

effect of finite breadth of levels gives too low a zero field polarization (15.27 percent), but on taking into account the finite breadth of levels using  $\tau=1.64 \times 10^{-8}$  sec. the zero field polarization comes out 16.25 percent in fair agreement

with the observed value. The heavy curve in Fig. 2 is computed for a nuclear moment of  $3/2$  with  $A=7.45 \times 10^{-4}$  cm $^{-1}$ ,  $\tau=1.64 \times 10^{-8}$  sec.<sup>5</sup>

<sup>5</sup> R. Minkowski, Zeits. f. Physik **36**, 839 (1926).

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## Nuclear Magnetic Moments from the Polarization of Resonance Radiation

### Sodium, $3^2S_{1/2}-4^2P_{3/2, 1/2}$

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The polarization of sodium  $3^2S_{1/2}-4^2P_{1/2, 3/2}$  ( $\lambda 3303A$ ) resonance radiation excited by unpolarized light has been measured as a function of the intensity of a magnetic field applied parallel to the direction of the incident light beam. Detailed Paschen-Back effect calculations have been carried out for the hyperfine multiplet which consti-

tutes a  $2^2S_{1/2}-2^2P_{3/2}$  transition for a nuclear moment of  $3/2(h/2\pi)$ . From the fitting of the observed polarization field strength curves we find the hyperfine structure separation constant for  $4^2P_{3/2}$  to be  $1.87 \pm 0.05 \times 10^{-4}$  cm $^{-1}$ . The polarization field strength curve shows very clearly the effect of proximity of hyperfine levels in  $4^2P_{3/2}$ .

**I**N many cases the splitting of atomic energy levels due to nuclear spin is on a scale so small that the resolution of the hyperfine components of a spectral line is difficult or impossible. The splitting of the  $3^2S_{1/2}$  level of sodium has been observed by Schüler<sup>1</sup> and measured by L. Granath and C. M. Van Atta,<sup>2</sup> who also obtained values for the hyperfine separation in  $3^2P_{1/2, 3/2}$ . The separation of the hyperfine levels making up the  $4^2P_{3/2}$  and higher states is too small to be resolvable. However, the effect of a magnetic field upon the polarization of resonance radiation affords an indirect means of measuring these separations. The method has the advantage that it is not limited by the resolving power attainable with interference spectrometers, nor by Doppler breadth. In fact it is only necessary that the separations to be measured should be of the order of magnitude of the natural breadth of the spectral line in question. The existence of structure on a scale smaller than this is not to be expected for one could scarcely recognize levels as distinct unless separated by an amount comparable with their own indefiniteness. That the depend-

ence of the polarization of resonance radiation upon the intensity of an applied magnetic field should afford a means of measuring separations in hyperfine multiplets is due to the fact that such multiplets undergo a Paschen-Back effect. Intensities in the Paschen-Back effect are functions of a single variable,  $g(J)(eH/4\pi mcA)$  where  $A$  is the separation constant of the hyperfine multiplet. From this, as we shall see, it follows that the polarization of resonance radiation is a function of  $H$  and  $A$  only through the ratio  $H/A$ , and upon this depends the possibility of obtaining the value of  $A$  from observation of the polarization as function of  $H$ , since the very laborious Paschen-Back effect calculations have to be carried through only for a suitably chosen set of values of  $H/A$ . If the dependence were upon  $H$  and  $A$  separately the task of computing would be so great that one would hesitate to undertake it.

### THEORY

Where the separation of the levels making up a hyperfine multiplet is large compared to the natural breadth of an energy level, we may calculate polarizations by Heisenberg's method. For this the relative intensities of the Zeeman components of the hyperfine multiplet are re-

<sup>1</sup> H. Schüler, Naturwiss. **16**, 512 (1926).

