The Nuclear Magnetic Moment of Caesium from the Polarization of Resonance Radiation

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Measurements on the polarization of resonance radiation in a magnetic field have been used to obtain the hyperfine separation constants of the $6\,^2P_{3/2}$ and $7\,^2P_{3/2}$ levels of caesium I. For a nuclear spin of 7/2 the hyperfine separation constant is 1.42×10^{-3} cm^{-1} for the $6\,^2P_{3/2}$ level and 4.86×10^{-4} cm^{-1} for the $7\,^2P_{3/2}$ level. From these values the nuclear magnetic moment is calculated as 2.40/1840 and 2.41/1840 Bohr magnetons, respectively. These agree well with the value 2.52/1840 obtained from the splitting of the lower $6\,^2S_{1/2}$ level. The theoretical and experimental curves for the polarization as a function of the magnetic field agree within experimental error when the shape of the exciting line in the source is taken into account.

INTRODUCTION AND THEORY

FROM an interval rule analysis of a large number of energy levels of the spark spectrum of caesium Kopfermann[1] has given $I=7/2$ as the most probable value for its nuclear spin. More recently Jackson[2] has confirmed this value in his work on the second doublet lines of the arc spectrum, λλ4555A and 4593A. His determination was based on a study of the intensity ratio of the two hyperfine components into which each of these lines are split due to the splitting of the ground level $6\,^2S_{1/2}$. From the measured splitting of this lower level and the value of the nuclear spin the $g(I)$ value for the nucleus can be calculated. It is of interest to compare the $g(I)$ calculated from this state with the values obtained from other states. With the exception of the $6\,^2P_{1/2}$ state for which a hyperfine separation has now been observed by Granath and Stranathan[3] the hyperfine splitting of the other states of the arc spectrum are too small to be resolved directly. It has been possible however to obtain the separation constants for both the $6\,^2P_{3/2}$ and the $7\,^2P_{3/2}$ levels indirectly by measuring the polarization of resonance radiation of the lines involving these levels in a magnetic field. This method has already been applied successfully to the $6\,^1P_1$ level of cadmium[4] and the $3\,^2P_{3/2}$ and $4\,^2P_{3/2}$ levels of sodium.[5]

The theory of this method of determining small separations has been discussed in detail in Parts I and II. That it is possible to determine small hyperfine separations by measuring the polarization of resonance radiation in a magnetic field depends on the fact that the upper hyperfine levels involved in the resonance line undergo a Paschen-Back effect, and that the intensities of the Zeeman components which determine the polarization depend on the applied magnetic field through the variable $\omega=g(J)eH/2mcA$; where A is the hyperfine separation constant of the upper levels. After calculating a theoretical curve of the polarization as a function of the magnetic field the value of A may be determined by fitting this curve to the experimental results, since A is the only variable parameter remaining in the function ω.

The theoretical polarization curves for nuclear spins of 3/2, 5/2, and 7/2 have been plotted in Fig. 1. The curve for $I=5/2$ was calculated by Ellett and Kellogg.[6] The abscissa for each has been so adjusted that the total splitting of the upper hyperfine levels is the same for each curve. These calculations have been made for the case of a magnetic field parallel to the direction of the incident light beam so that only perpendicular components are absorbed by the resonating atoms. The three curves coincide for large values of ω but differ appreciably for small values. This is to be expected for as the magnetic field is increased the interaction between the nucleus and the outer electron becomes small compared to the interaction of each with the field and as the interaction between the electron and the field is

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[1] Kopfermann, Zeits. f. Physik **73**, 437 (1932).

[2] D. A. Jackson, Proc. Roy. Soc. **A143**, 455 (1934).

[3] L. P. Granath and R. K. Stranathan, Phys. Rev. **46**, 317 (1934).

[4] N. P. Heydenburg, Phys. Rev. **43**, 640 (1933). Hereafter to be referred to as Part I.

[5] Ellett and N. P. Heydenburg, Phys. Rev. **46**, 583 (1934)—Part II. L. Larrick, Phys. Rev. **40**, 1041 (1932).

[6] Results were not published.

