where $-\Delta KE$ is positive and $-\Delta E_m$, a constant characteristic of each reaction, is either positive or negative.

On this basis Fig. 2 indicates

(4) that the total γ-ray energy emitted in the process rises rapidly as the kinetic energy of the bombarding neutron increases.

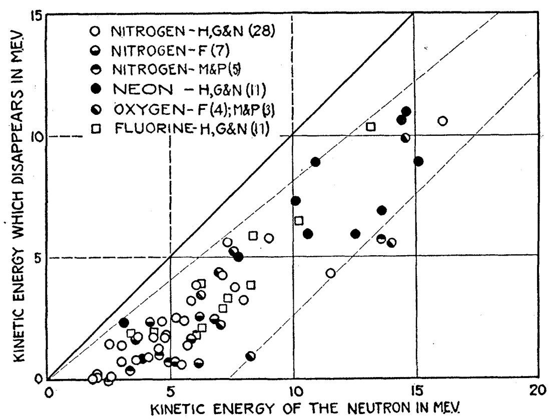

FIG. 2. H, G & N: Harkins, Gans and Newson. F: Feather. M & P: Meitner and Philipp.

A remarkable feature of Fig. 2 is

(5) the absence of points in the lower right hand part of the figure. Thus the γ-ray energy is always high if the neutron energy is high.

If the γ-rays represent quantized levels, then the values of the energy for any atomic species must lie, not on a single smooth curve, but on or near a series of horizontal straight lines, which would give a band as is found in Fig. 2. There may be, in addition, a continuous spectrum. As was pointed out earlier the present degree of experimental accuracy is not sufficient to resolve the definite energy values with any certainty.

The atomic nucleus is found to be a remarkably efficient machine for the conversion of large amounts of kinetic into γ-ray energy.

WILLIAM D. HARKINS
DAVID M. GANS

University of Chicago,
October 10, 1934.

[1] Fermi, Amaldi, D'Agostino, Rasetti and Segrè, Proc. Roy. Soc. **A146**, 483 (1934).
[2] Harkins, Gans and Newson, Phys. Rev. **44**, 945 (1933).

On the Modified Ritz Variation Method

Weinstein's results[1] on bounds for eigenvalues W of

$$(H-W)\psi=0 \tag{1}$$

may be derived and extended by variational arguments which avoid explicit reference to expansion theorems with their continuous spectrum difficulties. Using Weinstein's notation we assume that self-adjointness of H and functional behavior near the singularities and boundaries[2] allow partial integration to give

$$\int(\alpha H\beta)d\tau=\int(\beta H\alpha)d\tau \tag{2}$$

for functional forms α and β relevant to what follows. (Relation (2) appears to be necessary for application of the variation method.) Then for any constant V

$$I_m{}^1=\int[(H-V)^r\xi][(H-V)^{m-r}\xi]d\tau \quad \text{(integer } m\text{)}$$

will have the same value for all integers r in $0\leq r\leq m$.

We take variation re. ξ

$$\delta I_m{}^1=0 \quad \text{subject to } \int\xi^2 d\tau=1 \text{ and the boundary conditions} \tag{3}$$

as giving the discrete eigenvalues $\lambda^{(m)}$ of

$$[(H-V)^m-\lambda^{(m)}]\psi^{(m)}=0. \tag{4}$$

Substitution shows that $\psi^{(m)}=\psi$ and $\lambda^{(m)}=(W-V)^m$ satisfy (4). Moreover these solutions are exhaustive for (4) if the solutions of (1) form a complete set of functions (in the general sense of the term)—a property which is assured in general apparently. We have therefore from (3) that for "arbitrary" ξ

$$I_m{}^1\geq(W_n-V)^m, \tag{5}$$

where W_n is that discrete eigenvalue of (1) which gives the least $(W_n-V)^m$ for the chosen constant V (assuming that limit points of W's are avoided). There are two separate cases:

If $m=2l$ *is even* $|W_i-V|\leq(I_{2l}{}^1)^{1/2l}$, where W_i is that level closest to V. Here our desire is to have $I_{2l}{}^1$ as small as possible. Note that extremals re. V are given by $(\partial/\partial V)I_{2l}{}^1=0=-2lI_{2l}{}^1$ Weinstein's formulae represent the case $l=1$.

If $m=2l+1$ *is odd* $(-I_{2l+1}{}^1)^{1/(2l+1)}\leq V-W_0$,[3] provided that $I_{2l+1}{}^1\leq 0$; *or* $(I_{2l+1}{}^1)^{1/(2l+1)}\geq W_0-V$ provided that $W_0\geq V$. Note that here there is no extremal re. V since $(\partial/\partial V)I_{2l+1}{}^1=-(2l+1)I_{2l}{}^1=-(2l+1)\int[(H-V)\xi]^2d\tau\neq 0$ unless $(H-V)\xi=0$. The ordinary Ritz result represents the case $l=0$ with $V=\int\xi H\xi d\tau$.

J. K. L. MACDONALD

McLennan Physical Laboratory,
University of Toronto,
October 16, 1934.

[1] Weinstein, Proc. Nat. Acad. Sci. **20**, 529 (1934).
[2] See MacDonald, Phys. Rev. **43**, note (3), p. 830 (1933). Incidentally as a correction in Corollary 2.2, p. 832 replace s by $s+1$ and r by s.
[3] W_0 being the lowest eigenvalue of (1).

