$$\frac{1}{f(P)}=\left(\frac{1-P(H,\,p)/P_0}{P(H,\,p)/P_0}\right)^{-\frac{1}{2}}$$

be plotted against the pressure p, a straight line should result whose intercept is $mc/eHg\tau$ and whose slope, $(A\sigma_q^2/g)(mc/eH)$, where σ_q^2 is the quenching cross section and

$$A=2666.6\left(\frac{2\pi N}{kT}\cdot\frac{m_1+m_2}{m_1m_2}\right)^{\frac{1}{2}}.$$

If the values of $1/f(P)$ for the experiments on the line 4358 taken from Table II are plotted against the corresponding nitrogen pressures a straight line results which gives a value of $\tau=0.88\times10^{-8}$ sec. and of $\sigma_q^2=1.3\times10^{-16}$ cm^2. This result must be taken as only an approximation to σ_q^2 and τ, since in the theoretical treatment hyperfine structure was not considered. The experiments were performed, however, at such a magnetic field ($H=3$ gauss) that the value of $P(H)/P_0$, calculated on the basis of hyperfine structure, is not essentially different from that calculated neglecting hyperfine structure.

Finally, it must be emphasized that, in the experiments in which the mean life of the 7^3S_1 state was measured by observations on the three fluorescent lines, $\tau T/(\tau+T)$ was measured and not τ. Since, however, the nitrogen pressure was kept constant at 3 mm in these experiments, a short calculation shows that the values of the true mean life as measured by the three lines should have the same relation to each other as do the values given in Table I.

In conclusion, the authors wish to acknowledge their indebtedness to Dr. R. L. Garman, of the Chemistry Department of the Washington Square College who kindly made the intensity measurements with the microphotometer.

A Calculation of Mass Scattering Coefficients

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By means of graphical integration, the mass scattering coefficients of the substances carbon, aluminum, iron, tin and gold have been calculated over the wave-length range $\lambda=0.4$A to 1.1A. The results show: (1) the mass coefficients obtained agree reasonably well with those obtained experimentally by Coade, Mertz, Hewlett, Statz and Allen; (2) the mass scattering coefficients may be expressed by the relation, $\sigma/\rho=0.0230Z^{1.16}\lambda$, over a wave-length range $\lambda=0.4$A to 1.1A and upwards from atomic number 12; (3) the part of the scattering coefficient due to incoherent scattering averages 20 percent lower than that value expressed by the Dirac or the Klein-Nishina formula.

INTRODUCTION

EXPERIMENTAL values of the mass scattering coefficients of x-rays have been determined for a number of elements directly by Coade,[1] Mertz,[2] Statz[3] and Hewlett[4] and indirectly by Allen.[5] As yet no satisfactory theoretical calculations have been made that agree with the experimental values. It was the purpose of this study to calculate the value of these coefficients from the existing accurate knowledge of the intensity of x-rays, as a function of the scattering angle, scattered by gases and powdered crystals.

METHOD

The intensity of x-rays scattered by a single electron is given by the classical theory[6] as

$$I_{sc}=I_0(e^4/2m^2c^4R^2)(1+\cos^2\theta),$$

[1] E. H. Coade, Phys. Rev. **36**, 1109 (1930).
[2] P. Mertz, Phys. Rev. **28**, 891 (1926).
[3] Statz, Zeits. f. Physik **11**, 304 (1922).
[4] Hewlett, Phys. Rev. **20**, 688 (1922).
[5] Allen, Bull. Am. Phys. Soc. **8**, Nos. 5, 6.

[6] J. J. Thomson, *Conduction of Electricity Through Gases*, 3rd Ed., p. 321.

where R is the distance from the scattering electron to the receiver, e^4/m^2c^4 is 7.85×10^{-26}, I_0 is the intensity of the original x-ray beam and $\frac{1}{2}(1+\cos^2\theta)$ is the Thomson polarization factor.