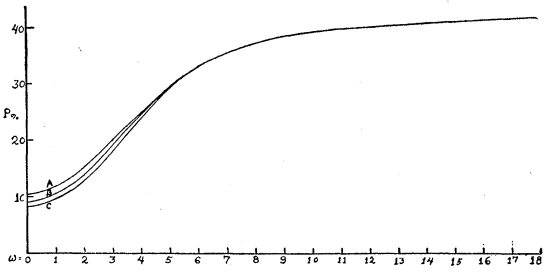

FIG. 1. Theoretical curves for the polarization of the transition $n\,^2P_{3/2}-m\,^2S_{1/2}$. Curve A, $I=3/2$; B, $I=5/2$; C, $I=7/2$.

much larger than that between the nucleus and field a change in the nuclear spin should have very little or no effect in this region.

For the polarization measurements to be described later the $^2P_{3/2}-{}^2S_{1/2}$ and the $^2P_{1/2}-{}^2S_{1/2}$ transitions are not separated, it is therefore necessary to take into account the latter component as well. In Part II it was proved that the polarization of this component is zero for all values of the nuclear spin and for all fields. The following equation has been derived giving the polarization for both components in terms of the polarization of the $^2P_{3/2}-{}^2S_{1/2}$ transition:

$$P(D_1+D_2)=3yP(D_2)/[3(y-1)-P(D_2)], \quad (1)$$

where $P(D_2)$ is the polarization of the $^2P_{3/2}-{}^2S_{1/2}$ transition and y is the product of the intensity ratio of the D_1 and D_2 lines in the source with the absorption ratio of these components for the atoms in the resonance chamber. For the case of the first doublet lines the ratio normally has the value 2 hence y is equal to 4 and the above equation reduces to

$$P(D_1+D_2)=12P(D_2)/(15-P(D_2)). \quad (2)$$

EXPERIMENTAL DETAILS

The experimental set-up was similar to that used in Parts I and II. The same type of source was used with a low pressure of hydrogen circulating through the tube, it was operated at a current of 0.5 ampere supplied by a 11,000 volt transformer. Cs vapor was supplied to the discharge tube from a side tube heated by an electric furnace so wound that the lower part of the side tube occupied by the Cs was cooler than the part nearer the capillary in which the discharge took place. For the first doublet lines the tube was operated so that only a very faint blue resonance appeared down the side tube and there was almost no coloring in the section of the capillary used to excite resonance radiation. Under these conditions a Fabry-Pérot picture showed that the λ8521A line was very broad and somewhat self-reversed. It was found however that it was impossible to operate the tube at a sufficiently low Cs vapor pressure to eliminate the effect of self-reversal entirely and still obtain resonance radiation of usable intensity.

For measurements on the second doublet lines the Cs vapor pressure in the capillary was increased so that the section which excited the resonance radiation had a fairly intense blue appearance. With the source operating in this manner a Fabry-Pérot analysis of the λ4555A line showed that it was broad having a half value breadth of 0.0795 cm^{-1} with no self-reversal present.

Due to the rather high vapor pressure of Cs at room temperature it was necessary to construct the resonance chamber in such a way that the part containing the Cs metal could be kept at a temperature between 10° and 15°C at which the vapor pressure of Cs is 2.5×10^{-7} and 6.0×10^{-7} mm of mercury, respectively. The Cs was distilled into a side tube 4 cm in diameter and 12 cm long, attached to the lower part of the resonance chamber. This side tube was kept at the proper temperature by a water bath.

For the second doublet lines a higher vapor pressure of Cs was necessary in order to get sufficient intensity of the resonance beam. The resonance tube was modified somewhat so that the chamber itself could be kept at a higher temperature than the side tube to eliminate condensation on the walls. Optimum temperatures were found to be 75°C for the resonance chamber and 55°C for the side tube containing the Cs. Under these conditions the resonance beam was found to be sufficiently intense without showing any appreciable secondary resonance radiation.

A Helmholtz coil placed with its axis parallel to the direction of the incident light beam produced the magnetic field at the resonance tube. A large coil was used to counterbalance the earth's field.

The polarization measurements were made by the Cornu method as described in Part I. Eastman type 1–P spectroscopic plates were found to be most satisfactory for photographing the first doublet lines. Eastman type 50 plates were used for the second doublet lines.

