THE

PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics

VOL. 46, NO. 8 | OCTOBER 15, 1934 | SECOND SERIES

Measurement of the Energy of a Beta-Ray of Radium B

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(Received June 4, 1934)

The energy of the most intense beta-ray line from radium B has been measured and found to be 2.6145×10^5 electron-volts, taking e/m to be 1.760×10^7 e.m.u. per gram. The value of $H\rho$ for this line was found to be 1931.8 gauss cm. The spectrum was obtained by the magnetic focussing method and the magnetic field was measured by the Cotton balance method. All measurements were made to within one part in ten thousand and it is hoped that the error in $H\rho$ is not greater than this. The value for the energy of course depends on the value of e/m and hence may be in error by about one part in one thousand.

INTRODUCTION

In 1924, Ellis and Skinner[1] determined the energies of the β-rays from radium B and they believed the results accurate to one part in five hundred. Recently further experiments have been done on these rays by Ellis.[2] The results of these experiments show that the 1924 values are too high by about one part in 150. For the most intense β-ray from radium B in 1924 Ellis and Skinner obtained a value of $H\rho$ of 1938 gauss cm. (H is the magnetic field strength, and ρ the radius of curvature of the path of the ray.) The recent results give the value of $H\rho$ for the same line as 1925.5 gauss cm. The present experiments were undertaken with the object of determining the value of $H\rho$ for this line with greater accuracy. All quantities have been measured to within one part in ten thousand and it is hoped that the value of $H\rho$ is not in error by more than this.

[1] Ellis and Skinner, Proc. Roy. Soc. **A105**, 165 (1924).
[2] Ellis, Proc. Roy. Soc. **A143**, 350 (1934).

APPARATUS

The β-ray spectrum of radium B was obtained by using the magnetic focussing method. The magnetic field was produced by a large permanent cobalt steel magnet. The poles of this magnet were square and 10 cm on each edge. The faces of these poles were ground until they were nearly flat. Pole pieces of mild steel were then placed in contact with the poles of the magnet. These pole pieces were cylindrical in shape, had a diameter of 15 cm and were 7.5 cm long. The surfaces of these pole pieces were flat to about one-thousandth of an inch. The faces of the pole pieces were adjusted parallel and then made vertical by means of a plumb line. The gap between them was 2.4 cm long. With the field used, 1282 gauss, the permeability of the pole pieces was about 1000 so that the field was assumed to be horizontal in the gap.

The strength of the magnetic field was measured by the Cotton balance method.[3] The Cotton balance consists essentially of two parallel

[3] Cotton and Dupouy, Congrès International d'Électricité, Vol. III, Sec. 2 E, 208 (1932).

vertical conductors connected at their lower ends by a horizontal conductor. These conductors are hung from a balance so that the horizontal conductor is perpendicular to the horizontal magnetic field to be measured. A measured current is passed through the conductors and the force on the horizontal conductor is measured with the balance. If the parallel conductors are exactly vertical, there is no vertical force on them so that the force measured by the balance is equal to the force on the horizontal conductor. The effective length of the horizontal conductor is equal to the distance between the two parallel vertical conductors. In this experiment these vertical conductors are supported by an optically worked glass plate so that they are very exactly straight and parallel. According to Cotton this method is capable of measuring the magnetic field with an accuracy much greater than one part in twenty thousand. It is therefore superior to the induction methods used by Ellis and others with which it is difficult to get an accuracy of one part in one thousand. The errors in the induction method are probably due to the difficulty of making coils of exactly known areas. In the Cotton balance the distance between the vertical conductors is comparatively easy to measure.

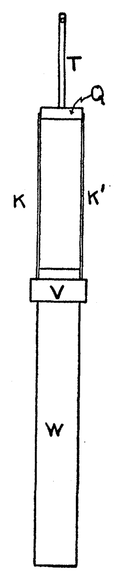

FIG. 1. Conductor support for Cotton balance.