According to the quantum theory this scattered radiation consists of two parts;[7] one part in which the wave-length of the x-rays scattered is changed, the modified or incoherent radiation, and one part in which the wave-length is the same, the coherent or unmodified radiation.

$$I_{sc}=I_{\text{coh}}+I_{\text{incoh}},$$

where

$$I_{\text{coh}}=I_0[e^4(1+\cos^2\theta)F^2/m^2c^42R^2], \qquad (1)$$

in which F is the atomic form factor[8, 9, 10, 11] which expresses the electron density in the atom, and

$$I_{\text{incoh}}=I_0[e^4(1+\cos^2\theta)/2m^2c^4R^2]\{(z-\sum_1^z E_r^2)/(1-\gamma(1-\cos\theta))^3\} \qquad (2)$$

as given by Compton,[12] where E_r is the structure factor of the electron in its proper shell in the normal atom as given by Pauling and Sherman[10] and $\gamma=h\nu/mc^2$ or by

$$I_{\text{incoh}}=I_0(e^4(1+\cos^2\theta)/2m^2c^4R^2)zG(v) \qquad (3)$$

as given by Heisenberg[13] in which $G(v)$ is a function of the wave-length of the coherent radiation and the atomic number of the scattering substance.

The total scattered radiation for z electrons in an atom, as a function of the scattering angle is given by

$$I_{sc}=I_0[e^4(1+\cos^2\theta)/2R^2m^2c^4]\{F^2+[z-\sum_1^z E_r^2]/(1+\gamma(1-\cos\theta))^3\} \qquad (4)$$

or by

$$I_{sc}=I_0[e^4(1+\cos^2\theta)/2R^2m^2c^4]\{F^2+zG(v)\}, \qquad (5)$$

depending on whether the incoherent radiation is expressed by the Compton or by the Heisenberg relation.

The Eqs. (4) and (5), of course, assume that the scattering substance is in a gaseous state, and that it is monatomic. At first sight one feels that the above relations for the intensity of the scattered radiation may not be applied to explain the results of x-rays scattered from solids such as experimentally measured by Coade, Mertz and Hewlett. However, A. W. Coven[14] and G. E. M. Jauncey and F. Pennell[15] have shown that, in the case of powdered crystals, the intensity of the scattered radiation is very well expressed by the Eq. (4) over a range of scattering angles from 10° to 180°. Below 10° this relation alone does not express the true intensity of the scattered x-rays. In this range of λ there is little error made in assuming that Eq. (4) does represent the scattered radiation since in the calculation of the scattering coefficient the scattered intensity is multiplied by the sine of the scattering angle and the contribution of this range is very small. Jauncey and Pennell[15] also found the electronic structure factor F to be the same in the powdered crystal as in the gas.

Jauncey and Harvey[16] on the other hand have shown in the case of radiation scattered from single crystals, that the scattering angle, at which the intensity of the radiation scattered from the single crystal differs from that scattered from a gas, may be as high as 40°. Therefore, in the case of single crystals, the Eq. (4) does not express the intensity of the scattered radiation to any degree of accuracy. From the description of the scattering substances employed by Coade, Mertz and Hewlett, it is probable that the scattering substances were not

[7] A. H. Compton, Phys. Rev. **35**, 925 (1930).
[8] James and Brindly, Phil. Mag. **12**, 81 (1931).
[9] L. H. Thomas, Proc. Camb. Phil. Soc. **23**, 542 (1927).
[10] Pauling and Sherman, Zeits. f. Krist. **81**, 1 (1932).
[11] D. R. Hartree, Proc. Camb. Phil. Soc. **24**, 89 (1928).
[12] A. H. Compton, Phys. Rev. **21**, 483 (1923).
[13] W. Heisenberg, Phys. Zeits. **32**, 737 (1931).
[14] A. W. Coven, Phys. Rev. **41**, 422 (1932).
[15] G. E. M. Jauncey and F. Pennell, Phys. Rev. **43**, 505 (1933).
[16] G. E. M. Jauncey and G. G. Harvey, Phys Rev. **38**, 1071 (1931).

single crystals and so far as scattering is concerned they could best be described as powdered crystals.