<sup>2</sup> L. Granath and C. M. Van Atta, Phys. Rev. **44**, 935 (1933).

quired, and they may be calculated, as Goudsmit and Bacher<sup>3</sup> have pointed out, by a simple modification of the method applied to ordinary multiplets by Darwin.<sup>4</sup> The essential point for the present application is that the relative Zeeman intensities in the Paschen-Back effect depend upon a single argument which, as noted

above, is essentially the ratio of the Larmor frequency to the separation constant of the multiplet. That intensities are function of  $g(J)(eH/4\pi mcA)$  is immediately evident upon inspection of the chains of equations for the Fourier coefficients of the perturbed functions. These chains are

$$\begin{aligned} -X_{M_J-1, M_I+1}(A/2)(J-M_J+1)(I+M_I+1) + X_{M_J M_I}(E-A M_J M_I - M_J \omega) \\ - X_{M_J+1, M_I-1}(A/2)(J+M_J+1)(I-M_I+1) = 0, \end{aligned} \quad (1)$$

where  $\omega = g(J)(eH/4\pi mc)$ . The roots of the secular determinant are seen to be functions of  $\Omega = \omega/A$  only, and consequently on substituting these roots back into the chains and solving for the  $X$ 's we find that they depend upon  $A$  and  $H$  through  $\Omega$  only.

As an illustration we may show that  $3^2S_{1/2} - n^2P_{1/2}$  resonance radiation is unpolarized whatever the value of the nuclear moment may be and whatever the intensity of the applied magnetic field so long as it produces no Paschen-Back effect of the gross multiplet. That is to say, we take the unperturbed characteristic functions to be of the form

$$\Psi_{M_J M_I}^J = \Psi_{M_J}^J P_I^{M_I} (\cos \theta) e^{iM_I \Phi},$$

where  $\Psi_{M_J}^J$  is the characteristic function for the atom with no nuclear spin. Strictly speaking the  $\Psi_{M_J}^J$  are not independent of  $H$ , but as long as the Zeeman separations produced by the applied field are very small compared to the separation of the  ${}^2P_{1/2, 3/2}$  levels we may treat them as though independent of  $H$ . The perturbed characteristic functions may then be written

$$\Psi_{M_F}^{JF} = \sum X_{M_J M_I}^{JF} \Psi_{M_J M_I}^J,$$

where the summation is over all values of  $M_J$  and  $M_I$  such that  $M_J + M_I = M_F$ . For either  ${}^2P_{1/2}$  or  ${}^2S_{1/2}$  the possible values of  $M_J$  are  $\pm \frac{1}{2}$ . The chains of equations reduce to two members, except when  $M_F = \pm(I + \frac{1}{2})$  when there is only one. Writing  $M_F = M_J + M_I$  these are

$$\begin{aligned} -X_{-\frac{1}{2}, M_F+\frac{1}{2}}(A/2)(I+M_F+\frac{1}{2}) + X_{\frac{1}{2}, M_F-\frac{1}{2}}[E - (A/2)(M_F - \frac{1}{2}) - \omega/2] = 0, \\ X_{-\frac{1}{2}, M_F+\frac{1}{2}}[E + (A/2)(M_F + \frac{1}{2}) + \omega/2] - X_{\frac{1}{2}, M_F-\frac{1}{2}}(A/2)(I - M_F + \frac{1}{2}) = 0. \end{aligned} \quad (2)$$

The normalizing equation is

$$(X_{-\frac{1}{2}, M_F+\frac{1}{2}})^2 (I+M_F+\frac{1}{2})! (I-M_F-\frac{1}{2})! + (X_{\frac{1}{2}, M_F-\frac{1}{2}})^2 (I+M_F-\frac{1}{2})! (I-M_F+\frac{1}{2})! = 1.$$

If we write  $\epsilon = 2E/A$ ,  $\Omega = \omega/A$  the roots of the secular equation are

$$\epsilon = (-1 \pm \Delta)/2; \quad \Delta = [1 + 4\Omega(2M_F + \Omega) + 4I(I+1)]^{\frac{1}{2}}.$$