An optically worked Pyrex glass plate $30\times 3\times 0.45$ cm was obtained from the Gaertner Scientific Company. The two long edges were plane and parallel to within one-five hundredth of a millimeter for a distance of 15 cm and then parallel to within one-one hundredth of a millimeter for the remainder of the distance. Thin silver strips of uniform thickness were fastened very closely to the sides of the plate with shellac and joined at the bottom. The strips were held very tightly against the glass in a specially constructed press while the shellac dried so that no appreciable thickness of shellac remained between the silver and the glass. Both sides of each silver strip formed a good plane mirror. At the top of the plate aluminum strips K and K' (Fig. 1) were held tightly against the silver by an ebonite block V. The upper ends of these aluminum strips were fastened to another ebonite block Q. From this second block extended two other aluminum strips T and T' which were attached to one arm of a sensitive balance. These aluminum strips fitted into supports on the balance which, when lowered on the knife edge of the balance, permitted the plate to swing freely both in the plane of the plate and in the plane at right angles to the surface of the plate. Light tin foil strips were attached to the aluminum strips K and K' at the block Q to supply current to the silver strips on the plate. These strips of foil were hung in vertical planes parallel to the horizontal component of the magnetic field (2.5 gauss) and brought out to an ebonite block placed in front of Q and at the same height as Q. The tin foil strips were about 10 cm long. The ends of the strips were at the same level but the middle points were about one centimeter below the ends. It can easily be shown that the very small resultant of the forces on the balance due to the action of the magnetic field on the current in these strips remains unchanged when the current is reversed so that no error is introduced by this force. Permanent connections were made to this third ebonite block by twisted leads coming from a reversing switch. The balance was mounted on a track above the magnet at such a height that the bottom of the glass plate came to the center of the pole pieces. By sliding the balance on the track it was possible to move the glass plate into or out of the field as desired.

The plate was surrounded by a brass box attached to the bottom of the balance case to prevent air currents from disturbing it. A mirror was mounted on the main knife edge of the balance and its deflections were observed by a lamp and scale about two meters from the mirror. The balance had a sensitivity of 19.5 mm per milligram and the scale could be read accurately to 0.2 mm by a hand magnifier. Since the value of the force measured was 200 milligrams, it could easily be measured to less than one part in ten thousand. The balance was so arranged that it could be rotated about a vertical axis through the center of the plate in order to determine the position of the plate for maximum force when the plane of the glass plate is perpendicular to the magnetic field. The plate was made vertical by a plumb line.

The current to the balance was supplied through a reversing switch which moved a lever and placed a 200 milligram weight on top of the glass plate when the current was reversed. The switch was attached to the base of the balance and was made so that the current in it gave no appreciable field. By the use of this arrangement the weight could be put on top of the glass plate and the current reversed without lifting the planes off the knife edges on the balance beam and without seriously disturbing the balance when the current was properly adjusted. The weight was of platinum and was standardized at the Bureau of Standards and had a value of 199.99 ± 0.01 milligrams. It had not been removed from its box since it was received.

The current to the glass plate was supplied by a lead storage battery. This battery consisted of two six volt 160 ampere hour capacity lead storage batteries connected in parallel. The batteries were first fully charged and then allowed to discharge until the voltage was nearly steady. Such a combination gave a very steady current.