The mass scattering coefficient, σ/ρ, defined as the fraction scattered per unit mass of a substance per second, is given by the expression,[17]

$$\sigma/\rho=\int_0^\pi I_{sc}2\pi R\sin\theta\cdot Rd\theta/I_0,$$

which leads to

$$\sigma/\rho=(\pi Ne^4/m^2c^4)\int_0^\pi(1+\cos^2\theta)\sin\theta\{F^2+[z-\sum_1^z E_r^2]/(1+\gamma(1-\cos\theta))^3\}d\theta \tag{6}$$

for the case of Compton incoherent scattering, where N is the number of scattering atoms per unit mass, or

$$\sigma/\rho=(\pi Ne^4/m^2c^4)\int_0^\pi(1+\cos^2\theta)\sin\theta\{F^2+zG(v)\}d\theta \tag{7}$$

for the case of Heisenberg incoherent scattering. Eqs. (6) and (7) do not lend themselves to an analytic integration (on account of the method of expression of the structure factor) and it is necessary to resort to graphical methods. The coherent and the incoherent parts of the integrals (6) and (7) were plotted separately in order to determine the amount each contributes to the scattering coefficient. Typical curves are shown in Figs. 1, 2 and 3. In these curves, the areas under the curves are direct measures of the scattering coefficients.

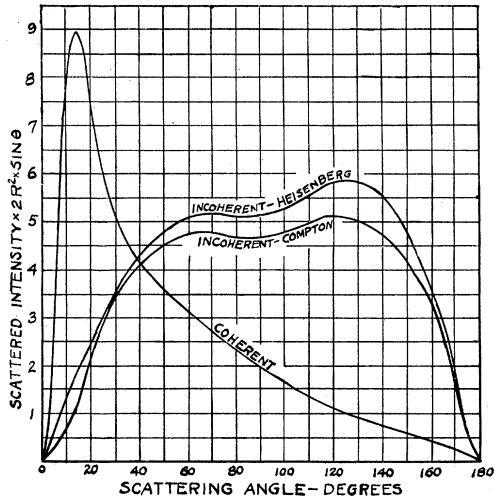

FIG. 1. The separate parts of integrals (6) and (7) *vs.* the scattering angle for carbon at a wave-length of 0.7A. The integral is evaluated in terms of $\pi e^2/m^2c^4$ as a unit.

This calculation of the mass scattering coefficient of course includes only the energy of the incident x-ray beam scattered in the form of radiation. Actually, the measured mass scattering coefficient may also include the energy lost in

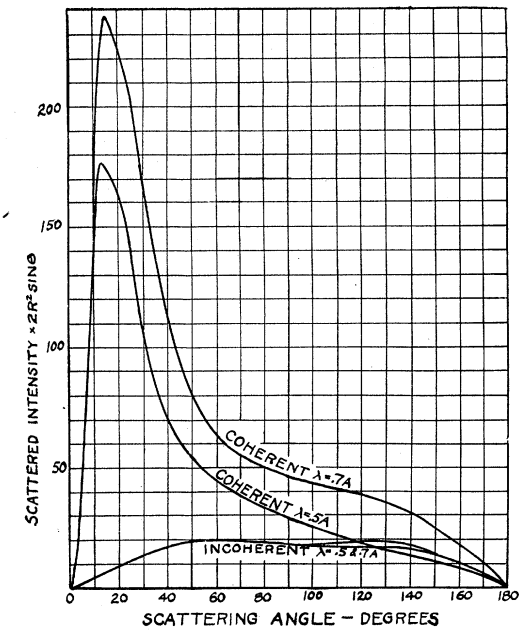

FIG. 2. The separate parts of integral (6) *vs.* the scattering angle for iron at a wave-length of 0.5 and 0.7A. The integral is given in terms of $\pi e^4/m^2c^4$.

[17] A. H. Compton, *X-rays and Electrons*, p. 60, 1927.

