forms α and β relevant to what follows. (Relation (2) appears to be necessary for application of the variation method.) Then for any constant V

$$I_m{}^1=\int[(H-V)^r\xi][(H-V)^{m-r}\xi]d\tau \quad \text{(integer } m)$$

will have the same value for all integers r in $0\leq r\leq m$.

We take variation re. ξ

$$\delta I_m{}^1=0 \quad \text{subject to } \int\xi^2d\tau=1 \text{ and the boundary conditions} \qquad (3)$$

as giving the discrete eigenvalues $\lambda^{(m)}$ of

$$[(H-V)^m-\lambda^{(m)}]\psi^{(m)}=0. \qquad (4)$$

Substitution shows that $\psi^{(m)}=\psi$ and $\lambda^{(m)}=(W-V)^m$ satisfy (4). Moreover these solutions are exhaustive for (4) if the solutions of (1) form a complete set of functions (in the general sense of the term)—a property which is assured in general apparently. We have therefore from (3) that for "arbitrary" ξ

$$I_m{}^1\geq(W_n-V)^m, \qquad (5)$$

where W_n is that discrete eigenvalue of (1) which gives the least $(W_n-V)^m$ for the chosen constant V (assuming that limit points of W's are avoided). There are two separate cases:

If $m=2l$ *is even* $|W_i-V|\leq(I_{2l}{}^1)^{1/2l}$, where W_i is that level closest to V. Here our desire is to have $I_{2l}{}^1$ as small as possible. Note that extremals re. V are given by $(\partial/\partial V)I_{2l}{}^1=0=-2lI_{2l}{}^1$ Weinstein's formulae represent the case $l=1$.

If $m=2l+1$ *is odd* $(-I_{2l+1}{}^1)^{1/(2l+1)}\leq V-W_0$,[3] provided that $I_{2l+1}{}^1\leq 0$; *or* $(I_{2l+1}{}^1)^{1/(2l+1)}\geq W_0-V$ provided that $W_0\geq V$. Note that here there is no extremal re. V since $(\partial/\partial V)I_{2l+1}{}^1=-(2l+1)I_{2l}{}^1=-(2l+1)\int[(H-V)\xi]^2d\tau\neq 0$ unless $(H-V)\xi=0$. The ordinary Ritz result represents the case $l=0$ with $V=\int\xi H\xi d\tau$.

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University of Toronto,
October 16, 1934.

[1] Weinstein, Proc. Nat. Acad. Sci. **20**, 529 (1934).
[2] See MacDonald, Phys. Rev. **43**, note (3), p. 830 (1933). Incidentally as a correction in Corollary 2.2, p. 832 replace s by $s+1$ and r by s.
[3] W_0 being the lowest eigenvalue of (1).

The Relation of the Primary Cosmic Radiation to the Phenomena Observed

In a recent paper[1] the writer, discussing the significance of J. Clay's[2] observations of the cosmic radiation down to depths of 270 meters below water, amplified a view suggested in former communications to the effect that the primary cosmic radiation of corpuscular kind produces no ions until by the production of showers (singles, doubles, and many-ray groups) it has fallen in energy below a certain critical value. On this view the primary corpuscular radiation is to be regarded as passing through our atmos-