values. However, the mean of all the values of C_3 is $(5.70\pm0.06)\times10^{-14}$.

This value of C_3 can be used in Eq. (3) to compute an absolute value of the Kerr constant for any experimental conditions desired. The value of B computed for a wave-length of 5890A, a temperature of 17.5°C and a pressure of one atmosphere is $0.249\pm0.003\times10^{-10}$. The experimental value obtained by Szivessy[5] is 0.24×10^{-10}, while Bruce's[7] preliminary measurements yielded 0.28×10^{-10}. The value of B computed from a relation between the Kerr constant and the light scattering coefficient Δ[12] is 0.29×10^{-10} with Parthasarathy's[13] value of Δ. In view of the difference in the value of this quantity as obtained by different observers, the agreement between experimental and theoretical values of B may be said to be fair.

[12] See Brigleb and Wolf, Forts. der Ch., Phys. u. Phys. Ch. **21**, 155 (1931).

Acknowledgments

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[13] Parthasarathy, Ind. J. Phys. **7**, 148 (1932).

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Intensities of Satellites of $L\beta_2$

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By use of the Siegbahn x-ray spectrograph, the intensities, relative to the parent line, of the satellites of the x-ray line $L\beta_2$, have been studied in the atomic number range $40<Z<53$, inclusive. The "integrated intensity" (total area of satellite structure, comprising four lines) is about 4 percent of that of $L\beta_2$ for Cb(41); rises to a maximum of 52 percent for Ag(47); and then drops rapidly to a value too small to be measured (less than 1 percent) for Te(52). These results are in disagreement with the theory of Druyvesteyn (Diss., Groningen, 1928), which predicts a continuous decrease in satellite intensity with increasing Z.

Although numerous experimenters have reported the presence of "satellites"—those fainter lines on the short wave-length side of the diagram lines in the x-ray spectrum—there has been little study of them other than determinations of wave-length. Data on wave-lengths are to be found in Siegbahn's *Spektroskopie der Röntgenstrahlen*. R. D. Richtmyer[1] in studying the satellites of the $L\beta_2$ line observed that they are not found above atomic number 53. He noted qualitatively that there was a rapid variation in their intensity with a change in atomic number. Later Richtmyer and Kaufman[2] found that the satellites of this line appear again at about atomic number 73. Coster[3] reported the reappearance of one of the satellites of $L\beta_2$ at atomic number 68.

Richtmyer and Taylor[4] investigated the satellites of the $L\alpha$ lines for copper by the ionization chamber method and estimated their intensity relative to that of the parent line. They took as a measure of intensity the ratio of the maximum ordinates of the energy distribution curves of the satellites and parent line, respectively; it is now known that the ratio of areas is preferable. Their work, however, was important as a first attempt at getting such intensities and they suggested that a further similar study might furnish a clue as to the origin of these non-diagram lines. DuMond and Hoyt[5] also studied the same satellites and their results confirmed those of Richtmyer and Taylor.

[1] R. D. Richtmyer, Phys. Rev. **38**, 1802 (1931).
[2] Richtmyer and Kaufman, Phys. Rev. **44**, 605 (1933).
[3] Coster, Phil. Mag. **43**, 1070 (1922).
[4] Richtmyer and Taylor, Phys. Rev. **36**, 1044 (1930).
[5] DuMond and Hoyt, Phys. Rev. **36**, 1702 (1930).

A theory was advanced by Wentzel[6] that these lines originate in a single-electron transition between doubly ionized states. Druyvesteyn[7] assumed for the several satellites he studied, the double ionization states according to Wentzel's theory and from this theory, determined satisfactorily the wave-lengths of satellites of $L\beta_1$ and $L\beta_2$. But the theory did not apply well to those of $L\alpha$. He also derived a formula for the probability of such double ionizations. Since his equation for wave-lengths checked roughly with his observations, it might be expected that the second formula, based on the same classical assumptions, would also fit experimental data.

Richtmyer[8] has proposed as an alternative theory, a two-electron transition between doubly ionized states. This hypothesis is sometimes called the "double jump" theory of satellite origin.