Results

The results for the first doublet lines are plotted in Fig. 2. The circle dots are the experimental points for the polarization as a function of the magnetic field. Three theoretical curves have been plotted on the same graph for comparison. Curve 2 is calculated using for the hyperfine separation constant A the value 1.42×10^{-3} cm^{-1}. Curves 3 and 4 are for A values, respectively, lower and higher than this. In calculating these curves the shape of the exciting line in the source has been taken into account. The λ8521 line has two hyperfine components resulting from the separation of the lower $6\,^2S_{1/2}$ level. It was found that under the operating conditions used these components were somewhat self-reversed. The contour (intensity-frequency curves) of the components taken from microphotometer tracings of Fabry-Pérot patterns have been plotted in Fig. 3. The shapes of the components were the same within experimental error. The two dashed curves represent extreme contours (plotted on a linear scale in cm^{-1} and corrected to represent intensities) taken from several different orders of interference. It was found that these curves could be represented very closely as the result of superimposing

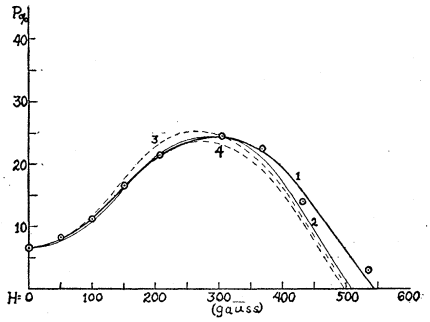

Fig. 2. Experimental and theoretical curves for the polarization of the first doublet lines. Curve 1 is the experimental curve. Curves 2, 3, and 4 are the theoretical curves with $A = 1.42 \times 10^{-3}$; 1.35×10^{-3}; and 1.50×10^{-3} cm^{-1}, respectively.

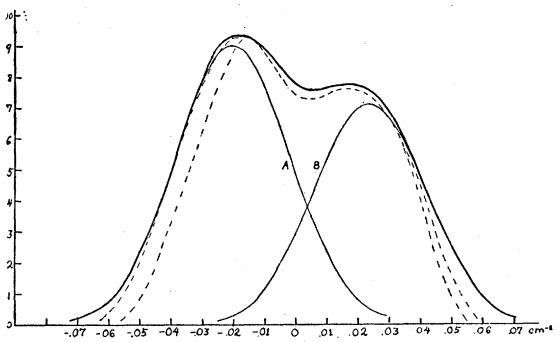

Fig. 3. Contour of the λ8521 line in the source. The dashed curves (experimental) are taken from Fabry-Pérot patterns. The heavy curve (theoretical) is the resultant of the two Doppler curves A and B.

two Doppler curves, the heavy line in Fig. 3 representing the superimposed curve. This fact simplifies considerably the calculation of the polarization. The method of taking into account the shape of the line in the source when the contour can be represented by Doppler curves has been discussed in Part I. The shape of the exciting line determines the relative population of the resonating atoms, the equation for determining these populations being:

$$I = \frac{B}{\nu_1 \nu_2} \exp\left[\frac{a_1^2 a_2^2 c^2 (\nu_2 - \nu_1)^2}{a_1^2 \nu_2^2 - a_2^2 \nu_1^2}\right] + \frac{B'}{\bar{\nu}_1 \nu_2} \exp\left[-\frac{\bar{a}_1^2 a_2^2 c^2 (\nu_2 - \bar{\nu}_1)^2}{\bar{a}_1^2 \nu_2^2 - a_2^2 \bar{\nu}_1^2}\right], \quad (3)$$

where $a_n^2 = M/2RT$, the subscript 1 applies to the atoms in the source and subscript 2 to the atoms in the resonance chamber. The two terms in this equation arise from the fact that two Doppler curves are superimposed.