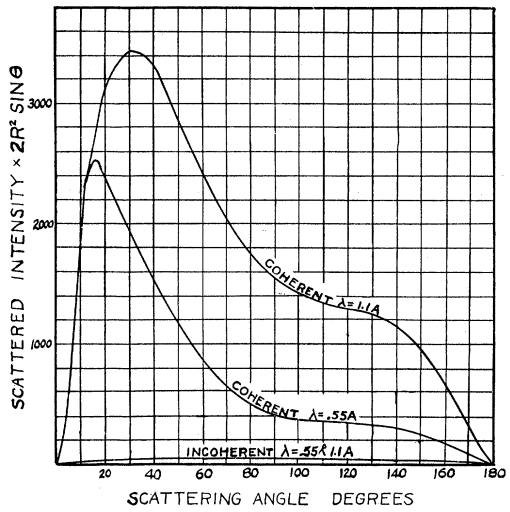

FIG. 3. The separate parts of the integral (6) *vs.* the scattering angle for gold at a wave-length of 0.55 and 1.1A. The integral is given in terms of $\pi e^4/m^2c^4$ as a unit.

the recoil electron. W. R. Harper[18] has shown that the ratio of the energy of the recoil electron to the total energy removed from the incident x-ray beam is about five percent in the case of hydrogen at $\lambda = 0.46$A. Since the present calculations are not accurate to more than three percent there is no serious error in calling these calculated coefficients the total mass scattering coefficients.

RESULTS

The mass scattering coefficients obtained from measurements of the areas under the curves show that, for elements of low atomic number (see Fig. 1), the contribution of the incoherent radiation is comparable with the coherent radiation. For elements of atomic number greater than 50 (see Figs. 2 and 3) the incoherent radiation contributes little to the scattering coefficient and may be neglected for the most part.

It is also obvious from the curves (see Figs. 2 and 3) that the contribution of the coherent radiation to the scattering coefficient increases as the wave-length increases while the part due to the incoherent scattering remains almost constant over the range of wave-lengths investigated. The curves also show as well as the integrated results that the part of the scattering coefficient represented by the Compton relation and that represented by the Heisenberg relation are practically the same.

Table I gives the results of the scattering coefficients in comparison with the experimental values of various observers for the elements, carbon, aluminum, iron, tin and gold, over a wave-length range from $\lambda = 0.42$A to $\lambda = 1.1$A. The agreement between the calculated and observed values is quite good for carbon and aluminum. For iron, tin and gold the agreement is not as good but is probably as good as that expected in the face of the possibility that the solid scattering substance in the experimental work may have consisted of some aggregates of rather large single crystals. It should also be remembered that scattering coefficients are rather difficult to measure experimentally and that disagreements of as much as 30 percent are not uncommon between various observers. The agreement of the calculated value and observed value in the case of carbon at $\lambda = 0.7$A (probably the most accepted value) is quite good.

VARIATION OF THE MASS SCATTERING COEFFICIENT WITH THE WAVE-LENGTH AND ATOMIC NUMBER

When the values of the mass scattering coefficients of the substances carbon, aluminum, iron, tin and gold are plotted against the wave-length (over the range 0.4 to 1.1A, Fig. 4) it is found that a linear relation fits well. Below 0.4A it is probable that the relation would not be linear since for this region the incoherent scattering is the more important and this is not linear as shown by the Dirac expression. Above

TABLE I. *The computed mass scattering coefficients compared with those observed by Coade, Hewlett and Allen.*

Substance	Wave-Length 0.42A	0.5A	0.6A	0.7A	1.0A	1.1A
Carbon				0.21 0.21(4)	0.244 0.28(1)	
Aluminum	0.21 0.16(5)	0.25 0.17(1)		0.35 0.21(1)		0.56 0.39(1)
Iron	0.42 0.28(5)	0.48 0.25(1)	0.59 0.36(1)	0.66 0.45(1)		1.03
Tin	0.86 1.40(5)	0.95 0.40(1)		1.39 0.78(1)		2.27
Gold	1.56	1.81 0.93(1)	1.18 0.60(1)			4.00

[18] W. R. Harper, Proc. Roy. Soc. **A141**, 686 (1933).