The  $X$ 's are

$$\begin{aligned} X_{\pm\frac{1}{2}, M_F \mp \frac{1}{2}}^{I+\frac{1}{2}} &= \left[ \frac{\frac{1}{2}\Delta \pm (M_F + \Omega)}{\Delta(I+M_F \mp \frac{1}{2})!(I-M_F \pm \frac{1}{2})!} \right]^{\frac{1}{2}}, \\ X_{\pm\frac{1}{2}, M_F \mp \frac{1}{2}}^{I-\frac{1}{2}} &= \left[ \frac{\frac{1}{2}\Delta \mp (M_F + \Omega)}{\Delta(I+M_F \mp \frac{1}{2})!(I-M_F \pm \frac{1}{2})!} \right]^{\frac{1}{2}}. \end{aligned} \quad (3)$$

<sup>3</sup> S. Goudsmit and R. F. Bacher, Phys. Rev. **34**, 1499 (1929).

<sup>4</sup> C. G. Darwin, Proc. Roy. Soc. **A115**, 1 (1927).

Labelling levels by their weak field quantum numbers  $F$  and  $M_F$  and writing transitions in the form  $\begin{bmatrix} F \rightarrow F' \\ M_F \rightarrow M_F' \end{bmatrix}$  the intensities of parallel transitions from  $F = I + \frac{1}{2}$  are

$$\begin{bmatrix} I + \frac{1}{2} \rightarrow I + \frac{1}{2} \\ M_F \rightarrow M_F \end{bmatrix} = \left[ \frac{(\frac{1}{2}\Delta + M_F + \Omega)^{\frac{1}{2}}(\frac{1}{2}\Delta' + M_F + \Omega)^{\frac{1}{2}} - (\frac{1}{2}\Delta - M_F - \Omega)^{\frac{1}{2}}(\frac{1}{2}\Delta' - M_F - \Omega)^{\frac{1}{2}}}{\Delta\Delta'} \right]^2, \quad (4)$$

$$\begin{bmatrix} I + \frac{1}{2} \rightarrow I - \frac{1}{2} \\ M_F \rightarrow M_F \end{bmatrix} = \left[ \frac{(\frac{1}{2}\Delta + M_F + \Omega)^{\frac{1}{2}}(\frac{1}{2}\Delta' - M_F - \Omega)^{\frac{1}{2}} + (\frac{1}{2}\Delta - M_F - \Omega)^{\frac{1}{2}}(\frac{1}{2}\Delta' + M_F + \Omega)^{\frac{1}{2}}}{\Delta\Delta'} \right]^2. \quad (5)$$

The sum of these is seen to be unity, and the same is readily found to be true for the sum of the parallel transitions from any sublevel of  $F = I - \frac{1}{2}$ . The total intensity (viewed perpendicular to the applied field) of the perpendicular components originating from any upper level is also readily found to be unity, as it obviously must be, since all levels have the same weight. Since the radiation from any upper magnetic sublevel shows no net polarization the line is unpolarized in resonance radiation. This result, it may be noted, is independent of the intensity distribution in the source.

The polarization of  ${}^2S_{1/2} - {}^2P_{3/2}$  resonance radiation for a nuclear moment of unity has been calculated by Larrick.<sup>5</sup> We have carried out similar calculations for  $I = 3/2$ . In this case the secular equation for  $M_F = 0$  is a biquadratic, while those for  $M_F = \pm 1$  are cubics. If the cubics are reduced to the so-called canonical form  $E^3 + aE + C = 0$  they become identical, so that the labor of calculation is considerably reduced. We find also that the  $X$ 's for  $M_F = \pm 1$  are related,  $X_{r,s}$  being the same as  $X_{-s,-r}$ . The reduction of the cubics to the same form and the relation between the  $X$ 's is a result of the symmetry introduced into the chains of equations when  $I$  and  $J$  are equal.