The Relation of the Primary Cosmic Radiation to the Phenomena Observed

In a recent paper[1] the writer, discussing the significance of J. Clay's[2] observations of the cosmic radiation down to depths of 270 meters below water, amplified a view suggested in former communications to the effect that the primary cosmic radiation of corpuscular kind produces no ions until by the production of showers (singles, doubles, and many-ray groups) it has fallen in energy below a certain critical value. On this view the primary corpuscular radiation is to be regarded as passing through our atmosphere with a density (measure by *numbers* of rays per square centimeter, per second) which is independent of altitude. The apparent increase of intensity with altitude becomes really an increase in the number of secondaries (showers) resulting from increase of the energy (not number) of the individual primary rays with altitude. The purpose of this note is to amplify further this view. To avoid complexity we shall retain only the fundamental elements and shall confine ourselves to the approximation that the range of the secondaries is small compared with the effective thickness of the atmosphere, so that the ionization at any altitude is determined by the number of secondaries produced per centimeter per second at that altitude. We shall consider that the showers are all of the same kind.

We shall first consider the simplest case, where the number of showers N_s produced per cc by the primaries falling in unit solid angle, at the angle θ to the vertical is

$$N_s=\alpha E, \tag{1}$$

where E is the energy of the rays and α is a constant. Since the rays lose energy by producing the showers, if x is an element of length measured along the path of the primary

$$dE/dx=-\beta N_s=-\beta\alpha E \tag{2}$$

when β is a constant.
Thus

$$E=Be^{-\beta\alpha x} \tag{3}$$

and, from (1) $N_s=\alpha Be^{-\beta\alpha x}$, so that the *observed* intensity in the direction determined by θ follows an exponential law with a coefficient of absorption $\beta\alpha$ although the primaries are *constant* with altitude as regards number per square centimeter per second.

We shall now take the next simplest case, where (1) is replaced by the more general form

$$N_s=\alpha_1 E+\alpha_2 E^2+\cdots \text{ etc.}, \tag{4}$$

in which α_2, α_3, etc. are such that the corresponding terms are small compared with the term $\alpha_1 E$. Eq. (2) then becomes replaced by

$$\frac{dE}{dx}=-\beta N_s=-\beta\alpha_1 E-\beta\alpha_2 E^2+\cdots \text{ etc.} \tag{5}$$

Solving this approximately by neglecting all terms involving higher powers of E than the first, we obtain, as in (3),

$$E=B_1 e^{-\beta\alpha_1 x}.$$

Substituting this in the terms of (5) involving E^2 and higher powers, we have

$$dE/dx+\mu E=-\beta B_1{}^2\alpha_2 e^{-2\mu x}-\beta B_1{}^3\alpha_3 e^{-3\mu x}+\cdots \text{ etc.},$$

where $\mu\equiv\beta\alpha_1$. The solution of this takes the form

$$E=Be^{-\mu x}+(\beta B^2\alpha_2/\mu)e^{-2\mu x}+(\beta B^3\alpha_3/2\mu)e^{-3\mu x}+\cdots \text{ etc.} \tag{6}$$

and N_s, and so the measured intensity for the direction concerned, takes the form

$$N_s=\alpha_1 Be^{-\mu x}+2\alpha_2 B^2 e^{-2\mu x}+\cdots \text{ etc.}$$

The significant thing is that, starting with a primary radiation whose intensity (as measured by numbers of rays) is independent of altitude, we have arrived at an expression for the measured intensity which involves a contribution from a series of terms with absorption coefficients μ, 2μ, 3μ, etc. However, our hypothesis as to the smallness of α_2, α_3, etc., would presumably render these "harmonics" unobservable, and the fundamental would correspond to that coefficient $\mu=0.5$ per meter of water, cited by Millikan as representative of 90 percent of the radiation observed.

Bearing upon variation of Stoss production in lead with altitude

As part of a general program of investigation of Stoss production, in this laboratory, Dr. and Mrs. Montgomery have measured the rate of production of Stösse in lead at the summit of Pike's Peak, at Colorado Springs, and at sea-level. The results, as yet unpublished, give a large increase with altitude, an increase much larger even than would correspond to the absorption coefficient of the softest component of the measured cosmic radiation. Now by considering matters in the light of the theory given above, I think it is possible to see, in a natural way, how this may come about. Suppose that in the fundamental Eq. (4) which tells the story of Stoss production as a function of energy, the coefficients α_1, α_2, α_3, are functions of the atomic number A of the material in which the Stösse are produced. Suppose, for example, α_2 were proportional to A^n, and the other coefficients were independent of A. Then it might well be that while the $\alpha_2 E^2$ term was negligible for air it might be all important for lead. The story of the variation of E with altitude would go on as before and would involve only the constants characteristic of air. When, however, we came to calculate the rate of production of the Stösse in lead, it would be the α_2 term which was now significant, so that the Stoss production at two altitudes would be determined by the ratio of the values of E^2 at those altitudes,[3] and so by the ratios of the squares of the right-hand sides of (6) for the two cases. The preponderant term involving $e^{-\mu x}$ in (6) will give rise to a term $e^{-2\mu x}$ so that the rate of Stoss production with altitude in lead will be twice that in air. If the α_3 term in (4) increased rapidly with the atomic number, the Stoss production in lead would show a rate of increase with altitude three times that for the main component of the apparent cosmic radiation. In general we may expect all of the α's to be functions of the atomic number so that the actual story may be expected to be more complicated in algebraic form, but not in quantitative significance.

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October 17, 1934.

[1] W. F. G. Swann, Phys. Rev. **46**, 432 (1934).
[2] J. Clay, Physica **1**, 363 (1934).
[3] To be meticulously exact we should say "for any given direction."