It was with the hope that a knowledge of intensities would help to make clear the origin of these lines that the present work was undertaken. A study was made of the $L\beta_2$ line and its satellites for elements in the atomic number range 40 to 53 inclusive and the ratio of the intensities of the satellites to that of the parent line was obtained. Stated briefly, the results are that a marked variation in relative intensity occurs with change in atomic number. Beginning at $Z=40$, there is an increase up to a maximum for atomic number 47 and then a rapid decrease until the satellites disappear entirely at $Z=53$. The results do not even roughly agree with values of relative intensities computed from Druyvesteyn's formula.

A series of thirteen x-ray plates was made with a Siegbahn vacuum spectrograph with a calcite crystal. Elements from zirconium, atomic number 40, to iodine, 53 (except 43) were studied. Wherever possible the element in metallic form was used on the removable anti-cathode. In a few cases, a powdered form of the element was rubbed or pounded into the roughened copper wedge of the target. The tube was run at a voltage approximately five times that necessary for the excitation of the parent line. Tube current and time of exposure were varied so as to produce a plate in which the maximum photographic density was in the range 0.3 to 0.5 since below that density, intensities are proportional to photographic density which is defined by the relation: Density $=\log(1/T)$ where T is the transmission of the plate. Microphotometer records were made from each plate and these

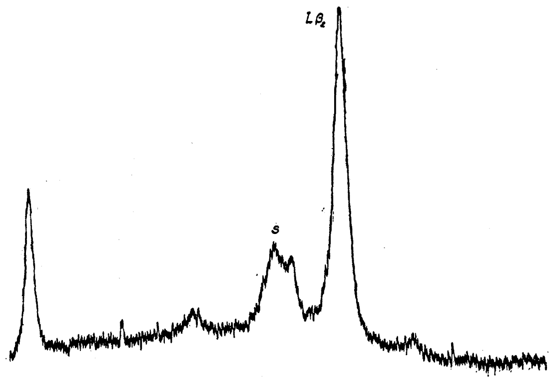

FIG. 1. Microphotometer record of $L\beta_2$ for Ag(47) showing the satellite structure at S.

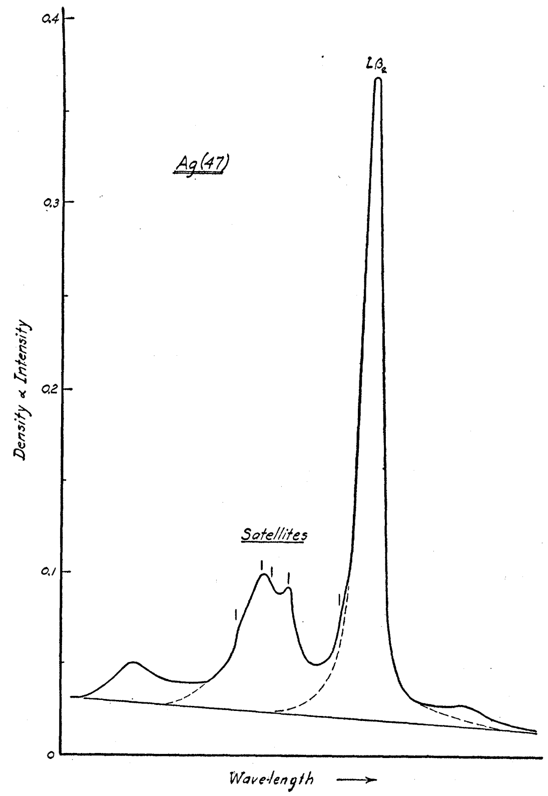

FIG. 2. Density plot for Ag(47).

[6] Wentzel, Ann. d. Physik **66**, 437 (1921).
[7] Druyvesteyn, Dissertation, Groningen, 1928.
[8] Richtmyer, Phil. Mag. **6**, 64 (1928).