The current to the plate was measured by means of a standard resistance and a Leeds and Northrup potentiometer with standard cell. The coils and slidewire of the potentiometer were tested for resistance variations and were found to be of the value designated. As a final check the three standard cells were checked against each other by means of the potentiometer and the potentials of the cells as measured by the instrument agreed with the certified values to less than one part in ten thousand. Two standard resistances furnished by Leeds and Northrup were used. One resistance was checked against the other in order to determine if any damage was done to either resistance during transit. The resistances each had a value of 4.0001 international ohms at 25°C correct to within 0.005 percent. The resistances had very small temperature coefficients which changed sign at about 25°C showing that the resistances had flat minimum values at this temperature. Since all measurements were made between 23°C and 25°C, it was not necessary to correct the resistances for temperature variations. It is scarcely necessary to mention that manganin standard resistances usually have the above characteristics which are in no way exceptional. These resistances were calibrated by the Bureau of Standards in February, 1934. The standard cells were calibrated by the Bureau of Standards in February, 1934, and were accurate to 0.01 percent at 24°C. Three such cells were used whose e.m.f.'s were, respectively, 1.01883, 1.01884, 1.01885 international volts. Since the relative values of these cells agreed with the Bureau's values to within one part in ten thousand it was assumed that the cells had not been damaged appreciably in transit. All measurements of length, current, etc., were done in two rooms whose temperature was maintained between 23°C and 25°C. With the small temperature variation it was unnecessary to correct the values of the resistance, standard cell potentials, or measurements of length for temperature variations to obtain the desired accuracy of one part in ten thousand. The current to the plate was adjusted to 0.25 international ampere in order to give the proper force in the magnetic field for the given weight.

The value of the acceleration of gravity used in determining the force from the standard weight was furnished by Dr. Blau of the Research Department of the Humble Oil and Refining Company, of Houston. This value of g is 979.28 $cm/sec.^2$ and is much more accurate than is necessary for the present experiments. It was determined by taking the value of g at a government station near Houston and correcting it for

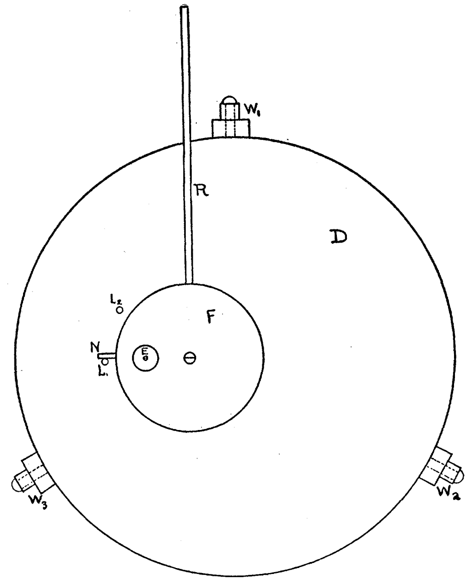

FIG. 2. Apparatus for measuring magnetic field variations.

the difference of position using the experimentally determined gravity gradients over the region in question.

The variations in the magnetic field were determined by means of a small coil of 1000 turns and diameter of 0.90 cm. This coil was attached to a high sensitivity low resistance galvanometer used as a fluxmeter which gave a deflection of 1 mm for a change of 0.094 gauss through the coil. The apparatus shown in Fig. 2 was designed to measure the variations in the field along the actual path of the β-rays. The large brass disk D was 15 cm in diameter and was fitted tightly over one of the pole pieces by screws through the blocks W_1, W_2, W_3. The coil E was fastened onto a smaller disk F which was so placed on the larger disk that if F were rotated 180° clockwise by the handle R the small coil followed the actual path of the β-rays in the magnetic field from M to O (Fig. 7). A small plug N was fastened to the side of F and by the use of a series of properly spaced stops screwed into the larger disk D it was possible to flip the coil through a small angle. Two of these stops, L_1 and L_2, are shown. The stops were so spaced that the coil could be flipped through an angle of 30° at a time and thus determine the difference in the strength of the field at two points on the electron path. The coil was placed initially at M and rotated along the path to O. By rotating the large disk D through 180° the field variations along the path from O to M' could also be mapped. The curve shown in Fig. 3 gives the angular variations of the field along the electron paths. δH represents the variations in the field from point to point in gauss, and θ is the angular distance along the path of the β-rays. The portion of the curve to the left of the axis gives the variations along the curved path OM and that to the right of the δH axis gives the variations in the field along the path OM' (Fig. 7).