The energies (in terms of  $A$ ) have been calculated for  $\Omega = 0, 1, 2, 3, 4, 6, 10, 15$ . The roots of the secular determinants were calculated to an accuracy of  $\pm 1$  in the fifth significant figure while the intensities<sup>6</sup> of the parallel transitions in the hyperfine multiplet were obtained to an accuracy of one part in  $10^4$ .

Having the intensities of the Zeeman compo-

nents making up the hyperfine multiplet we may compute the relative populations of excited states produced by radiation of given polarization and intensity distribution. Emissions from these excited states then give the relative intensities of the emitted radiation polarized in various azimuths. This procedure is unambiguous as long as the separations in the hyperfine multiplet are large compared to the breadth of a level. When this condition is no longer satisfied the specification of the state of polarization and frequency of an absorbed quantum no longer suffices to determine the excited atomic state resulting from its absorption. The degeneracy which thus arises is evidently not without effect upon the polarization of the resonance radiation for as the coupling with the nuclear magnetic moment approaches zero we should expect in some way to pass over to the situation prevailing when there is no nuclear moment and no hyperfine structure. How this occurs and how in such cases the polarization may be calculated has been discussed by Breit.<sup>7</sup>

In order to form an estimate of the importance of this effect we may bear in mind that the  ${}^2S_{1/2} - {}^2P_{1/2, 3/2}$  doublet excited to resonance by unpolarized incident light may show anywhere from 10.48 percent to 42.8 percent polarization in zero magnetic field, the first being the value when line breadth is completely negligible, the latter that when the coupling to the nuclear spin is so slight as to produce no effect. Now the observed value is only slightly greater than 10.48 percent so that we may expect to represent our results fairly well without taking into account the finite breadth of levels, especially in view of the fact that the separation of magnetic levels due to the applied field will remove

<sup>5</sup> Larrick, Phys. Rev. **46**, 581 (1934).

<sup>6</sup> These numerical results, together with those for  $I = 5/2$  are on file in the Physics Library of the University of Iowa and are available for loan to any one who may have use for them.

<sup>7</sup> G. Breit, Rev. Mod. Phys. **5**, 91 (1933) especially Art. 4, p. 117 ff.

the degeneracy due to breadth of levels sometime before the Paschen-Back effect is complete.

In Fig. 1 the broken curve represents the polarization of resonance radiation for a  ${}^2S_{1/2} - {}^2P_{1/2, 3/2}$  doublet excited by unpolarized incident radiation (in a magnetic field parallel to the direction of the incident light beam), as a function of the argument  $\Omega$  for the case where

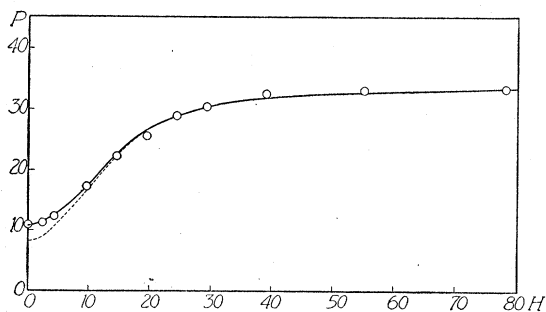


FIG. 1. Polarization as a function of magnetic field. Circles represent observations. Heavy curve computed with  $A(4^2P_{3/2}) = 1.87 \times 10^{-4} \text{ cm}^{-1}$ ,  $\tau = 3.13 \times 10^{-8} \text{ sec}$ . Broken curve computed neglecting effect of proximity of hyperfine levels.

the hyperfine separations are very large compared to the breadth of energy levels. The incident radiation is supposed to come from a source emitting broad lines with the intensity ratio in the doublet 2 : 1. On the same figure the circles are the observed values of polarization plotted against the intensity of the applied magnetic field.