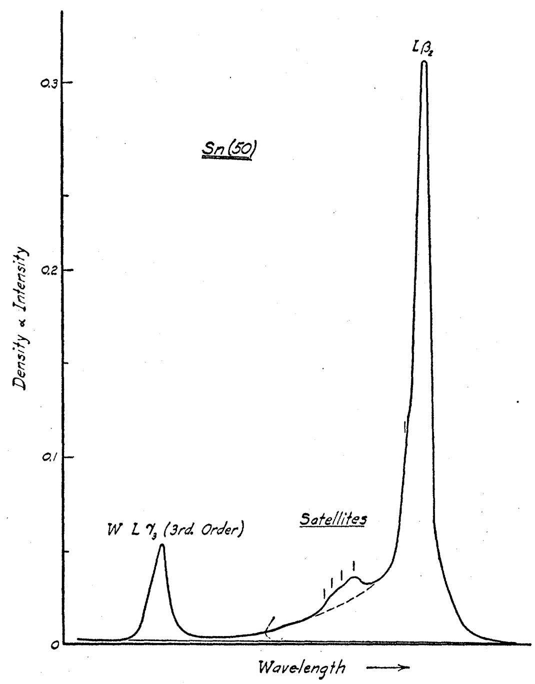

FIG. 3. Density plot for Sn(50).

records changed to intensity curves in the usual manner. Areas due to satellite structure and to the parent line were determined by planimeter. The ratio of these two areas gave the relative intensities sought. Since there is some doubt as to the form of the parent line on the short wavelength side, the area assumed for the satellites is at best only approximate. Therefore these results must be considered as only roughly quantitative. A more accurate method is needed for separating the satellite structure from that of the parent line.

Fig. 1 shows the microphotometer record for the plate for Ag (47) and Fig. 2 shows the intensity plot made from it. Note here the doubt as to the form of $L\beta_2$ and the guess made as to the boundary of the satellite area as shown by the dotted lines. For this element, we find the ratio of intensities of satellites and parent to be 52 percent, the maximum found in this atomic number range. To illustrate the rapid variation in relative intensity with atomic number, the density plot for Sn (50) is shown in Fig. 3. The results are shown in Table I and graphically by the full line in Fig. 4 where relative intensities are plotted against atomic number. The curve begins at about 5 percent for atomic number 40, rises gradually to a maximum in the vicinity of atomic number 47, then drops rapidly toward zero for atomic number 53.

Druyvesteyn's formula for the relative probability N_{LM}' of double (LM) to single (L) ionizations is:

$$N_{LM}' = An_M(Z-\alpha_M)^2/(Z-\gamma_M)^4, \quad (1)$$

where Z is the atomic number; γ_M is the total screening constant; α_M is the inner screening constant; n_M is the number of electrons in the M shell; A is a constant which depends upon the relative energies of the L and M ionized states and the initial energy of the cathode-ray electron as it strikes the target. To obtain the relative intensity R of satellites to parent line for a given element, the above ionization ratio should be corrected for the ratio P of probabilities of transitions between singly and doubly ionized states respectively. R is given by

$$R = PAn_M(Z-\alpha_M)^2/(Z-\gamma_M)^4. \quad (2)$$

Assuming that P does not vary rapidly from one element to another and that therefore to a first approximation it may be assumed constant, the value of PA was determined by using the observed value of R for Ag(47). The values of R for the other elements were then determined for the two cases where $\alpha_M = 10$ and $\alpha_M = 20$. Values of γ_M were taken from a curve in Pauling and Goudsmit.[9] The computed values of relative intensities are shown by the dotted lines in Fig. 4. It is readily seen that for no values of α_M do

TABLE I. *Values of ratio of intensities of satellite to parent line.*

Atomic number	Relative Intensities observed	computed for $\alpha_M=10$	$\alpha_M=20$
40	7.4%	151.0%	126.0%
41	4.3	126.0	108.5
42	11.2	107.0	94.0
44	26.8	78.0	73.0
45	32.8	67.7	64.5
46	46.9	59.1	57.8
47	52.2	52.2	52.2
48	13.2	45.2	46.1
49	8.9	40.6	42.2
50	3.5	36.6	38.5
51	2.8	32.9	35.1
52	—	29.6	32.5
53	0.0	27.1	29.9

[9] Pauling and Goudsmit, *Structure of Line Spectra*, p. 187.