The bottom of the glass plate attached to the balance for measuring the field came to O and extended for a distance of 1.5 cm on either side of O. Since the plate measured the average field over this distance and the field at O is desired, it was necessary to measure the variations in the field along the horizontal line through O. This was done by mounting the small coil on a bar which could be slid across the disk D. A plug in the side of the bar moved between stops 1 cm apart screwed into the surface of the disk. The curve of Fig. 4 gives these variations. The coil was placed at A, a distance of 1.5 cm from O and slid over a distance of one centimeter and the variation determined. The stops were then adjusted and the variation for the second centimeter obtained. A similar observation was made for the third centimeter. The average value of δH obtained from this curve is $+0.06$ gauss. The average field $\bar{H}$ measured by the glass plate is equal to the field at A plus the average δH, or $\bar{H} = H_A + 0.06$ gauss. Since the field at O is $+0.17$ gauss greater than H_A, we get finally $H_O = \bar{H} + 0.11$ gauss.

Hartree[4] has shown that if the magnetic field is not constant over the β-ray path, then the same point of impact would be obtained on the photographic plate by the electrons in a field of strength H given by $H = H_O + \frac{1}{2}\int \sin\theta\delta H\, d\theta$ where the integral extends over the path, H_O is the field at the β-ray source, θ is the angle turned through from the initial direction of propagation, and δH is the variation in the field from the point to point along the path. By plotting $\sin\theta\delta H$ against θ a curve is obtained for

[4] Hartree, Proc. Camb. Phil. Soc. **21**, 746 (1923).

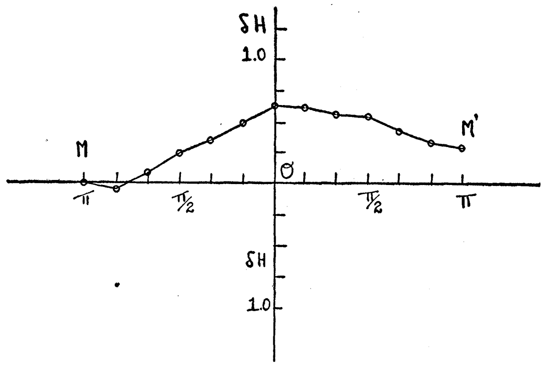

FIG. 3. Angular variations in magnetic field along paths of β-rays.

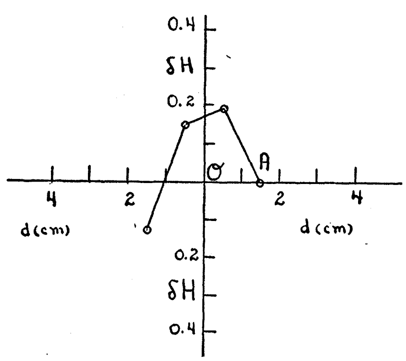

FIG. 4. Variations in magnetic field across center of field.

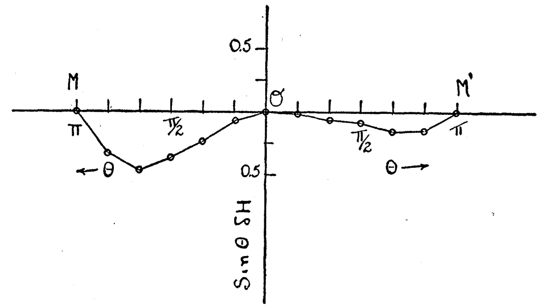

FIG. 5. Hartree correction curve for magnetic field variations.

