Note on the Structure of the Compton Modified Band

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The distribution-in-wave-length of the intensity of scattered x-rays about the center of the Compton modified band gives directly the distribution-in-component-velocity of the electrons producing the band. The two distribution functions are of the same form.

IN 1925 de Broglie,[1] Wentzel[2] and Jauncey,[3] applying the principles of conservation of energy and of momentum to the scattering of an x-ray quantum by an initially moving electron, independently arrived at a formula which for an initial electron velocity small compared with that of light may be written in the form

$$L=\lambda(\beta_x \text{ vers } \phi-\beta_y \sin \phi) \tag{1}$$

and

$$L=\lambda'-\lambda-(h/mc\lambda) \text{ vers } \phi, \tag{2}$$

where λ and λ' are, respectively, the wave-lengths of the primary and scattered quanta, βc is the initial velocity of the electron, $\beta_x c$ is the component of this initial velocity in the direction of propagation of the primary quantum (the x-axis), $\beta_y c$ is the component in the scattering plane and in a direction (the y-axis) perpendicular to $\beta_x c$, and ϕ is the angle of scattering. The quantity L is the difference between the wave-length scattered by a moving electron and that of a quantum scattered by an electron at rest. Jauncey[3] making use of Eq. (1) showed that, when x-rays are scattered by electrons moving with equal speeds βc but in random directions, the width of the Compton band is

$$\delta\lambda=4\lambda\beta \sin \tfrac{1}{2}\phi \tag{3}$$

with an almost constant intensity distribution in the band. Proceeding from this Jauncey[4] then considered the case for scattering by electrons which have a speed distribution. The electrons whose speeds are between βc and $(\beta+d\beta)c$ give rise to a band of width given by Eq. (3). By an awkward process of graphical integration the intensity distribution in the Compton band was worked out for scattering by electrons in elliptic Bohr orbits. Later, DuMond[5] and Chandrasekhar[6] applied this method to the scattering of x-rays by electrons which have the Fermi-Dirac speed distribution. They both arrived at a parabolic shape for the curve representing the intensity distribution in the Compton band.

However, it is not necessary to integrate as Jauncey[4] and DuMond[5] have done. There is instead a very simple relation between the component velocity distribution of the electrons and the intensity distribution in the Compton band. DuMond[5] has written Eq. (1) in the form

$$L=2\lambda(u/c) \sin \tfrac{1}{2}\phi, \tag{4}$$

where u is the component of the initial electron velocity along the bisector of the angle between the direction of propagation of the primary quantum and the reverse of that of the scattered quantum. As a consequence of the linear relation between L and u as shown by Eq. (4), it is immediately seen that, if $f(u)\cdot du$ is the number of electrons with velocity components between u and $u+du$, the number of scattered quanta whose L's are between L and $L+dL$ is proportional to $f(Lc/2\lambda \sin \frac{1}{2}\phi)\cdot dL$. Hence, if the intensity distribution $I(L)$ in the Compton band is found by an instrument such as DuMond's multicrystal spectrograph,[5] the component velocity distribution function of the electrons is obtained by plotting $I(L)$ not against L but against $Lc/2\lambda \sin \frac{1}{2}\phi$. We then have the curve of $f(u)$ plotted against u. *The structure of the Compton band thus gives directly the graph of the component velocity distribution function of the electrons producing the band.* Since the Fermi-Dirac theory as applied by Sommerfeld to the

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conduction electrons in metals demands that the component velocity distribution at ordinary temperatures be parabolic, the structure of the Compton band scattered by these electrons is therefore also parabolic.

In conclusion, the point of this note was implicit in the author's papers[3,4] of 1925 and in DuMond's paper[5] of 1929 but so far as the author knows has not been stated explicitly before.

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Effect of Electron Binding upon the Magnitude of the Compton Shift

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Precision measurements of the Compton shift for ninety degree scattering of the wave-lengths 0.435A, 0.496A and 0.631A by carbon and by beryllium have been performed, using the ring-target x-ray tube and a double spectrometer, under the assumption that the mean shift for two mutually supplementary angles of scattering equals the ninety degree shift. In all cases the observed shift is less than h/mc, the value required by the Compton formula, the maximum percentage discrepancy being 2.4 percent, observed in the case of λ0.631A scattered from carbon. These observations fulfill the predictions inherent in Wentzel's theory of scattering by bound electrons, and are consistent with the explicit shift formula previously deduced by the present authors, as regards dependence of the shift upon wave-length and upon ionization energy of the scatterer. They are also in satisfactory quantitative agreement with the theoretical shift formulas of F. Bloch.

Introductory

The announcement of the observation of scattered x-rays of modified wave-length by A. H. Compton[1] in 1923 was accompanied and supported by a theory so concise and acceptable, and so nicely confirmed in a number of laboratories, that less attention has been paid in subsequent years to possible minor inaccuracies in the original description than would have been devoted to the aftermath of a discovery less completely set forth in the first instance. The early shift measurements of Sharp[2] and of Kallmann and Mark,[3] agreeing satisfactorily with the Compton equation $\Delta\lambda=(h/mc)(1-\cos\varphi)$, operated to establish this equation as correct within the limits of possible observation. More recent experiments by Nutting[4] have resulted in values of the 90° shift which are from one to two percent lower than the value h/mc required by Compton's equation. Nutting also remarked that the mean of all determinations available in 1930 was somewhat below h/mc but considered this to be a result of experimental error and concluded that the experiments as a whole were in agreement with the Compton formula.

The incompleteness of the type of treatment employed by Compton had been considered, however, by Wentzel,[5] in whose theory of scattering we may discern the implicit requirement that the wave-length modification occasioned by scattering by bound electrons shall be less than that expected of the practically free electrons of Compton. Wentzel's treatment does not provide an explicit shift formula, comparable in simplicity to the Compton equation given above. Such an equation has been deduced by the present authors[6] from relatively simple energy and momentum considerations, and Bloch[7] has derived a similar and more complete relation upon the broader basis of wave mechanical theory. The present paper offers experimental evidence relevant to this question, in the form of results of precision measurements of the wave-length shifts accompanying the scattering of radiations of three different wave-lengths by carbon and by beryllium.

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