With a proper choice of scales for  $\Omega$  and  $H$  the observed points may be made to coincide with the computed curve fairly closely for  $\Omega > 4$ . Since  $\Omega = g(J)(eH/4\pi mcA)$  we conclude that the constant  $A$  for the hyperfine multiplet  $4^2P_{3/2}$  is of the order of  $2 \times 10^{-4} \text{ cm}^{-1}$ . We may now expect to improve the agreement and obtain a better value for  $A$  by taking into account the two effects so far neglected—first that due to finite breadth of energy levels and second that due to the fact that our source, while emitting fairly broad lines, is nevertheless not quite uniform across the

absorption lines. As both these “corrections” are small we may treat them as though independent—that is, in computing the correction for intensity distribution in the source we will neglect the effect of finite breadth of energy levels, while in computing the change effected by the finite breadth of energy levels we will regard the intensity as uniform across the absorption line. Taking the simplest one first we consider the effect of non-uniform intensity.

#### EFFECT ON NON-UNIFORM SOURCE

Because of the very small hyperfine separation in  $4^2P_{3/2}$  the line  $3^2S_{1/2} - 4^2P_{3/2}$  is effectively a close doublet whose separation is nearly that of the  $3^2S_{1/2}$  level. This according to Granath and Van Atta is  $0.0583 \text{ cm}^{-1}$ . We take the line form to be adequately represented by the superposition of two Rayleigh curves spaced  $0.058 \text{ cm}^{-1}$ , having maximum ordinates in the ratio of the statistical weights of the lower levels, 5 : 3, and a breadth corresponding to a source temperature of  $1000^\circ\text{K}$ . With this line form the energy absorbed by an absorption line of unit intensity whose center lies at a frequency distant  $\nu_1$  and  $\nu_2$ , respectively, from the centers of the components of the doublet will be proportional to

$$3 \exp(-\alpha\nu_1^2) + 5 \exp(-\alpha\nu_2^2),$$

in which

$$\alpha = Mc^2/2\nu_0R(T_r + T_s),$$

where  $\nu_0$  is a mean frequency,  $T_s$  and  $T_r$  the temperatures of source and resonance lamp. The factors 3 and 5 take account of the relative weights of the two hyperfine levels making up  ${}^2S_{1/2}$ . The difference in values of the polarization computed taking into account the actual intensity distribution in the source and that computed on the assumption of a uniform intensity varies from +1 in the third significant figure in zero field, to -4 in the fourth figure in a field of 30 gauss. As the correction is less than the experimental error it has been neglected.

#### FINITE BREADTH OF ENERGY LEVELS

Breit<sup>7</sup> has discussed the effect of finite breadth in detail for the case of zero applied field. He finds, for the case of incident light plane polarized along the  $x$  axis, observation along  $z$

$$I_x = \sum_{\phi, \mu} \left( \frac{C}{g_a} \right) (A_{\pi\phi\mu})^2 + \sum_{f, f', \phi > \phi'} \left( \frac{2C}{g_a} \right) \frac{Sp[(e\hat{x})^{\phi f} (e\hat{x})^{f\phi'} (e\hat{x})^{\phi' f'} (e\hat{x})^{f'\phi}]}{[1 + (2\pi\tau\nu_{\phi\phi'})^2]}, \quad (6)$$