each path, half the area under which gives the Hartree correction. Such a curve is shown in Fig. 5 and is obtained from the angular variation curve in the following manner. The point where the angular variation curve crossed the δH axis represents the field at O. The variations of δH with respect to O are then computed by taking the difference between the δH at O and that corresponding to some particular point on the path of the β-ray. Knowing the variations with respect to the field at O, the Hartree correction is easily plotted. The correction for the path OM computed from the curve is -0.385 gauss; that for the path OM' is -0.112 gauss. The average correction for the two paths is -0.248 or -0.25 gauss. The equivalent field is obtained from

$$H = H_O + \tfrac{1}{2}\int \sin\theta\delta H\, d\theta$$
$$= \bar{H} + 0.11 - 0.25$$
$$= \bar{H} - 0.14.$$

Thus -0.14 gauss represents the total correction which must be applied to the average field computed from the balance reading to obtain the equivalent field for the β-ray paths.

The equation for the motion of the β-rays along their paths in the magnetic field is

$$Hev = mv^2/\rho$$

whence

$$H\rho = \frac{mv}{e} = \frac{m_0}{e} \cdot \frac{v}{(1 - v^2/c^2)^{\frac{1}{2}}}.$$

The energy of such rays is obtained from these relations:

$$E = m_O c^2\{1/(1 - v^2/c^2)^{\frac{1}{2}} - 1\} \text{ ergs}$$
$$V = 299.8\{[(H\rho)^2 + (cm_0/e)^2]^{\frac{1}{2}} - (cm_0/e)\}\text{e. v.}$$
$$= 299.8\{[(H\rho)^2 + (1703.41)^2]^{\frac{1}{2}} - 1703.41\}.$$

The value of $\bar{H}$ is computed from the force and current used by the relation $\bar{H} = F/(i_1 + i_2)l$ where i_1 is the value of the current before reversal, i_2 the value of the current after reversal, l the mean distance between the centers of the silver strips on the two sides of the glass plate and F the force measured with the balance. The current through the balance was expressed in absolute electromagnetic units and the value of the force was determined from the standard weight and the acceleration due to gravity. In computing the energy of the β-rays the values $C = 2.9980 \times 10^{10}$ cm/sec. and $e/m = 1.760 \times 10^7$ e.m.u. per g were used.

The radius of curvature ρ was obtained by measuring the spectrum obtained on the photographic plates used in the apparatus (Fig. 7)

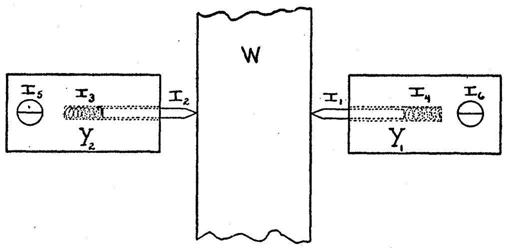

FIG. 6. Apparatus for measuring width of glass plate.

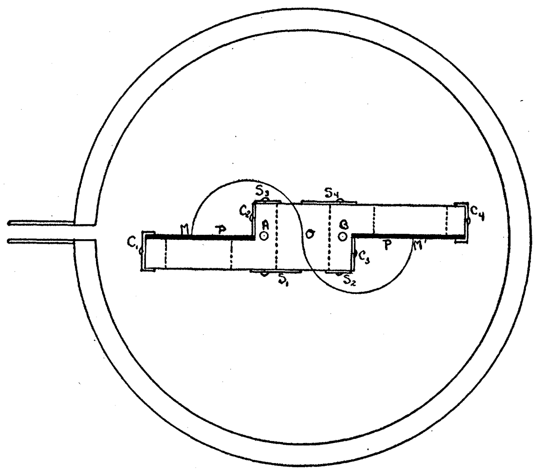

FIG. 7. Magnetic focussing apparatus for obtaining spectrum.

that is subsequently described. All measurements of length were made with a Gaertner comparator on which one scale division on the wheel corresponded to 0.0001 cm. Since only the ratio of lengths is required, it was not necessary to check the comparator against a standard of length. All measurements of length were made near the center of the screw of the comparator. For the desired accuracy of one part in ten thousand it was not necessary to consider possible errors in the screw.