In taking into account the unpolarized doublet component a value of y equal to 4 was used in Eq. (1). This implies using a 2 : 1 doublet intensity ratio for both the emitting atoms in the source and the absorbing atoms in the resonance chamber. It is doubtful that this large a ratio exists in the source because of the observed self-reversal. However the photographic plate used is known to have greater sensitivity for the λ8521 component than the λ8943 component. This effectively increases the doublet intensity ratio, and intensity measurements on these lines in the source revealed that this effective

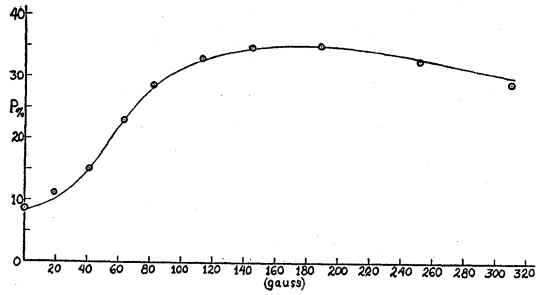

FIG. 4. Polarization curve for the second doublet lines. The theoretical curve is for an A value 4.86×10^{-4} cm^{-1}. The circled dots are the experimental points.

ratio is approximately 2 : 1. The theoretical polarization curve for A equal to 1.42×10^{-3} cm^{-1} is seen to agree well with the experimental curve up to about 350 gauss, beyond which the theoretical curve decreases too rapidly. However this region is not sensitive to changes in the A value but depends mostly on the contour of the exciting line. As the two dotted curves with A values 5 percent greater and less than 1.42×10^{-3} cm^{-1} lie definitely outside the limits of error of the experimental curve this value can be said to be accurate to within 5 percent.

In Fig. 4 is plotted the theoretical curve for the second doublet lines with a separation constant A equal to 4.86×10^{-4} cm^{-1}. This curve has been corrected for the line breadth in the source which causes the curve to change its slope for higher values of the field though here the effect is not as pronounced as in the first doublet line since the separation constant is much smaller. The circled dots are the experimental points. The doublet intensity ratio in the source was found to be 3.5 experimentally. The values for this ratio quoted by Korff and G. Breit[7] vary from 3.3 to 4.0. A value of 4 was taken for the absorbing atoms in the resonance bulb, therefore y in Eq. (1) is equal to 14. When the value of y is so large it no longer is very critical. For a field of 104 gauss, $P=31.81$ percent for $y=12$; and $P=32.04$ for $y=16$. As the agreement of the experimental points with the theoretical curve is good over the whole range of fields used the accuracy of the A value given for this level ($7\,^2P_{3/2}$) is about 3 percent.

The correction of the zero field polarization for overlapping of the hyperfine levels, discussed by Breit,[8] can be neglected since the hyperfine splittings observed here are much larger than those for Na. For either the first or second doublet the zero field polarization cannot be considered as sufficiently accurate to determine the value of the nuclear spin for Cs. For large values of the spin a change of unity produces only a small change in the calculated value of the polarization. Therefore from the present results it can only be said that the spin lies between 5/2 and 9/2. The value 7/2 may be considered as fairly well established from the results previously quoted.

TABLE I. *Magnetic moment of caesium.*

Level	$6\,^2S_{1/2}$	$6\,^2P_{3/2}$	$7\,^2P_{3/2}$
A	0.075 cm^{-1}	1.42×10^{-3}	4.86×10^{-4}
Z_i	55.	51.6	49.7
$g(I)$	0.72	0.68	0.69
$\mu(I)$	2.52	2.40	2.41

The final results are given in Table I. The $g(I)$ values have been calculated from the equations of Goudsmit[9] and Fermi.[10] The Z_i used were calculated from the experimental doublet separations by use of the equation

$$\Delta\nu = R\alpha^2 Z_0^2 Z_i^2/n^*l(l+1).$$

The $g(I)$ calculated for the different states are seen to agree very well. Due to the uncertainty in the theory used to calculate these values of $g(I)$ their close agreement should not be taken too seriously for the present.

In conclusion I wish to express my appreciation of the help given by Professor G. Breit, and to Dr. R. L. Garman of New York University for the microphotometer curves.

[7] Korff and G. Breit, Rev. Mod. Phys. **4**, 500 (1932).

[8] G. Breit, Rev. Mod. Phys. **5**, 122 (1933).

[9] Goudsmit, Phys. Rev. **43**, 636 (1933).

[10] E. Fermi and Segré, Zeits. f. Physik **82**, 729 (1933).