$$I_x + 2I_y = (CA^2/3g_a) \sum_{\phi} (2\phi + 1). \quad (7)$$

Here and in what follows  $M$  and  $f$  are magnetic and total spin quantum numbers for sub-levels of the normal state (here  $3^2S_{1/2}$ ),  $\mu$  and  $\phi$  corresponding quantities for the excited state ( $2P_{3/2}$ ). We reproduce these equations for the purpose of calling the reader's attention to the following points; Eq. (7) which is also valid when the applied field is not zero merely states that the total amount of energy absorbed does not depend on the degeneracy but merely upon the total weight of the system of nondegenerate levels into which a suitable perturbation will resolve the upper multiplet. Eq. (6) exhibits the intensity of the radiation polarized along  $x$  as the sum of two parts, a term  $\sum_{\phi\mu} (C/g_a)(A_{\pi\phi\mu})^2$  which is just what we would have obtained by the usual method of calculating populations of and emissions from the levels of the upper states, and which would tell the whole story if there were no degeneracy due breadth of levels; and another term, the balance of Eq. (6), which we will call  $\delta$ . The term  $\delta$  represents the change in intensity brought about by interference between the states which enter as linear combinations because of the degeneracy. It may be noted that we may write

$$I_x = I_x' + \delta; \quad I_y = I_y' - \delta/2,$$

where  $I_x'$  and  $I_y'$  are the values of  $I_x$  and  $I_y$  obtained when the breadth of levels is negligible. Let us consider the term  $\delta$  more closely. The summation

$$\sum_{f, f'} Sp[(e\dot{x})^{\phi f} (e\dot{x})^{f\phi'} (e\dot{x})^{\phi' f'} (e\dot{x})^{f' \phi}]$$

may, as Breit shows, be written

$$\sum_{\mu} \left| \sum_f (e\dot{z})_{\mu\mu}^{\phi f} (e\dot{z})_{\mu\mu}^{f\phi'} \right|^2;$$

that is, it involves the product of the probability amplitudes associated with transitions from two given upper levels  $\phi, \mu; \phi', \mu$  to *all common* lower levels  $f, m_f = \mu$ . That this term represents an interference effect, that is to say, that the relative phases of the probability amplitudes are of importance, is to be seen in the fact that the summation over the various possible lower levels is carried out before squaring. In the denominator of this term as it occurs in  $\delta$  is the expres-

sion  $1 + (2\pi\tau\nu_{\phi\phi'})^2$  characteristic of such interference effects and involving the frequency difference of the interfering probability amplitudes. In zero applied field this frequency difference is the same for all terms involved in the further summation over  $\mu$ . As this summation does not represent an interference (we are dealing now with the sum of squared terms) we may anticipate that we have merely to write for the more general case

$$\delta = \sum_{\mu, \phi > \phi'} \frac{|\sum_f (e\dot{z})^{\phi f} (e\dot{z})^{f\phi'}|^2}{[1 + (2\pi\tau\nu_{\mu\mu}^{\phi\phi'})^2]}.$$

In the solution of the problem of the Paschen-Back effect of the hyperfine multiplet we have calculated the probability amplitudes which enter into this expression and the energy values whose differences divided by  $h$  are the  $\nu_{\mu\mu}^{\phi\phi'}$ , so the calculation of  $\delta$  is merely a matter of numerical computation. A useful numerical check is afforded by the observation that if  $\tau$  is small enough so that  $(2\pi\tau\nu_{\mu\mu}^{\phi\phi'})^2$  may be neglected in comparison to unity we must get the same polarization as we would with zero coupling to the nuclear moment, in the present case 0.60. The polarization for the case of unpolarized incident light and field parallel to the light beam may be obtained from the above equations by use of the relation  $P_{\perp} = P_{||}/(2 - P_{||})$ , where  $P_{||}$  and  $P_{\perp}$  are the polarizations for polarized and unpolarized incident light, respectively.

In the quantity  $\delta$  there are two constants at our disposal,  $A$ , which enters through  $\Omega$  in the probability amplitudes and in the frequency differences, and  $\tau$ . We have seen that  $A \sim 2 \times 10^{-4} \text{ cm}^{-1}$ . Using this value of  $A$  as a first approximation we may give  $\tau$  a value such that the calculated polarization in zero field agrees with that observed (after correction for intensity distribution, if necessary). The entire curve may now be calculated with these  $A$  and  $\tau$  values, and if it shows systematic deviation from the observed curve the values of  $A$  and  $\tau$  may be changed in the sense required to secure better agreement. By this process we have arrived at  $A = 1.87 \times 10^{-4} \text{ cm}^{-1}$  with  $\tau = 3.13 \times 10^{-8} \text{ sec.}$  as giving the best fit. The upper heavy curve in Fig. 1 is drawn through points calculated with these  $A$  and  $\tau$  values.