The width of the glass plate without and with the silver was measured on the Gaertner comparator with the device shown in Fig. 6. Y_1 and Y_2 are brass blocks that are fastened to the bed plate of the comparator by blocks, I_5 and I_6. I_1 and I_2 are soft steel rods with rounded points that slide in holes in the blocks. Back of each of these rods in the holes is a light coiled spring I_3 or I_4. The glass plate W was fastened to the bed plate of the comparator and the blocks Y_1 and Y_2 adjusted until the points of the rods were just in contact with the edges of the plate but not pressed against the plate with any appreciable force. When the points were in contact with the edges of the plate, the image of these points could be seen in the plate. The cross hair of the travelling microscope of the comparator was then set on the junction of each of these points with its image and thus the width of the plate was measured. After the silver strip was applied to the plate similar measurements were again made. The silver strip had sufficient polish to give a good image of the points. This method gives the distance between the parallel reflecting planes and no error would be introduced if the points penetrated into the silver but the points were rounded and did not penetrate appreciably. In making these measurements, the plate W and the steel points were illuminated from above. Measurements were made on the width of the plate throughout its entire length, but the average of the measurements over only the first half of the length was used in computing the length of the current. This distance represents a portion of the plate, 7.5 cm of which are within the gap and 7.5 cm of which are outside the gap. Table I gives the measurements for the plate alone and for the plate with the strips for the same 15 cm of length. Each measurement represents an average of four readings. The average length of the current was obtained by taking the average distance between the centers of the silver strips and was 3.08253 cm which is probably accurate to within one part in ten thousand.

TABLE I. *Measurements of plate with and without silver strips.*

Distance from bottom of plate	Width of plate alone	Width of plate with strips
End	3.07105 cm	
1.5 cm	3.07109	3.09310 cm
3	3.07104	3.09434
5	3.07104	3.09424
7	3.07121	3.09416
9	3.07103	3.09453
11	3.07108	3.09312
13	3.07106	3.09445
	Av. 3.07107 cm	Av. 3.09399 cm

The average values of the widths with and without the silver strips are practically equal to the widths at 7 cm from the end of the plate where the magnetic field changes rapidly. The widths near the lower end of the plate and above 7 cm where the field is weak are not important. It was considered, nevertheless, proper to use the average width because small variations in the width may be due to errors in the measurements rather than to real variations in the width. The average deviation from the mean of the width without the strips is very small and that with the strips is 0.00050 cm so that the average deviation from the mean in the distance between the centers of the strips is 0.00025 cm or about one part in ten thousand.

In Table II is given a sample set of data observed for computing the value of H by means of the balance. E_1 and E_2 are the potentiometer readings across the standard resistance before and after reversing the current through the balance. The quantity δ represents the force that must be added to that of the standard weight to give the proper force. If the current had been of such a value as to give a force on reversal exactly equal to that of the standard weight, then δ would have been zero.

TABLE II. *Sample set of data.*

Reading of balance before reversing current = 24.05 cm
Reading of balance after reversing current = 23.43 cm
$\delta = 0.318$ milligram
$E_1 = 0.99270$ international volt
$E_2 = 0.99290$ international volt
$i_1 = 0.24816$ international ampere = 0.024813 abs. e.m.u.
$i_2 = 0.24821$ international ampere = 0.024818 abs. e.m.u.

$$\bar{H} = \frac{F}{(i_1+i_2)l} = \frac{(0.19999+0.000318)979.28}{(0.049631)\times 3.08253} = 1282.17 \text{ gauss}$$

Subtracting the correction from this value of H gives $1282.17 - 0.14 = 1282.03$ gauss.