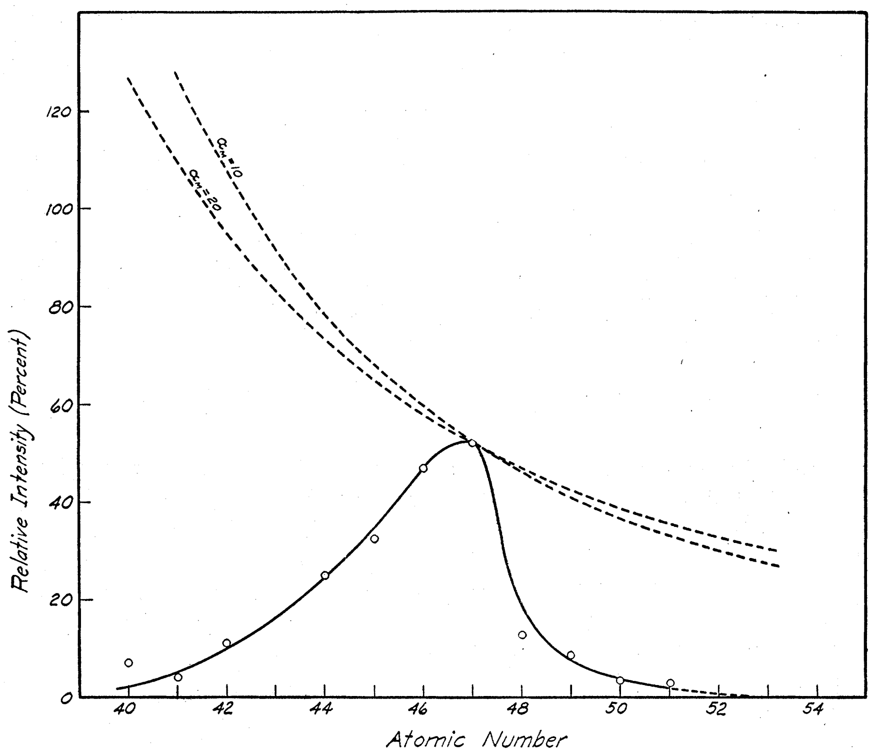

FIG. 4. Variation in intensities of satellites of $L\beta_2$ with atomic number (solid line). The dotted lines show relative intensities computed by Eq. (2). The arbitrary determination of the constant PA by use of the data for Ag(47) yields values of relative intensities many times greater than those predicted by Druyvesteyn's formula.

the theoretical curves even roughly coincide with the form of the observed curve. Obviously Eq. (2) does not even approximately agree with experimental facts.

In further disagreement with Eq. (2) is the work of Richtmyer and Barnes[10] who find the intensity of satellites of $L\beta_2$ for Au(79) to be 6.0 percent relative to that of $L\beta_2$. Eq. (2) predicts only a gradual decrease in relative intensity with increasing atomic number. It does not provide for the reappearance of the satellites and their increasing intensity in the high atomic number range. Similar facts have been observed for the satellites of $L\alpha$. Hirsh and Richtmyer[11] found a curve of intensities for $L\alpha$ similar to that in Fig. 4, with a maximum of 70 percent for $Z=45$. Richtmyer and Barnes[10] report the relative intensities of satellites of $L\alpha$ for Au(79) to be 5.0 percent. In each case the disappearance of the satellites is coincident with the completion of an electron shell.

Seemingly, the addition of outer electrons profoundly influences the production of satellites. At the present time however, there is no theory of satellite origin that can account for all the observed facts.

The writer wishes to express her sincere appreciation for the valuable advice and criticism given by Dr. F. K. Richtmyer during the progress of this research.

[10] Richtmyer and Barnes, Phys. Rev. **46**, 352 (1934).
[11] Hirsh and Richtmyer, Phys. Rev. **44**, 955 (1933).