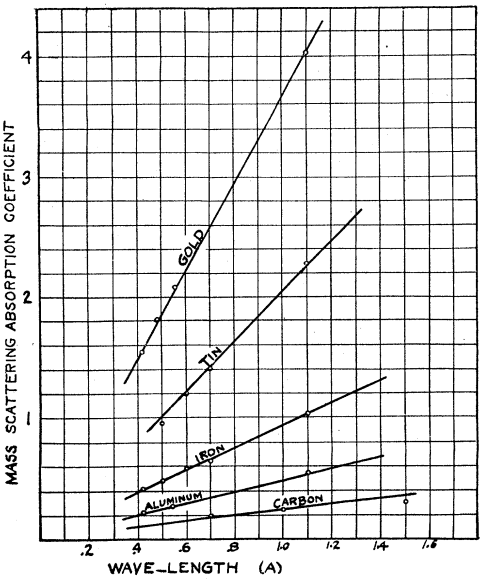

FIG. 4. Relation of the mass scattering coefficient with the wave-length.

1.1A the scattering is mainly coherent and in this range the coherent scattering increases more rapidly than is expressed by the linear relation.

Plotting the logarithms of the scattering coefficients of the various elements against the logarithms of the atomic numbers (Fig. 5) gives a straight line (at $\lambda=1$A) upon which the coefficients of all the elements fall, with the exception of carbon. The slope of this log-log plot is 1.16. It is obvious, then, that over a wave-length range from 0.4A to 1.1A and upward from atomic number 12, the scattering coefficient may be represented as $\sigma/\rho=AZ^{1.16}\lambda$, and A has the value 0.0230 when σ/ρ is the scattering coefficient per unit mass.

COMPARISON OF THE INCOHERENT PART OF THE MASS SCATTERING COEFFICIENT WITH THAT PREDICTED BY THE DIRAC FORMULA

The mass scattering coefficient of the incoherent part of the scattered radiation may be

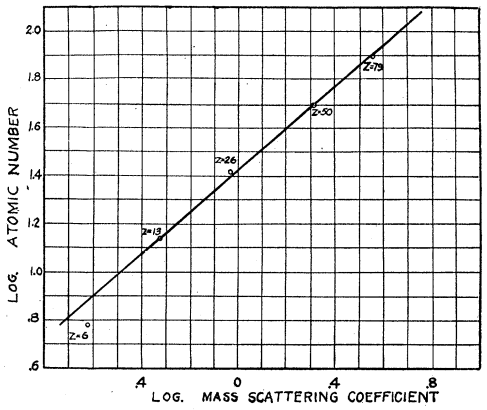

FIG. 5. Log-log plot of the mass scattering coefficient against the atomic number at a wave-length of 1.0A.

obtained by integrating under the incoherent curve only. The integration under the incoherent curve given by the Compton expression for the incoherent scattering for the various elements at a wave-length of 0.5A gives rise to the value of the coefficients in Table II. These values are in terms of the scattering per electron. The average value for the five elements under consideration is 454×10^{-27} against a value of 585×10^{-27} as given by the Dirac[19] expression or 587×10^{-27} as given by the Klein-Nishina[20] expression.

TABLE II. *Incoherent scattering per electron at $\lambda=0.5A$ as given by the integrated Compton expression, the Dirac formula and the Klein-Nishina formula.*

	Compton	Dirac	Klein-Nishina
Carbon	545×10^{-27}	585×10^{-27}	587×10^{-27}
Aluminum	510×10^{-27}		
Iron	400×10^{-27}		
Tin	437×10^{-27}		
Gold	380×10^{-27}		
	av. $=454\times10^{-27}$		

[19] P. A. M. Dirac, Proc. Roy. Soc. **A111**, 405 (1926).
[20] O. Klein and Y. Nishina, Zeits. f. Physik **52**, 853 (1929).