A diagram of the apparatus used in obtaining the spectrum of the β-rays from radium B is shown in Fig. 7. The brass block $C_1\,C_4$ has a small hole drilled through it at O in which the source was placed. The source consisted of a platinum wire 0.25 mm in diameter which had been activated by exposing it to the emanation from 20 milligrams of radium. $P\,P$ are ordinary process plates whose sensitive surfaces are in line with the source at O. These plates are held firmly in place by clamps C_1, C_2, C_3, C_4. S_1, S_2, S_3, S_4 are adjustable slit jaws. Slots were cut in the block as indicated by the dotted lines. By using a frame of this type two lines are obtained, one on each photographic plate. The lines at M and M' are produced by the β-rays which describe semi-circular paths as shown in the diagram. Since the outer edges of the lines are very sharp, the radius of curvature of these lines can be readily determined by subtracting the diameter of the wire at O from the distance MM' and dividing by four. This distance MM' was about six centimeters in the field used. The lines were near the middle of the photographic plates and the possible shift due to drying the film was considered negligible. It was measured with the comparator; one line was seen directly and the other through the photographic plate and the slot in the block. The frame was placed in a brass box on two pegs which came through the frame at A and B.

The box was tested for magnetic impurities. This test was made with a coil of diameter 2.5 cm and 2750 turns connected to a fluxmeter which gave a deflection of 1 mm for a change of field of 0.05 gauss through the coil. This coil was mounted in the center of the air gap of the permanent magnet while the brass box and cover were placed in the gap, one part on either side of the coil, and then withdrawn. The net change in the flux through the coil was zero as shown by the fluxmeter, indicating that the material did not change the field appreciably. The brass box surrounding the glass plate on the balance was also tested in a similar manner by placing the box about the coil and then removing it. It also had no effect on the field.

The box containing the frame was placed between the poles of the large permanent cobalt steel magnet and exhausted by a Hypervac pump. The plates were exposed for three hours, the life of the active deposit, and then developed. This was accomplished by immersing the entire frame in the developer. It was found that the plates gave the best results when developed in process developer for 4 or 5 minutes and fixed and washed in the usual way. After they had dried they were measured with the Gaertner comparator and the value of ρ, the radius of curvature, determined. The average value of ρ obtained in this manner may be used since the field over each path is nearly the same. The following values of ρ were obtained with different plates over a period of two months: 1.50704, 1.50669, 1.50684, 1.50705, 1.50667, 1.50675. It will be seen that they agree very closely. This indicates that the magnetic field remained very constant, which is no doubt due to the fact that the magnet was kept at nearly constant temperature.

The results listed in Table III give a series of values for ρ and H obtained at nearly the same times. The last figures in these values have little real significance but were retained to avoid errors in the computations. The values of H have had the correction -0.14 gauss applied to them. Using the average values of H and ρ obtained one gets a value for $H\rho=1931.80$ and an energy $E=2.6145\times10^5$ electron volts for the particular β-ray under consideration taking e/m equal to 1.760×10^7 e.m.u. per gram. The value for the energy found depends on the value of e/m assumed and so may be in error by about one part in one thousand.

TABLE III. *Values of H and ρ.*

H	ρ
1282.03 gauss	1.50668 cm
1282.04	1.50705
1282.08	1.50668
1281.92	1.50701
1282.04	1.50675
1282.00	1.50686
Av. 1282.02 gauss	Av. 1.50684 cm

The probable error in the value of H computed in the usual way is ±0.00016 and that in the value of ρ is ±0.000044 so that $H=1931.80\pm0.12$. This method of measuring H has the advantage of using only that portion of the magnetic field within 3 cm of the center of the pole pieces where the variations in the field were very small. Likewise by using two semicircular paths it was possible to employ twice as strong a magnetic field as if only one semicircular path had been used. Cotton mentions that his balance method seems to give values lower than induction methods for measuring magnetic field strengths by 18 parts in ten thousand. If this is true then the result obtained for $H\rho$ may be lower than that to be expected from induction methods.

The writer wishes to express his indebtedness to Professor H. A. Wilson for suggesting the problem and for his interest and guidance during the progress of the work.