## APPARATUS AND EXPERIMENTAL CONDITIONS

The interpretation of our results is based upon two principal assumptions regarding the experimental conditions. These are: first, that the vapor pressure of sodium in the resonance lamp is small enough so that errors due to secondary radiation, to depolarization by collision, and to change in the intensity distribution in the exciting beam as it passes through the resonance lamp are negligible; and second, that the vapor pressure in the source is so low that all effects due to reabsorption in the source are also negligible. We may test whether these conditions are met by progressively lowering the vapor pressure in resonance lamp and source. If we find that below a certain vapor pressure the polarization does not change over a considerable further range of reduction we may suppose that these sources of error have been eliminated. Our measurements of polarization were made with a resonance bulb temperature of 170°C which corresponds to a vapor pressure of sodium of  $2.36 \times 10^{-5}$  mm of mercury. With this vapor pressure it was possible to obtain a photograph of the resonance beam of sufficient intensity for measuring purposes with an exposure time of about 10 minutes. Lowering the temperature to 157°C at which the vapor pressure of sodium is  $1 \times 10^{-5}$  mm increased the necessary exposure time to about 30 minutes but did not cause any change in the polarization.

A photograph of the resonance beam with the slit between the bulb and camera removed showed a well-defined image of the beam with no blackening beyond the confines of the incident beam. This is evidence that the vapor pressure used was low enough that no secondary resonance radiation was present. Further evidence is derived from the observation that the image of the resonance beam was of nearly the same intensity at points where the exciting light entered and left the resonance bulb, showing that the intensity of the exciting light had not changed appreciably in passing through the absorbing sodium vapor in the bulb.

The source was a discharge tube of the type first used for resonance radiation work by one of the authors. Hydrogen is admitted through a capillary leak so adjusted that with continuous

pumping of the discharge tube with a Cenco Hyvac pump the hydrogen pressure is just high enough to eliminate overheating of the tube by positive ion bombardment. The side tube containing a small amount of sodium is heated by an electric heater so wound that the metallic sodium is in the coldest part of the tube. The temperature of the side tube was 270°C which for equilibrium conditions would correspond to a vapor pressure of sodium of  $4 \times 10^{-3}$  mm of mercury. However, the sodium vapor pressure in the discharge is certainly less than this since diffusion to cooler parts of the discharge tube, convection by the steady current of hydrogen flowing through the discharge and reaction with the quartz walls all remove sodium from the discharge. If the heater on the side tube is turned off the yellow color disappears from the source almost as soon as the side tube has become cool. The reaction of sodium with the quartz walls ultimately produces a slight yellow discoloration and reduces the ultraviolet transmission to such an extent that a new quartz section must be inserted after about ten hours of operation.

The temperature of the side tube could be lowered to 255°C corresponding to a decrease in pressure of fifty percent without changing the polarization and also raised to a temperature of at least 275°C without any effect. It is therefore safe to assume that the vapor pressure used in the discharge tube was below that at which appreciable self-reversal takes place.

## NUCLEAR MOMENT OF SODIUM

The calculation of nuclear magnetic moments from hyperfine structure  $A$  values has been discussed by Goudsmit<sup>8</sup> and by Fermi and Segré.<sup>9</sup> From Goudsmit's Eq. (6)

$$g(I) = \frac{aZ_i j(j+1)(l+\frac{1}{2})}{\Delta \nu l(l+1)} \times \frac{\lambda(lZ_i)}{k(jZ_i)}$$

We can calculate  $g(I)$  provided we know  $Z_i$ . The best that we can do at present is to use the value of  $Z_i$  obtained from the ordinary spin doublet formulas and the results of such calculations are presented in Table I. (The  $A$  values

<sup>8</sup> S. Goudsmit, Phys. Rev. **43**, 636 (1933).

<sup>9</sup> E. Fermi and E. Segré, Zeits. f. Physik **82**, 729 (1933).

TABLE I. Hyperfine structure splitting constants ( $A$ ) with corresponding nuclear magnetic moments and  $g$  factors together with values of effective nuclear charge used in computing them.

	$3^2S_{1/2}$	$3^2P_{3/2}$	$4^2P_{3/2}$
$A$	0.029	$6.6-7.5 \times 10^{-4}$	$1.87 \times 10^{-4}$
$g(I)$	1.35	1.5-1.7	1.33
$\mu(I)$	2.02	2.25-2.6	1.99
$Z_i$	11	7.66	7.55

for  $3^2S_{1/2}$  and  $3^2P_{3/2}$  are taken from Granath and Van Atta<sup>2</sup> and the preceding paper of Larrick.) As both Goudsmit and Fermi point out this  $Z_i$  value is not quite correct, since the hyperfine structure constant depends essentially on  $1/r^3$  while the spin doublet separation depends upon  $(1/r)(dV/dr)$ . The values of the nuclear moment calculated from the splittings of  $3^2S_{1/2}$  and  $4^2P_{3/2}$  are in good agreement, but differ by almost thirty percent from that based on the  $3^2P_{3/2}$  separation. This is no doubt due to some extent to the  $Z_i$  values used, and also to errors in polarization measurements. In the determination of  $A$  values from polarization *vs.* field strength curves the greatest weight should be attached to those points lying on the steeply sloping part of the curve, as it is here that a small error in the measurement of polarization will least affect the result. In the present case these are the points lying in the region 8-30 gauss. Of five points in this region only one is badly off the curve, that at 19.38 gauss, which corresponds to an  $A$  value of  $2.05 \times 10^{-4} \text{ cm}^{-1}$ , about ten percent

higher than that which we have chosen as giving the best fit. The results probably warrant writing  $A(4^2P_{3/2}) = 1.87 \pm 0.05 \times 10^{-4} \text{ cm}^{-1}$ .

The value of  $A(3^2P_{3/2})$  is less certain, the points on Larrick's polarization field strength curve (see the preceding paper) scatter so that the  $A$  value is uncertain by about 10 percent. The present writers have measured one point in the critical region of this curve photographically and find 31.2 percent polarization in a field of 39.2 gauss applied parallel to the electric vector of plane polarized exciting radiation. To these values corresponds  $A(3^2P_{3/2}) = 6.6 \times 10^{-4} \text{ cm}^{-1}$ . Larrick's observed values at 19.4 and 48.5 gauss will lie very close to a curve drawn for this value of  $A(3^2P_{3/2})$ , and we are inclined to suppose that a curve run by the more accurate photographic method will give an  $A$  value somewhat lower than he has obtained, but not very much lower than  $6.6 \times 10^{-4} \text{ cm}^{-1}$ .

*Note added in proof.* Regarding the computation of  $g(I)$  values of the  $^2P$  states (Table I supra), Breit writes that the value of  $Z_i$  which must be used is 8.9, or practically 9, but that the use of a  $Z_i$  has a sense only insofar as the central field picture applies. The value 8.9 is the ratio  $(Z_{i\Delta})^2/Z_{iS}$  where  $Z_{i\Delta}$  is the  $Z_i$  of the Landé doublet splitting formula and the  $Z_{iS}$  is the  $Z_i$  in Goudsmit's formula for the hyperfine splitting in terms of  $g(I)$ . He remarks that it is possible that this picture is sufficiently inaccurate to make the difference between  $Z_i=7$  and  $Z_i=9$  meaningless.